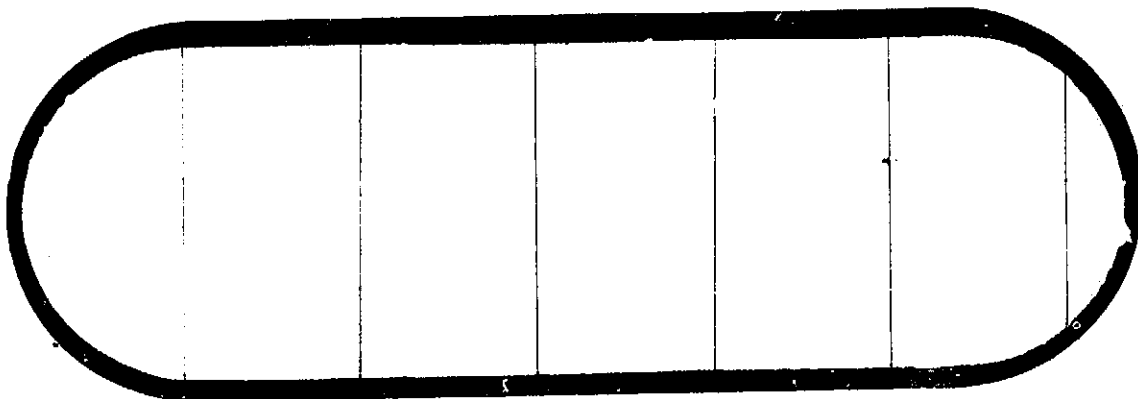


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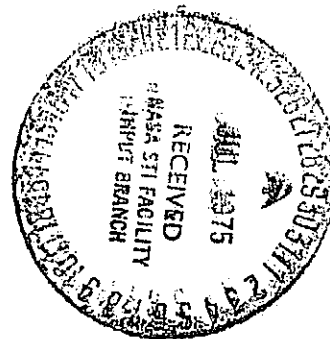
# ***BOEING***



(NASA-CR-143891) BOPACE 3-D (THE BOEING  
PLASTIC ANALYSIS CAPABILITY FOR  
3-DIMENSIONAL SOLIDS USING ISOPARAMETRIC  
FINITE ELEMENTS) Final Report (Boeing  
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## **BOPACE 3-D**

**(THE BOEING PLASTIC ANALYSIS CAPABILITY  
FOR 3-DIMENSIONAL SOLIDS USING  
ISOPARAMETRIC FINITE ELEMENTS)**

**FINAL REPORT  
CONTRACT NAS8-30615**

**April 15, 1975**

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## 1.0 INTRODUCTION

BOPACE 3-D is the acronym for the Boeing Plastic Analysis Capability for 3-dimensional solids. It is an extension of the original BOPACE 2-D version [1]. BOPACE 3-D is a finite element computer program, which provides a very general family of 3-dimensional isoparametric solid elements, and includes a new algorithm for improving the efficiency of the elastic-plastic-creep solution procedure. This document describes the BOPACE 3-D program, and includes theoretical, user and programmer oriented sections.

The BOPACE programs have been developed by The Boeing Company for the NASA Marshall Space Flight Center, in order to meet the evident need for an advanced thermal-elastic-plastic-creep structural analyzer. Although BOPACE development has been strongly influenced by the requirements for analysis of engines, in particular the space shuttle main engine, the programs have been kept quite general and they are applicable to many types of nonlinear structures. The philosophy for BOPACE program development has been based on the following requirements.

- 1) Analysis of very high temperature and large plastic-creep effects.
- 2) Treatment of cyclic thermal and mechanical loads.
- 3) Improved material constitutive theory which closely follows actual behavior under variable temperature conditions.

- 4) A stable numerical solution approach which avoids cumulative errors.
- 5) Capability for handling up to 3000 degrees of freedom.

Although the finite-element method was first applied to plasticity in the early 1960's, and several good programs for nonlinear analysis have since been developed, numerous improvements were indicated in order to satisfy the above requirements. For example, some other available programs assumed linear or bilinear plastic hardening, accumulated errors by failing to satisfy equilibrium at each step, or did not completely account for the effects of variable temperature on the elastic, plastic and creep relations. The stated requirements have been effectively met by the current BOPACE program versions. The BOPACE research and development efforts have led to an improved hardening theory for cyclic plasticity, a method for representing general cases of load reversal, and advanced techniques for improving the accuracy and controlling convergence of highly nonlinear solutions.

Two options have been provided in the BOPACE 3-D program: 1) The use of 64K words of core to solve up to 1500-DOF problems, and 2) The use of 128K words of core to solve up to 3000-DOF problems. The program is written in FORTRAN IV and is available on both the IBM 360/370 and the UNIVAC 1108 machines. The BOPACE 3-D document consists of three major parts:

- Part I. Theoretical Manual
- Part II. User Manual
- Part III. Programmer Manual



Recognition is due to D. L. Beste and M. W. Ice for their BOPACE 3-D programming efforts, to W. H. Armstrong for some of the initial 3-D element development, and to S. Wahlstrom for providing his isoparametric elastic stiffness generation routines. The Gauss linear equation solver package was originally developed by J. L. Ballinger.

THE **BOEING** COMPANY

BOPACE 3-D

PART I. THEORETICAL MANUAL

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## 2.0 MATERIAL CONSTITUTIVE THEORY

The basic purpose of classical constitutive theory in an elasto-viscoplastic program such as BOPACE is to provide incremental relations between stresses and strains. BOPACE uses these relations with the finite-element stiffness method to provide a convenient and efficient approach for solution of an important class of nonlinear problems.

BOPACE 3-D employs isoparametric solid elements, with numerical integration over a number of material points within each element. It accounts for elastic, plastic, thermal and creep deformations, and the nonlinear dependence of all deformations on temperature. The material constitutive theory includes a combined isotropic/kinematic plastic hardening theory, and a generalized approach to cyclic load reversal. The BOPACE constitutive theory is developed by a tensorial approach which provides all relations in a form which is invariant under coordinate transformations.

### 2.1 ELASTICITY EQUATIONS

This section defines the cumulative and incremental forms of the relations for temperature-dependent elasticity, which are used in BOPACE 3-D.

General Concepts and 3-D Relations - The basic cumulative stress-strain relation, for either temperature-dependent or temperature-independent elasticity, is

$$\sigma_{ij} = D_{ijkl}^e \epsilon_{kl}^e \quad (2.1-1)$$

where  $\sigma$  and  $\epsilon^e$  are the  $3 \times 3$  tensors of stress and elastic (recoverable) strain, respectively, and  $D^e$  is the tensor of elastic coefficients which may depend on temperature. For convenience we will use the equivalent single-subscript notation

$$\sigma_i = D_{ij}^e \epsilon_j^e \quad (2.1-2)$$

where subscripts  $i$  and  $j$  range over all nine of the tensor components.

For 3-dimensional analysis the relation 2.1-2 is taken as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \\ \sigma_{yx} \\ \sigma_{zx} \\ \sigma_{zy} \end{Bmatrix} = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}^e \\ \epsilon_{yy}^e \\ \epsilon_{zz}^e \\ \epsilon_{xy}^e \\ \epsilon_{xz}^e \\ \epsilon_{yz}^e \\ \epsilon_{yx}^e \\ \epsilon_{zx}^e \\ \epsilon_{zy}^e \end{Bmatrix} \quad (2.1-3)$$

where

$$D_{11} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix}$$

and

$$D_{22} = D_{33} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-2\nu & 0 & 0 \\ 0 & 1-2\nu & 0 \\ 0 & 0 & 1-2\nu \end{bmatrix}$$

Here  $E$  is Young's modulus and  $\nu$  is Poisson's ratio.

Note that the elasticity matrix in Equation 2.1-3 is consistent with the tensorial definition of shear strains (e.g.  $\epsilon_{xy}^e = \gamma_{xy}^e/2$ , where  $\gamma_{xy}$  is the engineering definition of shear strain). Tensorial definitions are used in the BOPACE program in order to easily formulate constitutive theories which are invariant with respect to coordinate transformations. Although such invariance can be achieved by a careful use of engineering strain definitions, such definitions are probably responsible for some of the invariance difficulties which have been reported in the plasticity literature.

The last three of Equations 2.1-3 are somewhat redundant and may be discarded to give an abbreviated 6-component form (e.g.  $\sigma_{yx} = \sigma_{xy}$  and  $\epsilon_{yx} = \epsilon_{xy}$ ). It should be emphasized, however, that in performing later summations all non-zero values of the nine tensor components must be accounted for.

Incremental Relations - For the case of temperature-independent elasticity the incremental stress-strain relations are simply

$$\Delta\sigma_i = D_{ij}^e \Delta\epsilon_j^e \quad (2.1-4)$$

where  $\Delta$  denotes an incremental quantity and  $D^e$  is the appropriate elasticity matrix.

When temperature dependence is considered, the incremental relation may be written either as

$$\Delta\sigma_i = \sigma_i^1 - \sigma_i^0 = D_{ij}^{e1} \epsilon_j^{e1} - D_{ij}^{e0} \epsilon_j^{e0} \quad (2.1-5a)$$

or

$$\Delta\sigma_i = \Delta D_{ij}^e \epsilon_j^{e0} + D_{ij}^{e1} \Delta\epsilon_j^e \quad (2.1-5b)$$

where the superscripts 0 and 1 denote quantities evaluated at the beginning and end of the increment, respectively, and  $\Delta D^e = D^{e1} - D^{e0}$  is the change in elasticity matrix from beginning to end of the increment. The first term in Equation 2.1-5b accounts for stress change due only to

change in elastic properties, while the second term accounts for additional stress change due to the increment of elastic strain.

## 2.2 THERMAL STRAIN

Alternate Formulations - The conventional description of thermal strain is given by

$$\Delta \begin{Bmatrix} \epsilon_{xx}^t \\ \epsilon_{yy}^t \\ \epsilon_{zz}^t \end{Bmatrix} = \gamma \Delta T \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \quad (2.2-1)$$

where  $\epsilon^t$  denotes thermal strain,  $T$  is the temperature, and  $\gamma$  is the thermal coefficient of expansion which may be a function of temperature.

An alternate integrated description of thermal strain is

$$\begin{Bmatrix} \epsilon_{xx}^t \\ \epsilon_{yy}^t \\ \epsilon_{zz}^t \end{Bmatrix} = \epsilon^t(T) \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \quad (2.2-2)$$

where here  $\epsilon^t$  gives the thermal strain directly as a function of temperature. If only incremental thermal strains are of interest,  $\epsilon^t$  may be taken as zero at any convenient reference temperature.

BOPACE Formulation - BOPACE uses the direct form 2.2-2. This form is preferred over that involving a thermal expansion coefficient because accumulated errors in thermal strain are not introduced. These errors could arise with the form 2.2-1, in case  $\gamma$  varied with temperature and the specified heating and cooling sequences used different temperature increments. BOPACE takes the structural fabrication temperature as the reference temperature for zero thermal strains.

### 2.3 PLASTICITY

This section defines the incremental elasto-plastic relations used in the BOPACE program. (See also Section 2.6 for the elasto-plastic iterative algorithm.) BOPACE employs a new concept of combined isotropic and kinematic hardening, and accounts for temperature-dependent elasto-plastic behavior as well as a generalized form of cyclic load reversal. In order to develop the constitutive theory in a straightforward manner, discussion of the effects of temperature-dependent elasticity on the elasto-plastic relations is deferred until Section 2.5.

Definitions - The following nomenclature is defined.

- $\sigma$  = total stress
- $\alpha$  = stress center (of yield surface in kinematic hardening)
- $s$  = deviatoric (total - hydrostatic) stress



- $a$  = deviatoric stress center
- $\hat{s}$  =  $s - a$  = relative deviatoric stress
- $\epsilon^e$  = elastic (recoverable) strain
- $\epsilon^p$  = plastic (time-independent non stress-inducing) strain
- $\epsilon^c$  = creep (time-dependent non stress inducing) strain

General Concepts - The basic concepts in most elasto-plastic theories are those of a yield surface, the dependence of yield on only the deviatoric stress components, incompressibility under plastic strains, and normality of the incremental plastic-strain vector to the yield surface. The definition of a particular theory requires assumptions for three basic constituents:

1. a surface relating the stress components at yield
2. a flow rule defining a direction for the incremental plastic-strain vector.
3. a hardening rule.

Yield Surface - BOPACE employs the Huber-Mises yield surface [2], defined by the relative deviatoric stresses as

$$F = \hat{s}_i \hat{s}_i - \hat{s}_i^0 \hat{s}_i^0 = 0 \quad (2.3-1)$$

where the summation is again taken over all nine tensor components of  $s$ . The  $\hat{s}_i^0$  are components of a point on the yield surface at a known condition of temperature and plastic deformation, e.g., from a uniaxial test.

Equation 2.3-1 holds whenever the material is plastic, i.e. whenever the components of  $s$  are on the yield surface. Function  $F$  may be thought of geometrically as defining a hypersphere in 9-dimensional deviatoric stress space. Alternately, when expressed in the 3-D space of principal stresses, this yield surface can be shown to be an open-ended circular cylinder whose axis passes through the origin and makes equal angles with each of the three principal stress axes. The Huber-Mises yield surface is generally used to describe plasticity in metals because it agrees reasonably well with test results and it gives a smooth surface which is convenient for calculations.

Flow Rule - BOPACE uses the Prandtl-Reuss flow rule, which is the usual rule associated with the Huber-Mises yield surface. The assumptions are that the material is incompressible under plastic flow, and that increments of plastic strain are normal to the yield surface at the stress point.

These assumptions provide the relation

$$\Delta \epsilon_i^p = \lambda \hat{s}_i \quad (2.3-2)$$

where  $\lambda$  is a flow parameter (or plastic proportionality constant).

Basic Hardening Concepts - An elastic-plastic material which work hardens in the plastic range is commonly analyzed using either of two classical hardening theories. Isotropic hardening [3], which assumes a uniform expansion of the yield surface during plastic flow, accounts for change in size of the hysteresis loop during cycling. Kinematic hardening [4], which assumes a rigid translation of the yield surface in the direction of the plastic strain increment, accounts for the pronounced Bauschinger effect which is evident in cyclic behavior of most metals. In general, an actual cyclic behavior can be more accurately described by a combination of isotropic and kinematic hardening. A combined hardening theory has been given by Hodge [5] for materials which satisfy the Tresca yield condition. Because a better representation for most metals is provided by the Huber-Mises yield surface, a corresponding combined hardening theory [6] has been developed for the BOPACE program.

Hardening Parameters - A simple combined hardening theory such as that presented in Reference 6 makes two basic assumptions:

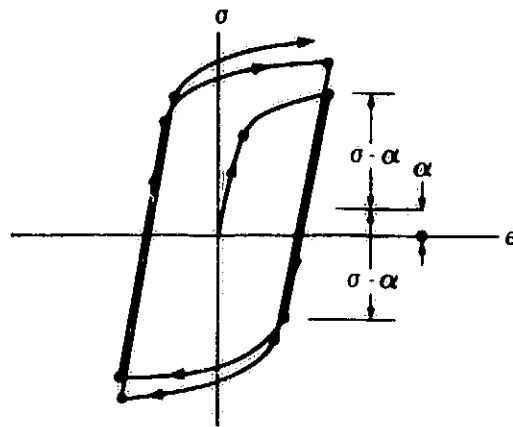
- 1) Size of the yield surface is a function of a cumulative hardening parameter,  $\kappa$ . This means that the isotropic hardening, i.e. the incremental change in size of the yield surface, depends on the initial value of  $\kappa$  and its change  $\Delta\kappa$ .
- 2) Yield surface translation is related (but only in an incremental manner) to a kinematic hardening parameter,  $\kappa^k$ . The kinematic hardening, i.e. the incremental translation of the yield surface, depends on the initial value of  $\kappa^k$  and its change  $\Delta\kappa^k$ .

For a simple uniaxial load case, the yield surface size at any time is measured by one half the algebraic difference between the current yield stresses in tension (positive) and compression (negative), while the cumulative kinematic hardening is measured by one half the algebraic sum of the yield stresses in tension and compression.

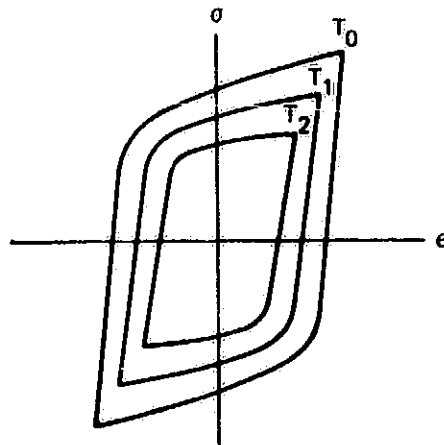
It will be evident in the discussion to follow that isotropic hardening can be related to  $\kappa$  on either a cumulative or incremental basis, while kinematic hardening can be related to  $\kappa^k$  only on an incremental basis. In addition to the parameters  $\kappa$  and  $\kappa^k$ , hardening is also a function of temperature.

Figure 2.3-1 shows hysteresis loops for the first two strain-controlled cycles of a typical material which exhibits combined isotropic and kinematic hardening. Here  $\sigma$  denotes yield stress and  $\alpha$  denotes yield stress center. The Bauschinger kinematic hardening effect is apparent in that the initial yielding in tension causes a reduced yield stress in compression, i.e. a shift of the yield center by an amount  $\alpha$ . Successive yielding in compression causes a reduced yield stress in tension, and so forth. Isotropic hardening causes the increase in size of the hysteresis loop with continued cycling. The hysteresis loops for many materials become stabilized after a number of cycles, and they may begin to decrease in size as further deformation causes a softening effect.

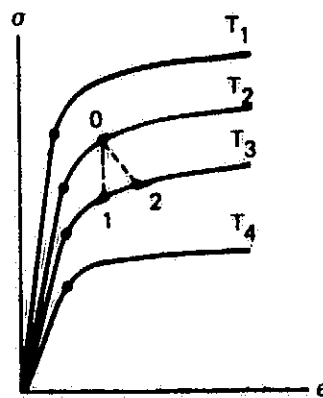
Figure 2.3-2 shows the stabilized hysteresis loops for a material at various temperature levels. (Different strain ranges are used to separate the loops for purpose of illustration.) The hysteresis loop of a material



**Figure 2.3-1. Combined Hardening Behavior (Non-stabilized)**



**Figure 2.3-2. Variable Temperature Hysteresis Loops (Stabilized)**



**Figure 2.3-3. Variable Temperature Hardening Effects**

typically decreases in size with increasing temperature. Note that the size of the yield surface will vary in a similar manner with temperature. Also the rates of isotropic and kinematic hardening with respect to plastic deformation vary with temperature.

The isotropic hardening parameter  $\kappa$  may be appropriately taken as either the cumulative plastic work density, or as the sum of increments of effective plastic strain. The kinematic hardening parameter  $\kappa^k$  must account for the Bauschinger effect in cyclic loading, and it may be taken as an adjusted value of  $\kappa$ . As long as no load reversal occurs and the loading is proportional,  $\kappa^k$  is simply equal to  $\kappa$ . However,  $\kappa^k$  must be set to zero at the start of each increment in which a complete load reversal occurs. (A complete load reversal occurs when the incremental plastic strain vector has a direction exactly reversed from that of the previous plastic increment). For an incomplete load reversal, the BOPACE program computes the starting value for  $\kappa^k$  by multiplying the existing accumulated value of  $\kappa^k$  by the factor  $(1 + \text{COSINE})/2$ , where COSINE is the Cosine of the angle between successive incremental plastic strain vectors. At the end of each increment,  $\kappa^k$  becomes  $\kappa^k + \Delta\kappa$ .

Because the Bauschinger effect varies with cumulative deformation in certain materials (e.g. it may become more pronounced as plastic cycling continues), BOPACE allows an additional option for the kinematic hardening to be defined as a product of two functions. The first is a function of  $\kappa^k$  and defines the shape of the kinematic hardening, while the second is an additional factor which depends on  $\kappa$  and defines the magnitude of the kinematic hardening.

In order to implement the BOPACE hardening theory, it must be determined how the size of the yield surface varies with temperature. In addition, the dependence of isotropic and kinematic hardening on the parameters  $\kappa$  and  $\kappa^k$  must be determined. This is accomplished by performing cyclic tests at several levels of constant temperature. After the cyclic hardening behavior is thus determined at different constant temperatures, an assumption must be made for variable temperature cycling. The hardening effects of variable temperature are illustrated in Figure 2.3-3. As long as temperature remains constant, plastic hardening behavior is defined by following the shape of a stress-strain curve at the given temperature, say to the point 0 on the  $T_2$  curve. If temperature changes to  $T_3$ , and then plastic deformation continues, an initial point must be determined on the  $T_3$  curve from which the new yield surface size and initial hardening slopes may be determined. This transfer from curve  $T_2$  to curve  $T_3$  requires a definition of the basis for hardening, i.e. the definition of the parameters  $\kappa$  and  $\kappa^k$ . BOPACE allows the option of either plastic work or the sum of increments of effective plastic strain to be used as the hardening basis. The strain and work options correspond to the respective points 1 and 2 in Figure 2.3-3.

The hardening relationship determined from a series of cyclic tests may depend somewhat on the strain range used in a particular test. If strain range is a significant factor the test conditions should duplicate the approximate expected strain range for which an analysis is to be made. The choice between plastic strain and plastic work as a basis for the hardening parameters  $\kappa$  and  $\kappa^k$  may depend to a large extent on

which basis provides the better overall representation of cyclic behavior at various strain ranges.

Multiaxial Hardening Rule - The kinematic hardening rule employed in BOPACE is that due to Prager [4]. It gives the increment of yield surface translation in terms of the incremental plastic strains, as

$$\Delta \alpha_i = C_{ij} \Delta \epsilon_j^p = \frac{2}{3} c I_{ij} \Delta \epsilon_j^p \quad (2.3-3)$$

where  $c$  is the kinematic contribution to the slope of the uniaxial stress vs. plastic-strain curve, and  $I$  is the identity matrix. An alternate hardening rule due to Ziegler [7] is preferred by some plasticity analysts because the form of Ziegler's rule does not change with reduction in the number of spatial dimensions, and it is therefore supposed to simplify the calculations. Prager's rule is considered more acceptable from a physical point of view, however, and it presents no difficulty when all components of the required tensors are retained as they are in the BOPACE programs. Note that for Prager's kinematic hardening rule, the deviatoric stress center is equal to the stress center, i.e.  $a_i = \alpha_i$ .

The isotropic hardening, i.e. change in size of the yield surface due to plastic deformation, is defined for a proportional test loading by

$$\hat{\Delta s}_1^0 = R_{ij}^0 \Delta \epsilon_j^p = \frac{2}{3} r I_{ij} \Delta \epsilon_j^p \quad (2.3-4)$$



where  $r$  is the isotropic contribution to the slope of the uniaxial stress vs. plastic-strain curve.

The necessary condition that stresses remain on the yield surface is satisfied by taking the differential of Equation 2.3-1. The condition is  $\Delta F = 0$ , which to a first order approximation can be shown to give

$$\hat{s}_i \Delta \sigma_i - \hat{s}_i \Delta \alpha_i - \hat{s}_i^0 \Delta \hat{s}_i^0 = 0 \quad (2.3-5)$$

or

$$\hat{s}_i \Delta \sigma_i - A \lambda = 0 \quad (2.3-6)$$

where

$$A = \frac{1}{\lambda} \hat{s}_i \Delta \alpha_i + \frac{1}{\lambda} \hat{s}_i^0 \Delta \hat{s}_i^0 = C_{ij} \hat{s}_i \hat{s}_j + R_{ij}^0 \hat{s}_i^0 \hat{s}_j^0 \quad (2.3-7)$$

The key to a successful combined hardening theory is the proper determination of the hardening variable  $A$ . The BOPACE program uses hardening tables which give the yield-surface size and the surface translation as functions of the hardening parameters  $\kappa$  and  $\kappa^k$ . These are two-dimensional tables for each material whose ordinates and abscissas are, respectively, temperature and hardening parameter. Given the initial values of  $\kappa$  and  $\kappa^k$  at the beginning of an increment, and estimated values for  $\Delta \kappa$  and  $\Delta \kappa^k$ ,

the corresponding increments of isotropic and kinematic stress increase are obtained from the hardening tables. (Hardening due to temperature change is included by adding it to the isotropic stress increment.) The hardening slopes  $c$  and  $r$  are then computed, by dividing the incremental stress increases by the estimated increment of effective plastic strain. This procedure gives average values for the slopes  $c$  and  $r$  during the increment, and tends to produce an accurate and stable numerical iterative process. Note that it is the isotropic and kinematic stress increases, rather than the slopes  $c$  and  $r$ , which are directly related to the hardening parameters. The choice of a test value for  $\hat{s}^0$  in Equation 2.3-7 is arbitrary, as long as it is a point on a yield surface of size corresponding to  $\hat{s}$ , i.e. a surface with equal values of temperature and parameter  $\kappa$ . It is convenient in BOPACE to take  $\hat{s}^0$  equal to  $\hat{s}$ .

Incremental Stress-Strain Relation - The incremental stress-strain relation now follows the development of References 8 and 9. Take

$$\Delta \sigma_i = D_{ij}^e \Delta \epsilon_j^e = D_{ij}^e \Delta \epsilon_j^{e+p} - D_{ij}^e \hat{s}_j \lambda \quad (2.3-8)$$

where  $D^e$  is the appropriate matrix of elastic constants.

Then

$$A \lambda = \hat{s}_i \Delta \sigma_i = \hat{s}_i D_{ij}^e \Delta \epsilon_j^{e+p} - \hat{s}_i D_{ij}^e \hat{s}_j \lambda \quad (2.3-9)$$

which gives

$$\lambda = \hat{s}_i D_{ij}^e \Delta \epsilon_j^{e+p} / (A + \hat{s}_k D_{kl}^e \hat{s}_l) \quad (2.3-10)$$

Substituting Equation 2.3-10 into Equation 2.3-8 provides the desired relation

$$\Delta \sigma_i = \left( D_{ij}^e - \frac{D_{ik}^e \hat{s}_k \hat{s}_l D_{lj}^e}{A + \hat{s}_m D_{mn}^e \hat{s}_n} \right) \Delta \epsilon_i^{e+p} \quad (2.3-11)$$

or

$$\Delta \sigma_i = (D_{ij}^e + D_{ij}^p) \Delta \epsilon_j^{e+p} = D_{ij} \Delta \epsilon_j^{e+p} \quad (2.3-12)$$

D is the elasto-plastic Jacobian (tangent-stiffness) matrix relating incremental stresses to incremental elastic+plastic strains. In effect, it separates the elastic and plastic strains and determines the incremental stress corresponding to the incremental elastic strain.  $D^p$  is the stiffness reduction due to plastic flow, and becomes zero for the case of infinite hardening, i.e.  $A = \infty$ , or equivalently the total slope (c+r) of the stress vs. plastic-strain curve is infinite.

Effective Stress-Strain and Plastic Work - The concepts of "effective stress" and "effective strain" are related to plastic work, and are used in a limited way in the development of constitutive theory for the BOPACE program.

Because they can easily be misapplied, especially in the presence of kinematic hardening, the use and limitations of the concepts are briefly discussed here for the Mises plasticity theory.

Due to characteristics of the Prager hardening theory, the following statements of equivalence and proportionality should first be noted.

$$\Delta a_i = \Delta \alpha_i \approx \Delta \epsilon_i^p \approx \hat{s}_i \neq s_i \quad (2.3-13)$$

Because of the incremental nature of kinematic hardening,  $s_i$  and  $s_j$  are in general not proportional.

The Mises effective stress  $\bar{\sigma}$  is defined by

$$\bar{\sigma}^2 = \frac{3}{2} s_i s_i \quad (2.3-14)$$

The incremental and cumulative values for plastic work,  $W^p$ , are given by

$$\Delta W^p = \sigma_i \Delta \epsilon_i^p \quad (2.3-15a)$$

and

$$W^p = \sum \sigma_i \Delta \epsilon_i^p \quad (2.3-15b)$$

where  $\Sigma$  denotes summation over all increments. For the special case of proportional loading (i.e. loading in which all stress components are increased proportionately) followed by a constant stress level (i.e. no plastic hardening), the cumulative plastic work is given by

$$W^P = \sigma_i \epsilon_i^P \quad (2.3-15c)$$

As a matter of convenience in computing plastic work, an increment of effective plastic strain,  $\Delta \bar{\epsilon}^P$ , has historically been defined by

$$(\Delta \bar{\epsilon}^P)^2 = \frac{2}{3} \Delta \epsilon_i^P \Delta \epsilon_i^P \quad (2.3-16)$$

At this point, however, care must be exercised in using the historical calculation for plastic work. If kinematic hardening were zero, then  $\hat{s}_i = s_i$ , and because  $\Delta \epsilon_i^P$  is proportional to  $\hat{s}_i$  the use of Equations 2.3-14 and 2.3-16 would give plastic work as

$$\Delta W^P = \bar{\sigma} \Delta \bar{\epsilon}^P \quad (2.3-17a)$$

and

$$W^P = \Sigma \bar{\sigma} \Delta \bar{\epsilon}^P \quad (2.3-17b)$$

If in addition, the condition were one of proportional loading and constant stress, then by defining the cumulative effective plastic strain,  $\bar{\epsilon}^P$ , in the same manner as  $\Delta\bar{\epsilon}^P$ , we would have

$$W^P = \bar{\sigma} \bar{\epsilon}^P \quad (2.3-17c)$$

Of course the Equations 2.3-17 in general are not valid, because of the presence of kinematic hardening and non-proportional loading. Thus plastic work must be computed from Equation 2.3-15a and b, rather than from the product of effective stress and strain quantities.

The quantity  $\bar{\epsilon}^P$  serves little purpose in a general plasticity analysis, although it is a tensorially invariant quantity and does provide a measure of net residual deformation. For a rational measure of deformation history, either the plastic work,  $W^P$ , or the sum of increments of effective plastic strain,  $\sum \Delta\bar{\epsilon}^P$ , is appropriate. The difference in concept between the quantities  $W^P$  and  $\sum \Delta\bar{\epsilon}^P$  should, however, be recognized.

## 2.4 CREEP

Stages - Metals characteristically exhibit the three stages of primary, secondary and tertiary creep. Figure 2.4-1 shows these stages in a typical creep history under conditions of constant temperature and stress. Because creep rate varies considerably during the different stages, the description of actual creep histories is considered to be essential for an accurate analysis. The BOPACE program accounts for

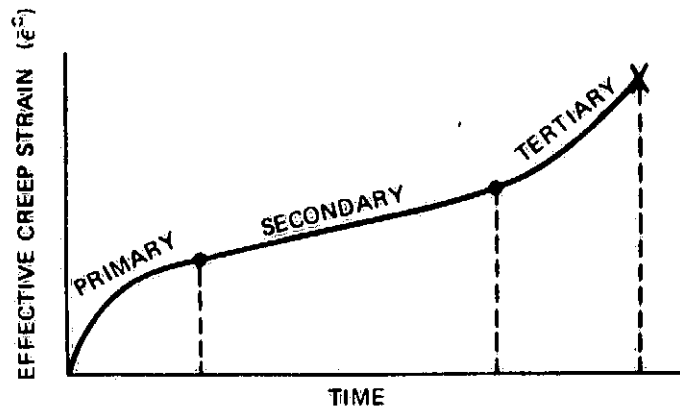


Figure 2.4-1. Typical Creep Stages

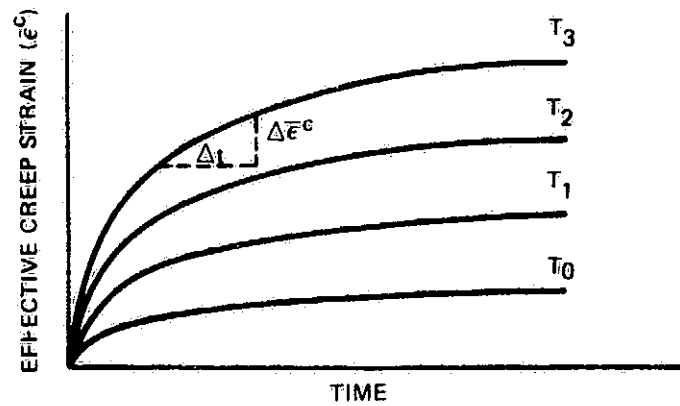


Figure 2.4-2. BOPACE Creep Representation ( Example for Variable Temperature and Constant Stress)

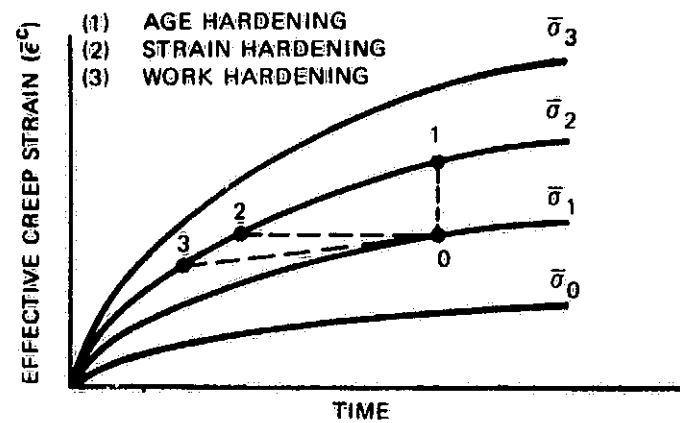


Figure 2.4-3. BOPACE Creep Hardening Options (Example for Constant Temperature and Variable Stress)

the creep time history by allowing the user to define, by a series of input points, the shape of the effective-creep-strain vs. time curve for each material.

Temperature and Stress Effects - Creep rate in most metals is very dependent upon temperature and stress level. The BOPACE approach to creep analysis provides a reasonable description of temperature and stress effects, while avoiding excessive storage and computational requirements. For each material, BOPACE requires a reference creep curve shape which gives the relative variation of effective-creep-strain vs. time for the various stages considered. This shape is assumed to be valid for all the temperatures and stress levels of the particular material. A table of creep factors for the material is then specified as a function of temperature and effective stress, and a portion of the actual creep curve is determined by multiplying the reference creep curve by the appropriate factor using the average temperature and stress during the increment. Figure 2.4-2 shows portions of typical creep curves for the special case of constant stress level and variable temperature. Note that according to BOPACE assumptions these curves have the same shape.

Hardening - As long as the temperature and stress level remain constant, an increment of creep is determined by following the corresponding creep curve for the given time increment. However, if temperature or stress level changes, an initial point must be identified on the corresponding



new creep curve in order to determine the new creep rate. This transfer from one curve to another requires an assumption for creep hardening, which in BOPACE is defined by a single hardening parameter,  $\kappa^C$ . BOPACE allows the option of either age, strain, or work hardening, for which  $\kappa^C$  is defined respectively as the accumulated time, sum of increments of effective creep strain, or creep work. Consider, for example, these options in Figure 2.4-3 for a case of constant temperature. Creep during the preceding increments has progressed to the point 0 on the  $\bar{\sigma}_1$  curve. The average effective stress during the present increment is  $\bar{\sigma}_2$ , which gives the initial points 1, 2 and 3, respectively, for the options of age, strain and work hardening. Incremental creep for the current increment is then determined by continuing along the  $\bar{\sigma}_2$  curve from the appropriate initial point, for a distance equal to the specified creep time increment. In the general case both temperature and stress will vary from one increment to the next, but the hardening option still determines in the same manner how the transfer is made between the creep curves.

Load Reversal - The main use of the creep-hardening parameter  $\kappa^C$  comes into play during a load reversal. When a complete reversal occurs,  $\kappa^C$  is set to zero and the initial point on the creep curve is taken as that corresponding to a zero value of  $\kappa^C$ . (A complete load reversal occurs if the incremental creep-strain vector has a direction exactly reversed from that of the preceding creep increment.) For an incomplete load reversal, the BOPACE program computes the starting value for  $\kappa^C$  by multiplying the existing value of  $\kappa^C$  by the factor  $(1 + \text{COSINE})/2$ , where COSINE is the Cosine of the angle between successive incremental creep

strain vectors. Parameter  $\kappa^C$  then accumulates as before, i.e. at the end of each increment  $\kappa^C$  becomes  $\kappa^C + \Delta\kappa^C$ .

Multiaxial Flow Rule - The incremental creep-strain vector has historically been taken normal to a Mises type of surface which passes through the stress point. When kinematic plastic hardening is considered, this surface could be taken either as the actual translated yield surface, or as an untranslated surface which passes through the stress point but whose center remains at the origin. The appropriate choice of surface is not clear, and the multiaxial creep flow rule is therefore defined on the basis of programming simplicity. BOPACE 3-D defines multiaxial creep under elastic conditions by

$$\Delta\epsilon_i^C = \left(\frac{3}{2} \Delta\bar{\epsilon}^C / \bar{\sigma}\right) s_i \quad (2.4-1)$$

where  $\Delta\bar{\epsilon}^C$  is the increment of effective creep strain defined by

$$(\Delta\bar{\epsilon}^C)^2 = \frac{2}{3} \Delta\epsilon_i^C \Delta\epsilon_i^C \quad (2.4-2)$$

while  $\bar{\sigma}$  and  $s$  are evaluated at the beginning of the increment.

Creep which occurs under plastic conditions is taken in the same direction as that of the plastic strain increment (see Section 2.6).

## 2.5 COMPLETE STRESS-STRAIN RELATIONS

In Sections 2.1 to 2.4, the basic theory used in BOPACE for elasticity, thermal strains, plasticity and creep has been discussed. The present section describes the complete stress-strain relations, and the manner in which simultaneous elastic, plastic, thermal and creep strains are accounted for. The combined effects of temperature-dependent elasticity and plasticity are included.

General 3-D Relations - For temperature-dependent behavior, an elasto-plastic incremental stress-strain relation follows from Equations 2.1-5b and 2.3-8:

$$\Delta \sigma_i = \Delta D_{ij}^e \epsilon_j^{e0} + D_{ij}^{el} \Delta \epsilon_j^{e+p} - D_{ij}^{el} s_j \lambda \quad (2.5-1)$$

Here the first term accounts for stress change due to change in elastic properties, while the second and third terms account for stress change due to change in elastic strain. Following Equation 2.3-9,

$$A \lambda = \hat{s}_i \Delta \sigma_i = \hat{s}_i \Delta D_{ij}^e \epsilon_j^{e0} + \hat{s}_i D_{ij}^{el} \Delta \epsilon_j^{e+p} - \hat{s}_i D_{ij}^{el} s_j \lambda \quad (2.5-2)$$

where again

$$A = C_{ij} \hat{s}_i \hat{s}_j + R_{ij}^0 \hat{s}_i^0 \hat{s}_j^0 \quad (2.5-3)$$

For the general case of temperature-dependent plasticity,  $R^0$  accounts for isotropic hardening due to both plastic deformation and temperature. Then

$$\lambda = \frac{\hat{s}_i \Delta D_{ij}^e \epsilon_j^{e0} + \hat{s}_i D_{ij}^{el} \Delta \epsilon_j^{e+p}}{A + \hat{s}_k D_{kl}^{el} \hat{s}_l} \quad (2.5-4)$$

Substituting Equation 2.5-4 into 2.5-1 gives

$$\Delta \sigma_i = \left( \Delta D_{ij}^e - \frac{D_{ik}^{el} \hat{s}_k \hat{s}_l \Delta D_{lj}^e}{A + \hat{s}_m D_{mn}^{el} \hat{s}_n} \right) \epsilon_j^{e0} + \left( D_{ij}^{el} - \frac{D_{ik}^{el} \hat{s}_k \hat{s}_l D_{lj}^{el}}{A + \hat{s}_m D_{mn}^{el} \hat{s}_n} \right) \Delta \epsilon_j^{e+p} \quad (2.5-5)$$

or, using abbreviated notation

$$\Delta \sigma_i = (\Delta D_{ij}^e + \Delta D_{ij}^p) \epsilon_j^{e0} + (D_{ij}^{el} + D_{ij}^{pl}) \Delta \epsilon_j^{e+p} = \Delta D_{ij}^e \epsilon_j^{e0} + D_{ij}^l \Delta \epsilon_j^{e+p} \quad (2.5-6)$$

Thus the increment of stress can be determined as the sum of two products: an incremental matrix times the initial elastic strains, plus an end-of-increment matrix times the incremental elastic+plastic strains.

This formulation was used by the initial BOPACE program in the iterative stress-strain algorithm for temperature dependent materials, and is developed here for the sake of clarity. The present BOPACE 3-D program,

however, employs an improved iterative algorithm, which allows an additional benefit by substituting the simpler Equation 2.1-5a for 2.1-5b. Details of the new algorithm are discussed in Section 2.6. For either approach, the formation of the tangent stiffness matrix is based on Equation 2.3-12, with quantities evaluated at a single appropriate temperature. (In updating the matrix the temperature used is that at the end of the increment).

## 2.6 IMPROVED ALGORITHM FOR INELASTIC CALCULATIONS

Summary of Basic Concepts - The iterative residual-force procedure is often employed with an incremental solution for inelastic (plasticity and creep) problems, in order to avoid accumulated error. Each iteration in the residual-force procedure involves the following two stages.

- 1) **Equilibrium and Compatibility:** Given the current residuals (unbalanced forces or stresses), the equilibrium and compatibility equations are applied in order to predict an improved configuration (of displacements and strains).
- 2) **Separation of Strains:** Given the current strains, some algorithm based on the inelastic material theory is applied in order to separate the strains into their elastic, plastic and creep portions, and thus provide the resulting stresses.

When this procedure has converged to the correct result, the following conditions will be met.

- 1) Forces in equilibrium
- 2) Displacements compatible
- 3) Plastic strain increment satisfies normality rule
- 4) Size of yield surface consistent with deformation history
- 5) Translation of yield surface consistent with deformation history

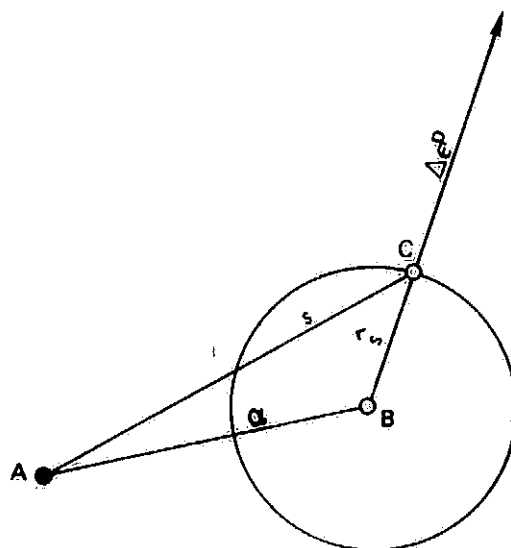
The overall BOPACE solution technique based on the residual-force procedure is summarized in Section 4. The purpose of the present section is to discuss the details of a new algorithm which has been developed and incorporated into BOPACE, for improving the convergence and accuracy of the inelastic stress-strain calculations. This algorithm defines the implementation of stage 2 (separation of strains) in the residual-force iterative procedure.

Background - The theory already presented in Sections 2.1 through 2.5 may be employed for both stages of the iterative procedure, and in fact equations of the type 2.5-5 were used for all stress-strain calculations in the initial version of BOPACE. Convergence difficulties resulted from the use of this approach in stage 2, however, when the incremental inelastic strains were large relative to the cumulative elastic strains. These difficulties were substantially eliminated by properly controlling the direction defined for the incremental inelastic strains. (The

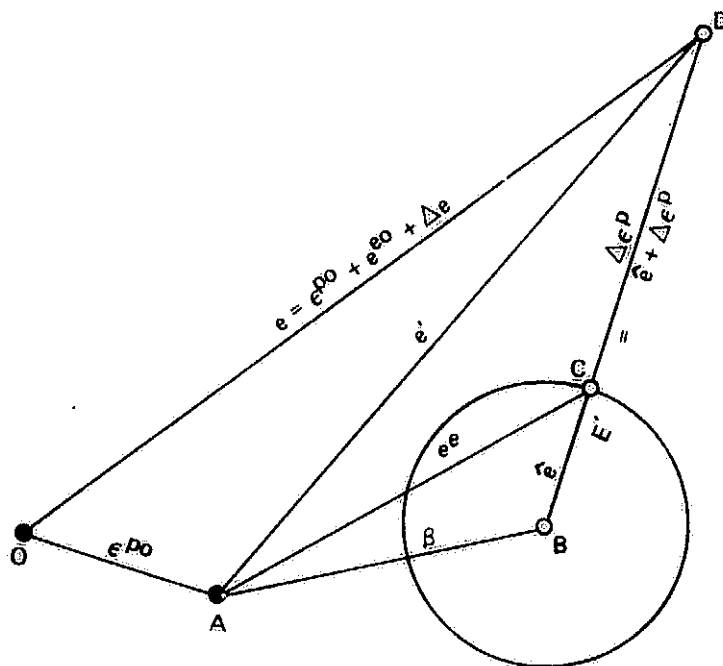
reason for the difficulties and the method of control were presented in Reference 13). Another quite different approach is based on a "strain-space" concept, and was presented by Barsoum in Reference 14 with the claim of a significant improvement in efficiency. That approach has therefore also been developed and evaluated for BOPACE usage. Because the method as presented in Reference 14 assumes kinematic hardening only, it was extended to incorporate the combined isotropic and kinematic hardening provided by BOPACE. In addition, some further techniques for accelerating convergence were identified and incorporated into the method. The resulting modified version of BOPACE has shown promising results. For example, a small test problem involving 15 load increments and solved using BOPACE solution option 5, required 25% fewer iterations and 50% fewer stiffness matrix updates than when run on the previous version. Although further comparisons are needed to prove the effectiveness of the new algorithm, it is expected to supersede the previous BOPACE capability.

Basic Definitions and Comparison of Algorithms - The new inelastic algorithm involves calculations in the "deviatoric strain space", rather than the more conventional "deviatoric stress space" used in previous BOPACE programs. For the sake of clarity, the previously used stress-space algorithm will again be summarized here, and the elastic-plastic quantities used in the new strain-space algorithm will be defined and compared with previous quantities.

Note: ● = Point which is fixed during increment  
○ = Point which moves during increment



a) QUANTITIES IN DEVIATORIC STRESS SPACE



b) QUANTITIES IN DEVIATORIC STRAIN SPACE

Figure 2.6-1. Graphical Representation of Elastic-Plastic Quantities



As described in Section 2.3, the definition of a plasticity theory requires assumptions for three basic constituents: a yield surface, a flow rule, and a hardening assumption. BOPACE development is based on the Mises yield surface, and this surface is represented by a hypercircle in 9-dimensional deviatoric stress space, as shown in Figure 2.6-1a. The surface is defined by the equation

$$F = \hat{s}_i \hat{s}_i - \hat{s}_i^0 \hat{s}_i^0 = 0 \quad (2.6-1)$$

where  $s$  is the deviatoric stress,  $\hat{s} = s - \alpha$  is the relative deviatoric stress and defines the isotropic hardening,  $\alpha$  is the surface translation and defines the kinematic hardening, while  $\hat{s}^0$  is a reference value of  $\hat{s}$  and must be known as a function of plastic deformation (e.g. from a uniaxial test). Point A in Figure 2.6-1a is the origin of the deviatoric stress space, point B is the current center of the yield surface, and point C represents the current state of deviatoric stress. A stress point on the surface corresponds to a plastic state. According to the Prandtl-Reuss flow rule, the direction of the incremental plastic strain,  $\Delta \epsilon^P$ , is normal to the yield surface at the current deviatoric stress state,  $s$ . A solid circle (●) in Figure 2.6-1 denotes a point which remains fixed throughout the increment, while an open circle (○) denotes a point which moves during the increment. In order to achieve greater accuracy and allow larger load increments, BOPACE evaluates moving points such as B and C at the midpoint of the plastic increment. Additional details of the BOPACE stress-space algorithm are discussed in Section 2.3.

For the new strain-space algorithm, the three basic constituents of the plasticity theory remain unchanged, and direct use is made of the stress-space theory and nomenclature. However, we now work with a yield surface and associated quantities in strain-space. Thus we compute the deviatoric elastic strain,  $e^e$ , in terms of the deviatoric stress,  $s$ , by

$$e_i^e = s_i / G \quad (2.6-2)$$

where  $G = E/(1+\nu)$  is a tensorial shear modulus. Similarly we define a "strain center",  $\beta$ , in terms of the stress center,  $\alpha$ , by

$$\beta_i = \alpha_i / G \quad (2.6-3)$$

Then the relative deviatoric strain,  $\hat{e}$ , is defined by

$$\hat{e}_i = e_i^e - \beta_i = (s_i - \alpha_i) / G = \hat{s}_i / G \quad (2.6-4)$$

The geometrical interpretation of the new algorithm involving these quantities is provided by a sketch in 9-dimensional deviatoric strain-space, shown in Figure 2.6-1b. There point 0 is the origin, defining the initial undeformed (zero strain) state. Subsequent deformation is caused by a series of load increments, resulting in elastic and plastic strains. A superscript 0 is used to denote the value of a quantity at the beginning of the load increment. Thus, point A defines the cumulative plastic strain,  $\epsilon^{p0}$ , which exists at the beginning of the current increment.

(Because of the plastic incompressibility assumption, the plastic strains themselves are deviatoric strains). All other points in Figure 2.6-1b refer to locations at some time during the current increment. In particular, we will be mainly concerned with the location of these points at a defined reference time. This reference time may be taken at the end of the increment, following the approach of Barsoum [14], or greater accuracy may be obtained at the expense of some additional variable storage by taking the reference time at the midpoint of the plastic increment, as is done in the new BOPACE algorithm. Point D defines the total cumulative deviatoric strain,  $e$ , during the increment. The circle is associated with the Mises yield surface, but is a hyper-circle in the deviatoric strain space. A strain point within the surface corresponds to an elastic state, while a strain point outside the surface corresponds to a plastic state. The size of this circle is defined by its radius  $\hat{e}_i$  ( $\hat{e}_i = \hat{s}_i/G$ ), whereas the Mises stress-space surface has radius  $\hat{s}$ . The center of the circle is at point B ( $B_i = \epsilon_i^{p0} + \beta_i = \epsilon_i^{p0} + \alpha_i/G$ ), whereas the center of the Mises stress-space surface has components  $\alpha_i$ . During plastic deformation, the strain-space surface may undergo both expansion (due to isotropic hardening), and translation (due to kinematic hardening). The cumulative deviatoric elastic strain,  $e^e$ , is defined by the vector AC ( $e_i^e = s_i/G$ ). From these comparisons it should be apparent that the basic quantities in Figures 2.6-1a and b, respectively, can be made to coincide, if points A are superimposed and all dimensions in 2.6-1b are divided by the factor  $G$ . The incremental plastic strain,  $\Delta\epsilon^p$ , is defined by the

vector CD. It is normal to the circle because of the Prandtl-Reuss flow rule, and is therefore colinear with the radius  $\hat{e}$  to point C. The vector  $BD = \hat{e} + \Delta \epsilon^p$  is denoted by  $E'$ . The symbols  $e'$ ,  $e^e$ ,  $\Delta \epsilon^p$  and  $E'$  are consistent with their usage in Reference 14.

Computation Procedure - We now define the new strain-space algorithm for implementing stage 2 of the residual-force iterative procedure. The problem which must be solved can be stated in terms of the various strain vectors. At the beginning of the increment, we have known values for  $\epsilon^{p0}$  (which remains constant during the increment), and for  $\beta$ ,  $e^e$ , and  $\hat{e}$ . These have been determined such that they are all consistent, i.e., such that the appropriate vectors meet at single points A, B and C. The current estimate for the value of  $e'$  at the reference time is also known from stage 1 of the iterative procedure. We must then determine values for  $\beta$ ,  $e^e$ ,  $\hat{e}$  and  $\Delta \epsilon^p$  at the reference time, consistent with the convergence requirements. Stated somewhat differently, we are given the locations of points A and D at the reference time, and the locations of points B and C at the beginning of the increment. We must then compute the locations of B and C at the reference time, consistent with the convergence requirements.

The basic steps of the stage 2 algorithm are summarized by the following.

- 1) Given values at beginning of increment for:

$$\alpha^0 = \text{stress center}$$

$\hat{s}^0$  = relative deviatoric stress

$e^{e0}$  = elastic strains

2) Given  $\Delta\epsilon$  = total (elastic + plastic) strain increment from stage 1.

3) Compute values at reference time, based on estimated incremental deformation, for:

$\Delta\alpha$  = kinematic hardening increment

$\hat{\Delta s}$  = isotropic hardening increment

4) Compute:  $\epsilon_i = \epsilon_i^{e0} + \Delta\epsilon_i$  = initial elastic strain + total strain increment

$e_i^i$  = corresponding deviatoric value

5) Compute:  $\beta_i = (\alpha_i^0 + \Delta\alpha_i)/G$

$\hat{e}_i = (s_i^0 + \hat{\Delta s}_i)/G$

6) Compute  $E_i^i = e_i^i - \beta_i$

7) Compute  $\lambda = (|E^i| - |\hat{e}|)/|E^i|$  = plastic proportionality constant

8) Compute  $\Delta\epsilon_i^p = \lambda E_i^i$  = incremental plastic strain at reference time.

Adjust  $\Delta\epsilon_i^p \leftarrow \Delta\epsilon_i^p$  times (ratio of total to reference time increment), to obtain total plastic strain increment. Compute

$\Delta\epsilon_i^e = \Delta\epsilon_i - \Delta\epsilon_i^p$  = incremental elastic strain.

9) Compute end of increment values for:

$\epsilon_i^e = \epsilon_i^{e0} + \Delta\epsilon_i^e$  = cumulative elastic strain

$\sigma_i = D_{ij} \epsilon_j^e$  = cumulative stress

10) Use  $\sigma$  to compute residual forces and error norm, and return to stage 1 if convergence has not been achieved.

The strain-space algorithm presented above corresponds to that given by Barsoum [14] except that here a combined isotropic and kinematic hardening is provided and a reference (midpoint) time calculation of the incremental variables is used to improve accuracy. As noted by Barsoum, greater consistency and better convergence are obtained by utilizing an algorithm in strain space rather than in stress space. This is because the stress-space calculation fixes the  $\Delta \epsilon^P$  vector along the direction of the current  $\hat{s}$  vector, rather than simultaneously fixing the directions of  $\hat{s}$  and  $\Delta \epsilon^P$  consistent with the given total strain increment  $\Delta \epsilon$ . The stress-space iteration can cause large oscillations in the location of point C, resulting in divergence if  $\Delta \epsilon^P$  is large relative to the cumulative elastic strain.

Although a strain-space algorithm eliminates most of the inconsistencies and tendencies toward divergence, it should be noted that an inconsistency still exists in the plastic hardening quantities. This is because  $\Delta \alpha$  and  $\Delta \hat{s}$  are based on the estimated increment of plastic deformation, which will not in general be consistent with the actual deformation. Thus if another iteration were performed using the same value for the total strain increment  $\Delta \epsilon$ , different results would be obtained due to change in  $\beta$  and  $\hat{e}$ .

This difficulty is eliminated in the present approach by properly modifying the calculation of  $\lambda$  in step 7. For this calculation, we use the parameters  $c$  and  $r$  associated with kinematic and isotropic hardening, respectively, in the expressions

$$\begin{aligned}\Delta\alpha_i &= \frac{2}{3} c \Delta\epsilon_i^p \\ \Delta s_i &= \frac{2}{3} r \Delta\epsilon_i^p\end{aligned}\tag{2.6-5}$$

We then see that

$$\begin{aligned}E_i' &= e_i' - \beta_i = e_i' - (\beta_i^0 + \Delta\beta_i) \\ &= e_i' - \beta_i^0 - \Delta\alpha_i/G = e_i' - \beta_i^0 - \frac{2}{3} c \Delta\epsilon_i^p / G\end{aligned}\tag{2.6-6}$$

Replacing  $\Delta\epsilon_i^p$  in this equation by  $\lambda E_i'$ , we may solve for  $E_i'$ :

$$E_i' = (e_i' - \beta_i^0) / (1 + \frac{2}{3} \lambda c / G)\tag{2.6-7a}$$

In a similar manner we may obtain

$$\hat{e}_i = \hat{e}_i^0 + \frac{2}{3} \lambda r E_i' / G \quad (2.6-7b)$$

The plastic proportionality constant, as already defined, is

$$\lambda = (|E'| - |\hat{e}|) / |E'| \quad (2.6-7c)$$

It is apparent from Equations 2.6-7 that the expression for  $\lambda$  is nonlinearly dependent upon  $\lambda$  itself, and this is the reason why a consistent  $\lambda$  is not solved for directly. An accurate value for  $\lambda$ , however, can easily be obtained by a "linear intersection method." In this method we take the approximate value of  $\lambda$  from step 7, and substitute into the Equations 2.6-7 to obtain a new computed value  $\lambda_{c0}$ . We then assume a value of  $\lambda + \Delta\lambda$ , where  $\Delta\lambda$  is a small change (perhaps  $.01\lambda$ ), and again substitute into Equations 2.6-7 to compute another value  $\lambda_{c1}$ . The two pairs of assumed and computed  $\lambda$  values are plotted in Figure 2.6-2. The correct value for  $\lambda$  lies on the 45-degree line (since there the assumed and computed values would be equal), at the intersection of this line with the line connecting the two plotted points. This corrected value of  $\lambda$  is obtained by the following adjustment of  $\lambda$  from step 7.



$$\lambda \leftarrow \lambda + \Delta\lambda(\lambda - \lambda_{c0})/(\lambda_{c1} - \lambda_{c0} - \Delta\lambda) \quad (2.6-8)$$

The incorporation of this adjustment into the strain-space algorithm provides consistent values for all quantities in stage 2 of the iterative process, and results in improved convergence.

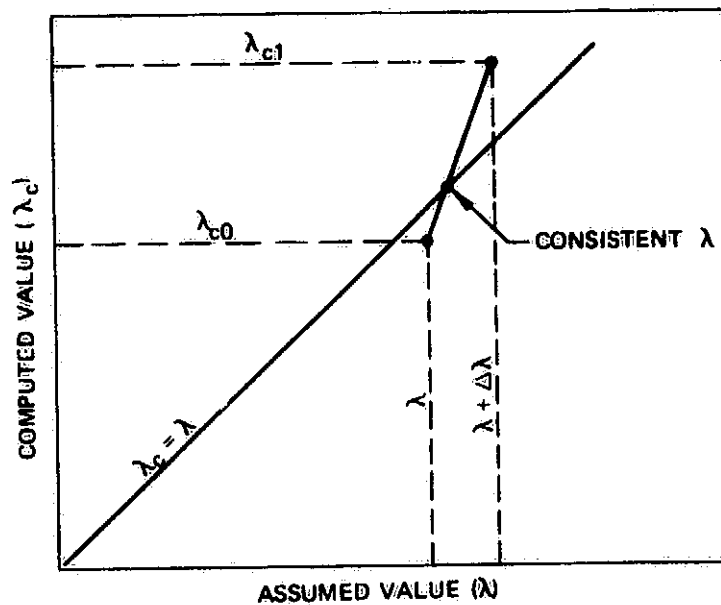


Figure 2.6-2. Linear-Intersection Calculation for  $\lambda$

Extensions and Refinements to the Basic Algorithm - The strain-space algorithm as presented here is employed in BOPACE for plastic analysis. In addition, the BOPACE algorithm treats creep strains in a manner similar to that for the plastic strains. For cases where the material is elastic at the beginning of an increment and then reaches the plastic yield point at some intermediate time during the increment, greater accuracy is obtained by dividing the calculations into two parts. In such cases the initial creep is taken in the direction of the initial deviatoric stress, and creep which occurs after the yield point is taken in the same direction as the plastic strain increment. Other refinements, such as temperature dependent elastic-plastic-creep and generalized load reversal, are treated as discussed in Section 2.

### 3.0 Axisymmetric Finite Element Formulation

The BOPACE axisymmetric program provides a family of solid isoparametric ring elements with curved boundaries. The formulation of these elements is essentially a specialization to two dimensions of the BOPACE 3-D formulation. The simplest ring element in this family is the basic 4-node quadrilateral, and higher order elements are defined by adding additional edge nodes to the basic 4 corner nodes. BOPACE allows an arbitrary number of nodes (from 4 to 10) per element, and each edge may contain an optionally different number of arbitrarily spaced nodes (from 2 to 5).

This section discusses the elemental-level axisymmetric formulation, including the isoparametric formulation and displacement functions, the calculation of strain, force, and stiffness quantities, and the numerical integration process. Further details of the general approach are provided in the BOPACE 3-D theory manual.

#### 3.1 Isoparametric Formulation and Shape Functions

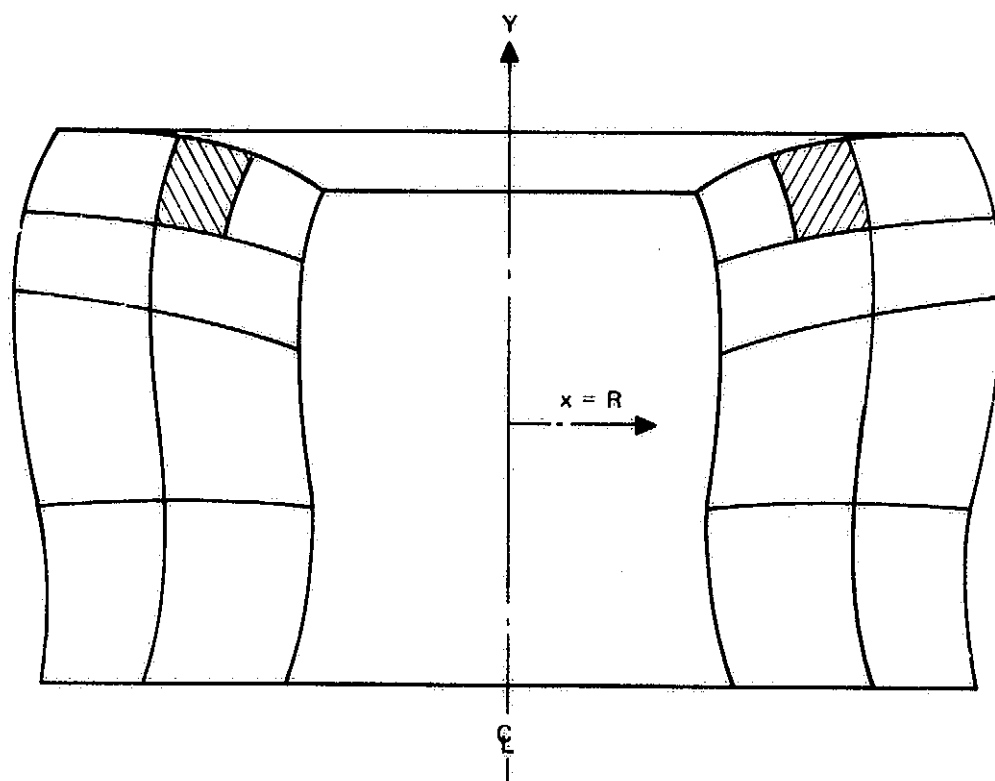
In a manner similar to that for the 3-D isoparametric brick, the axisymmetric ring element requires the definition of shape functions over a parent element. These functions map the geometry of the parent element into an element on the cross section of the body, and also interpolate the field quantities at any point within the element in terms of the nodal quantities.

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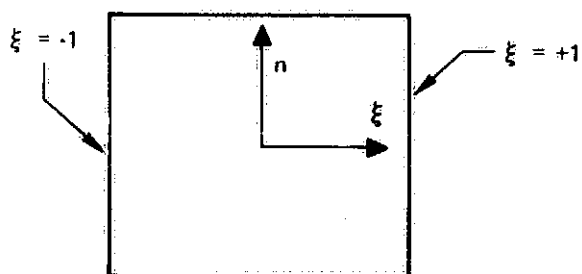
BOPACE Axisymmetric Ring Elements - The BOPACE isoparametric ring type of element is illustrated in Figure 3.1-1. In Figure 3.1-1a an axisymmetric solid is shown idealized by ring elements, with the nodes completely defined by their x-y coordinates on the cross section (each node is actually an axisymmetric nodal ring). The radial coordinate R coincides with the x-axis on the section. A z-axis is taken normal to the x-y axes, forming a right-hand Cartesian system and allowing all nonzero stress and strain components (xx,yy,zz,xy) to be represented. Figure 3.1-1 b and c show the parent and actual forms, respectively, for a cross section of one of the ring elements. The parent element is a 2 x 2 square, having an associated Cartesian coordinate system  $\xi-\eta$  with origin at the center of the square and axes normal and parallel to the faces. The element has 4 corner nodes, plus an optional number of arbitrarily spaced edge nodes. The actual element has generally curved boundaries (each edge is a curve defined by the polynomial through its nodes), and is referenced to the Cartesian coordinate system x-y. The x-y-z system is the basic global system for an entire cross section of the body, and provides the reference frame for definition of all stresses, strains, etc.

Element Geometry and Displacements - Each element node has an associated shape function given in terms of the  $\xi-\eta$  coordinates, i.e. at the  $n$ th node the shape function is denoted by  $N^n(\xi,\eta)$ . These shape functions define the geometry of the actual element by a point-wise mapping back onto the parent element. Thus in the actual element the x-coordinate of a point is given by

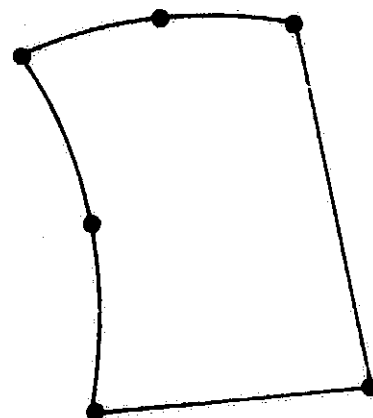
$$x = x^n N^n(\xi,\eta) \quad (3.1-1a)$$



(a) AXISYMMETRIC SOLID IDEALIZATION BY RING ELEMENTS



(b) PARENT SQUARE



(c) ACTUAL RING ELEMENT CROSS SECTION

Figure 3.1-1. BOPACE Isoparametric Ring Element

Here  $(\xi, \eta)$  are the coordinates of the corresponding point on the parent element, and  $x^n$  is the x-coordinate of the nth node, with summation implied over n. A similar expression is used for the y-coordinate. Displacements u-v (in the x-y directions, respectively) of a point in the actual element are defined in the same manner. For example the u-displacement of a point is defined by

$$u = u^n N^n(\xi, \eta) \quad (3.1-1b)$$

where  $u^n$  is the u-displacement of the nth node.

Displacement Derivative Calculations - For the BOPACE stress analysis the spatial displacement derivatives must be expressed in terms of the nodal displacements. To accomplish this, we first define a matrix  $\bar{g}$  of the nodal shape functions, differentiated with respect to the  $\xi$ - $\eta$  coordinate:

$$\bar{g} = \begin{bmatrix} \frac{\partial N^1}{\partial \xi} & \frac{\partial N^2}{\partial \xi} & \text{---} & \text{---} & \text{---} & \text{---} & \frac{\partial N^n}{\partial \xi} \\ \frac{\partial N^1}{\partial \eta} & \frac{\partial N^2}{\partial \eta} & \text{---} & \text{---} & \text{---} & \text{---} & \frac{\partial N^n}{\partial \eta} \end{bmatrix} \quad (3.1-2)$$

When the displacement derivatives are used in the program they must be taken with respect to the x-y coordinates, and a transformation relation between derivatives in the two coordinate systems must therefore be supplied. This relation is given by the Jacobian matrix J, which is defined by

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} \end{bmatrix} \quad (3.1-3)$$

A transformation is applied to the  $\bar{g}$  matrix of displacement function derivatives, of the form

$$g_{ij} = J_{im}^{-1} \bar{g}_{mj} \quad (3.1-4)$$

This inverse Jacobian transformation produces the desired form of the partial derivatives matrix  $g$ , which is

$$g = \begin{bmatrix} \frac{\partial N^1}{\partial x} & \frac{\partial N^2}{\partial x} & - & - & - & \frac{\partial N^n}{\partial x} \\ \frac{\partial N^1}{\partial y} & \frac{\partial N^2}{\partial y} & - & - & - & \frac{\partial N^n}{\partial y} \end{bmatrix} \quad (3.1-5a)$$

Using this  $g$  matrix, and the basic matrix of shape functions  $f$ , defined by

$$f = \begin{pmatrix} N^1 & N^2 & - & - & - & N^n \end{pmatrix} \quad (3.1-5b)$$

a composite matrix is then defined by

$$G = \begin{bmatrix} g & o \\ o & g \\ f/R & o \end{bmatrix} \quad (3.1-6)$$

where the radius  $R$  at any point is computed using the nodal radii and the shape functions ( $R=R^i N^i$ ). If we now define the vector  $\theta$  of displacement derivatives by

$$\theta = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{u}{R} \end{pmatrix} \quad (3.1-7a)$$

and arrange the vector  $q$  of nodal displacements in the form

$$q = ( u^1 \ u^2 \ u^3 \ u^4 \ - \ - \ - \ u^n, \ v^1 \ - \ - \ - \ v^n ) \quad (3.1-7b)$$

we may write the important relationship between displacement derivatives and nodal displacements, as

$$\theta_i = G_{ij} \ q_j \quad (3.1-8)$$

In constructing the BOPACE ring element program logic it is actually more convenient to rearrange the  $q$ -vector so that the  $u$ - $v$  displacements at a particular node are grouped together. This also requires rearrangement of the columns of the  $G$  matrix. However the calculation and storage of the  $g$  and  $f$  matrices at integration points occurs in the simple form of Equations 3.1-5, and required operations involving the  $G$  matrix are performed simply by an appropriate indexing procedure, taking full advantage of the evident sparsity in the given form of  $G$ .

General Considerations for the Shape Functions - The ring element shape functions are simply a specialization to two dimensions of the 3-D element functions. In fact the same element generation routines are shared by the BOPACE 3-D and axisymmetric programs, by eliminating the  $\zeta$  and  $z$  coordinates for the axisymmetric generation. The functions are again separated into two types - corner and edge functions. The definition of the ring element functions should be obvious from the BOPACE 3-D definitions, and therefore no further explanation is given here.



### 3.2 Strain, Force and Stiffness Quantities

Strains - The Lagrangian strain,  $\epsilon$ , is defined in terms of material displacement derivatives,  $\theta$ , at a point in the body, by

$$\epsilon_i = AO_{ij} \theta_j + \frac{1}{2} Al_{ijk} \theta_j \theta_k \quad (3.2-1)$$

Here  $AO$  and  $Al$  are constant coefficients which define the strain tensor, with  $Al$  providing the (geometrically) nonlinear portion of the strain. The axisymmetric BOPACE program has been designed to allow a very efficient incorporation of geometric nonlinearities, although the present version is restricted to treatment of material nonlinearities only. Only the  $AO$  term in Equation 3.2-1 is therefore retained, and the expanded strain-displacement relation for axisymmetric BOPACE is written as

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ v_x \\ v_y \\ u/R \end{Bmatrix} \quad (3.2-2)$$

Forces - The principle of virtual work is valid for arbitrary non-linear materials, and it provides a simple basis for deriving the element force and stiffness relations. The equivalence of external and internal virtual work relates the generalized nodal forces  $p$ , and displacements  $q$ , in the element equilibrium equation

$$\delta q_i p_i = \int_V \delta \epsilon_i \sigma_i dV \approx \int_V \delta \epsilon_i (\sigma_i^* + DO_{ij}^* \Delta \epsilon_j) dV \quad (3.2-3)$$

which holds along any equilibrium path in the neighborhood of the reference equilibrium (\*) configuration. (The approximation to the integral in Equation 3.2-3 occurs when the first order incremental stress-strain relation involving the DO matrix is used. This does not lead to inaccurate results in the solution process because of the equilibrium check and residual-force corrective iterations which are performed). Here  $\delta \epsilon$  and  $\delta q$  are kinematically consistent variations, and from Sections 3.1 and 3.2

$$\delta \epsilon_i \equiv AO_{ij} \delta \theta_j = AO_{im} G_{mj} \delta q_i \equiv B_{ij} \delta q_i \quad (3.2-4)$$

The stress components for the axisymmetric element correspond to the strain components, and are  $(\sigma_{xx} \sigma_{yy} \sigma_{zz} \sigma_{xy})$ .

The theoretical implementation of Equation 3.2-3 requires the use of the second Piola-Kirchoff stress associated with Lagrangian strain, with integration performed over the undeformed volume. However, the small strain assumption is used in the BOPACE formulation, so that the stress may be taken as the usual engineering or true stress.

Substituting for  $\delta \epsilon$ , and realizing that Equation 3.2-3 must be satisfied for arbitrary variations  $\delta q$ , provides the basic equilibrium equation for forces, as

$$P_i = \int_A 2\pi R B_{ai} \sigma_a dV \approx \int_A 2\pi R G_{mi} AO_{am} (\sigma_a^* + DO_{ab}^* \Delta \epsilon_b) dA \quad (3.2-5a)$$

or at a reference equilibrium (\*) configuration, where  $\sigma = \sigma^*$ , we have

$$P_i^* = \int_A 2\pi R G_{mi} A O_{am} \sigma_a^* dA \quad (3.2-5b)$$

The  $2\pi R$  factor in these equations results from integration around the circumference, which converts the volume integral to an integral over the cross sectional area. Thus the force used in all calculations is the total "ring" force around the circumference. The radius  $R$  is obtained using the nodal radii and shape functions (see Section 3.1) by the interpolation formula  $R = R^i N^i$ .

Stiffness - Differentiating Equation 3.2-5a and evaluating at the reference configuration ( $\Delta \epsilon = 0$ ,  $\theta = \theta^*$ , etc.) provides the first order incremental equilibrium equation

$$\dot{P}_i^* = K O_{ij}^* \dot{q}_j^*$$

where

$$K O_{ij}^* = \int_A 2\pi R B_{ai} D O_{ab}^* B_{bj} dV = \int_A 2\pi R G_{mi} A O_{am} D O_{ab}^* A O_{bn} G_{nj} dA \quad (3.2-7)$$

The "tangent stiffness" matrix  $K O^*$  is thus a function of the current elastic-plastic stress-strain matrix  $D O^*$ . The incremental form of Equation 3.2-6 is of course

$$\Delta P_i = K O_{ij}^* \Delta q_j \quad (3.2-8)$$

and is the basis for axisymmetric BOPACE solutions. Although one could compute  $B_{ai} = AO_{am} G_{mi}$ , it is more efficient in the program logic to use  $AO$  and  $G$  separately. In generating the tangent stiffness by Equation 3.2-7, a matrix  $\bar{K}_{mn} = AO_{am} DO_{ab}^* AO_{bn}$  is first computed, after which the product  $G_{mi} \bar{K}_{mn} G_{nj}$  is formed. This procedure takes maximum advantage of sparsity in the  $G$  matrix, and also allows a simple extension for including geometric nonlinearities.

### 3.3 Numerical Integration

The integrals which define force and stiffness quantities in the axisymmetric BOPACE program are calculated by numerical Gauss-point integration and are taken over the volume (or area) of the actual element. The mechanics of the integration process, however, are best accomplished over the parent square element, where there are simple integration limits in terms of the  $\xi$ - $\eta$  Cartesian coordinate system. Integration is therefore taken in the form

$$\int_A f \, dA = \int_{-1}^{+1} \int_{-1}^{+1} f(\xi, \eta) |J| \, d\xi d\eta \quad (3.3-1)$$

where  $f$  is the function to be integrated (including the  $2\pi R$  factor resulting from circumferential integration), and  $|J|$  is the Jacobian determinant which corrects for the fact that a differential area ( $dx dy$ ) in the actual element is equal to  $|J| \, (d\xi d\eta)$ .

The integral is evaluated numerically by substituting for it a sum over a number of Gauss integration points:

$$\int_A f \, dA = \sum_{i=1}^m \sum_{j=1}^n f(\xi_i, \eta_j) W_{ij} \quad (3.3-2)$$

Here  $m, n$  are the numbers of integration points in the  $\xi, \eta$  directions, respectively (total number of points =  $m \times n$ ), and  $W_{ij}$  is a weighting factor for each point which includes the value of the Jacobian determinant.

The Gauss integration scheme is used because it provides higher accuracy for a given number of points than some other methods, through an optimum selection of the point locations. (The use of  $m$  Gauss points allows the exact integration of a polynomial of degree  $2m - 1$ ). The BOPACE axisymmetric program currently uses a 4-point ( $2 \times 2$ ) Gauss formula for all integrations, although the integration logic includes options for other Gauss-point formulas.

#### 4.0 SOLUTION METHOD

##### 4.1 BASIC SOLUTION REQUIREMENTS

The exact elasto-plastic-creep analysis of a structure requires the satisfaction, at all points in the structure, of three requirements:

- 1) Equilibrium of stresses
- 2) Compatibility of strains
- 3) Satisfaction of constitutive theory, which is summarized by the appropriate incremental stress-strain relation

The following paragraphs summarize the BOPACE solution approach as it relates to satisfying these three requirements.

Stress-Strain Relation - The incremental stress-strain calculation is defined by Equation 2.1-5a, and implemented by the algorithm presented in Section 2.6. The stress-strain relation is satisfied exactly in the BOPACE solution procedure, provided the increment is sufficiently small so that incremental quantities can be treated in a differential manner. Because of the numerical integration employed over the 3-D isoparametric element volume, the stress-strain relation may be satisfied in only an approximate manner at points other than the integration points.

Compatibility - Compatibility is satisfied exactly within each 3-D isoparametric element as a result of the finite-element derivation. In the global sense, i.e., over the entire structure, compatibility is also satisfied exactly, by merging the element degrees of freedom into global degrees of freedom and thereby establishing the equality of displacements at appropriate adjacent nodes.

Equilibrium - Equilibrium in general is satisfied only approximately within a 3-D isoparametric element, because of its variable stress state. Stresses are also not necessarily in equilibrium between adjacent elements, although all stress equilibrium is satisfied in the limit as the finite-element mesh is refined. For any mesh representation of the structure, global equilibrium is satisfied in BOPACE in an average sense, because equilibrium is established between the generalized nodal forces defined according to the usual finite-element procedure.

#### 4.2 COMPARISON OF COMMON SOLUTION METHODS

The common stiffness methods used for solution of elasto-plastic problems can be classified by three general types:

- 1) The pure "tangent stiffness" method
- 2) The "constant-stiffness residual-load" method
- 3) "Combined" methods

Tangent-Stiffness Method - The pure tangent-stiffness method obtains the solution for each load increment by a single solution of the incremental equilibrium equation:

$$\Delta P_i = K_{ij}^J \Delta Q_j \quad (4.2-1)$$

in which  $\Delta P$  and  $\Delta Q$  are the global incremental forces and displacements, respectively, and  $K^J$  is the Jacobian (tangent-stiffness) matrix. This is the type of solution used in NASTRAN's "piecewise linear analysis," for example. There is no equilibrium check, and no iteration is performed to improve the incremental solution. The matrix  $K^J$  is determined by evaluation or extrapolation at previous solution points. Because in an actual structure the stress-strain slopes, creep rates, direction of the incremental plastic and creep strain vectors, etc., will generally vary within an increment, the pure tangent-stiffness approach can result in a substantial departure from the true force-displacement path unless load increments are kept quite small.

Constant-Stiffness Residual-Load Method - This solution method [9] employs an iterative procedure. In each iteration the residual (unbalanced) forces are computed based on the current estimate for the incremental configuration, and are then applied to the constant elastic stiffness matrix in order to solve for displacement corrections. The approach is computationally efficient because it requires the formation and decomposition of only a single stiffness matrix, but it is not directly applicable to highly nonlinear structures because of convergence difficulties.



Combined Methods - Various combined methods have been employed for solution of elasto-plastic problems, for example that described in Reference 12. These involve the use of an equilibrium check through the calculation of unbalanced forces, as well as various procedures for updating the approximate Jacobian matrix.

BOPACE Approach - BOPACE uses a combined approach for solution, with the iterative procedure consisting of two stages:

- 1) Improvement of the solution configuration by using the Jacobian matrix to reduce the residual nodal forces.
- 2) Calculation of residual forces based on the estimated configuration and "exact" constitutive theory.

Several user controlled options are available in BOPACE for updating the Jacobian matrix.

#### 4.3 CALCULATION OF UNBALANCED FORCES

It is assumed for the present discussion that the exact solution configuration is known at the start of a particular load increment. (Actually the BOPACE program takes any unbalanced forces which might remain from the previous increment and adds them to the present load increment, in order to achieve greater accuracy.) For a given iteration within the present increment, i.e., for a given estimate of the incremental

solution, it is necessary to compute the corresponding unbalanced forces. This section summarizes the steps involved in computing these forces, including determination of strains, stresses, and forces. A flowchart for these calculations is given in Figure 4.3-1.

Strains - For the given estimate of incremental global nodal displacements,  $\Delta Q$ , the corresponding element nodal displacements,  $\Delta q$ , are obtained by coordinate transformations at the nodes, involving appropriate direction Cosines. For the BOPACE 3-D program, all element displacements are referred to the basic X-Y-Z Cartesian coordinate system. Strains,  $\Delta \epsilon$ , are then computed at each integration point by using the relations 3.1-8 and 3.2-2:

$$\Delta \epsilon_i = A0_{im} G_{mj} \Delta q_j \quad (4.3-1)$$

The strain  $\Delta \epsilon$  is the total (physical) strain increment at the point:

$$\Delta \epsilon_i = \Delta \epsilon_i^e + \Delta \epsilon_i^p + \Delta \epsilon_i^t + \Delta \epsilon_i^c \quad (4.3-2)$$

The thermal strains,  $\Delta \epsilon^t$ , are determined as described in Section 2.2.

Subtracting these strains from the total strain, gives:

$$\Delta \epsilon_i^{e+p+c} = \Delta \epsilon_i^e + \Delta \epsilon_i^p + \Delta \epsilon_i^c = \Delta \epsilon_i - \Delta \epsilon_i^t \quad (4.3-3)$$

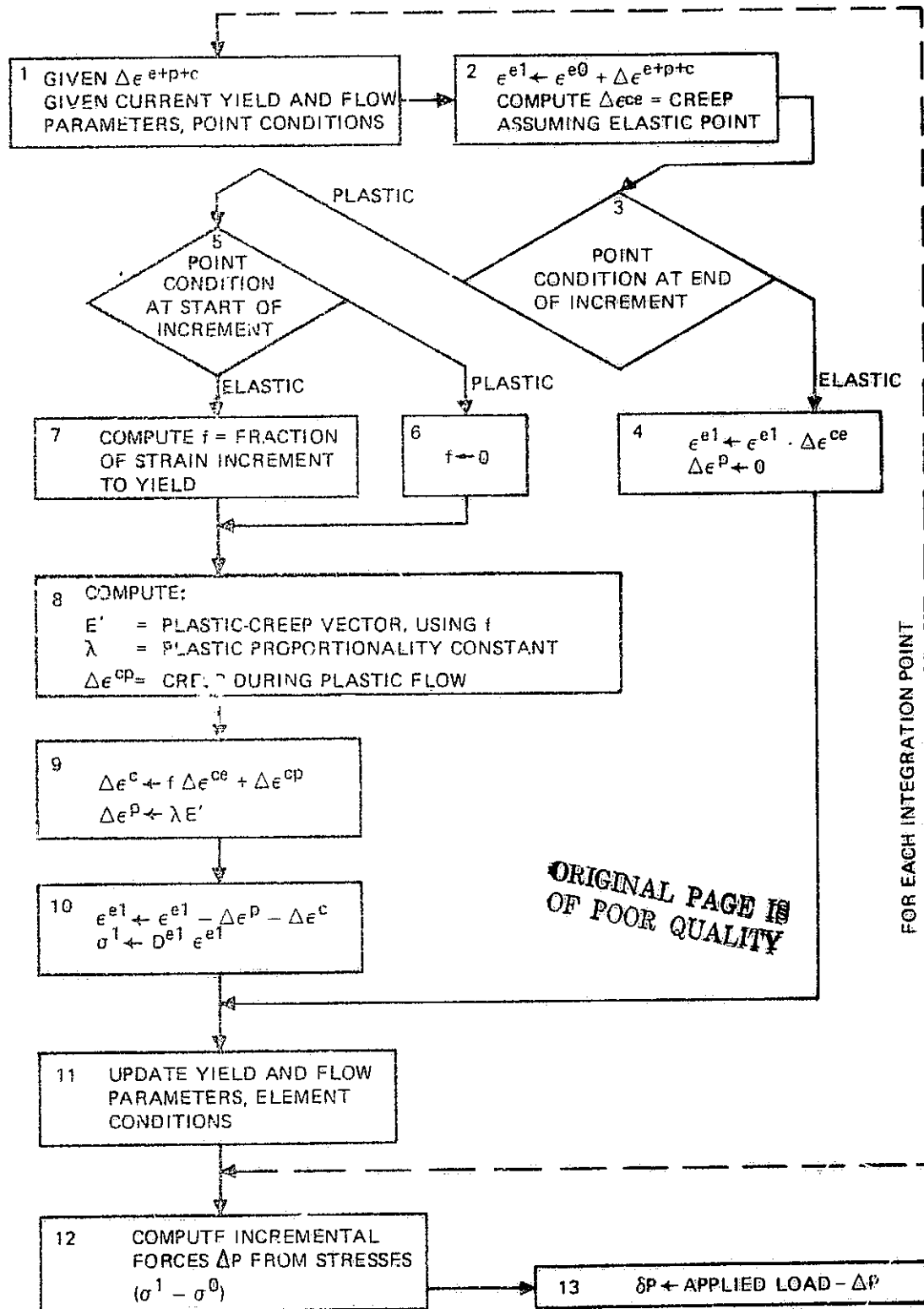


Figure 4.3-1. BOPACE Unbalanced-Force Calculations

Stresses - Elastic strains are then determined using the elasto-plastic-creep algorithm presented in Section 2.6. With the elastic strains known at the end of the increment, the incremental stresses are computed:

$$\Delta\sigma_i = D_{ij}^{e1} \epsilon_i^{e1} - D_{ij}^{e0} \epsilon_i^{e0} \quad (4.3-4)$$

where  $\epsilon^{e0}$  and  $\epsilon^{e1}$  are the known cumulative elastic strains at the beginning and end of the increment, respectively. The calculation of  $\epsilon^{e1}$  may need to be modified, depending on which of three conditions exists at the particular integration point:

- Condition I    Point is elastic at end of load increment, i.e., either the point remains elastic or unloading occurs. Compute stress and elastic strains. Plastic strains are zero.
- Condition II    Point is plastic throughout load increment. Compute stresses, elastic and plastic strains by algorithm of Section 2.6.
- Condition III    Point is initially elastic, but becomes plastic at some point during the load increment. Find intermediate time at which yielding occurs (this requires solving a simple quadratic equation). Compute stresses and elastic strains up to that time. Compute stresses and strains beyond yielding as for Condition II.

The condition at the beginning of the increment is known for each point. The condition at the end of the increment is assumed, for the first iteration, to be Condition I. The end condition is re-evaluated during each iteration, using either the material yield value or the plastic-strain vector. For an elastic point, it is determined whether or not the current material yield has been exceeded. For a plastic point, the plastic strain vector (normal to the yield surface) is observed; an outward vector ( $\lambda > 0$ ) implies a plastic condition, while an inward vector ( $\lambda < 0$ ) implies elastic unloading.

Element Nodal Forces - The force-stress relation for the BOPACE 3-D elements is given by Equations 3.2-5:

$$\Delta p_i = \int_V B_{ai} \Delta \sigma_a dV \quad (4.3-5)$$

where  $V$  is the element volume,  $B$  is the strain-displacement matrix, and  $p_i$  are the element nodal forces.

Global Nodal Unbalanced Forces - Incremental global forces,  $\Delta P$ , are obtained from the incremental element forces, by adding nodal contributions from all elements and applying coordinate transformations. The global unbalanced forces,  $\delta P$ , are then determined by subtracting these computed (internal) incremental forces from the applied (external) incremental loads:

$$\delta P_i = (\Delta \text{load})_i - \Delta P_i \quad (4.3-6)$$

#### 4.4 IMPROVING THE SOLUTION

The basic global relation for incremental forces and displacements corresponds to the element relation 3.2-8:

$$\Delta P_i = K0_{ij}^* \Delta Q_j \quad (4.4-1)$$

where the incremental global displacements  $\Delta Q$  are the total physical displacements (including thermal, elastic, plastic and creep effects).  $K0^*$  is the elasto-plastic tangent stiffness (Jacobian) matrix for the increment.

In order to improve a given displacement configuration, the displacement corrections  $\delta Q$  corresponding to unbalanced forces  $\delta P$ , are obtained in BOPACE by solving a set of linear equations of the form

$$\delta P_i = K_{ij}^J \delta Q_j \quad (4.4-2)$$

The matrix  $K^J$  is also a Jacobian (tangent-stiffness) matrix, or some approximation to the Jacobian, but is used for displacement corrections rather than a one-step solution for the displacements of the entire increment. The purpose of this section is to discuss the procedure for relating Equations 4.4-1 and 4.4-2, and describe BOPACE options for updating the Jacobian.

Procedure - In the iterative BOPACE approach, the only global solution employed is the displacement-correction relation 4.4-2. The best approximation for the Jacobian is

$$K^J = K^{e1} + K^{p1} \quad (4.4-3)$$

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where  $K^{e1}$  and  $K^{p1}$  are evaluated at the end of the current load increment using Equation 3.2-8 with the respective material matrices  $D^{e1}$  and  $D^{p1}$  (see Equation 2.5-6). The effects of change in elastic properties ( $\Delta D^e$  and  $\Delta D^p$ ), as well as the effects of thermal and creep strains, are computed at the integration point level by the algorithm of Section 2.6, and accounted for by the unbalanced forces. Thus Equation 4.4-1 is satisfied in an iterative fashion.

Updating the Jacobian - In order to account for possible large-scale elastic unloading of the structure under cyclic load conditions, one or more initial iterations are performed for each load increment using only the elastic portion,  $K^{e1}$ , of the  $K^J$  matrix. Succeeding iterations use the total  $K^J$  matrix.

Initially the  $K^J$  matrix is taken to be the usual elastic stiffness matrix for the structure, with elastic properties evaluated at the fabrication temperature. Whenever convergence is not achieved within a specified number of iterations, the Jacobian matrix is updated. BOPACE allows five options for updating the matrix  $K^J$  and/or its component matrices  $K^{e1}$  and  $K^{p1}$ :

- 1) Use only elastic matrix  $K^{el}$  with no updating. This option corresponds to the constant-stiffness residual-load method, and is most effective for problems with small plastic strains and elastic properties which do not vary much with temperature.
- 2) Update only  $K^{el}$ . This option is best for problems with small plastic strains and elastic properties which vary considerably with temperature.
- 3) Update only  $K^{pl}$ . This option is best for problems with large plastic strains and elastic properties which do not vary much with temperature.
- 4) Update total  $K^J$  matrix, but not its component matrices. This option may be used for problems with large plastic strains and elastic properties which vary somewhat with temperature.
- 5) Update both  $K^J$ , and  $K^{el}$  and  $K^{pl}$  matrices. This is the most effective option for problems with large plastic strains and elastic properties which vary considerably with temperature.



#### 4.5 SUMMARY OF BOPACE SOLUTION METHOD

An outline of the BOPACE solution method is given in the flowchart of Figure 4.5-1. In step 1, the Jacobian is initialized to the elastic stiffness matrix, based on elastic properties at the fabrication temperature.

At the start of each load increment (step 2) the residual forces  $\delta P$  are set equal to the increment of applied loads. Also, if any residual forces remain from the previous load increment, these are added to  $\delta P$ . The estimate for incremental displacements,  $\Delta Q$ , is set to zero.

The iteration loop involves successive improvement of the solution, by solving for displacement corrections using the unbalanced forces and the Jacobian, and then recomputing the unbalanced forces corresponding to the new displacement configuration. The displacement corrections  $\delta Q$  are determined in step 3, and in step 4 the improved incremental configuration  $\Delta Q$  is updated by addition of  $\delta Q$ . Although convergence of this iterative process is usually quite good, BOPACE has a feature for modifying the process if convergence is not occurring. This involves using only a specified fraction of the computed correction, e.g.,  $\Delta Q \leftarrow \Delta Q + 0.5 \delta Q$ . This would increase the numerical stability but could tend to slow down convergence.

In step 5 the strain-displacement relations are used to compute the total incremental strains  $\Delta \epsilon$  from displacements  $\Delta Q$ . In step 6 the thermal strains are subtracted from total strains to give the elastic+plastic+creep strains required for the calculation of stresses. Step 7

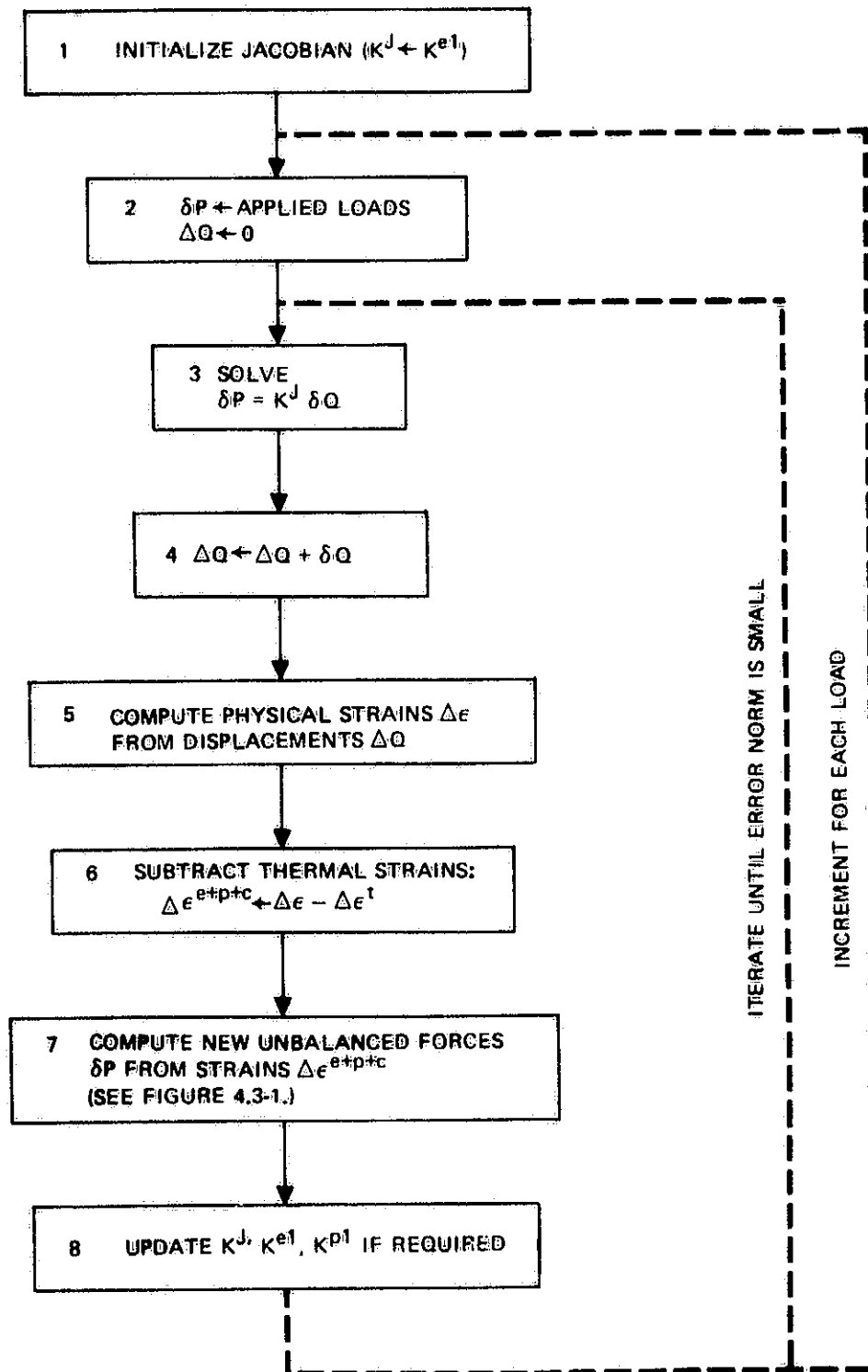


Figure 4.5-1. BOPACE Solution

involves the major iteration algorithm, in which the strain is separated into elastic, plastic and creep components. Stresses are determined according to the algorithm of Section 2.6, and the corresponding unbalanced forces are computed.

If the maximum allowable iterations have been exceeded, step 8 is used to update the Jacobian matrices according to the specified updating option. The Jacobian update is based on the current estimates of the yield surface and flow parameters for each integration point at the end of the present increment. Iteration is stopped when a residual error norm (determined by a ratio of residual forces to applied forces) is sufficiently small.

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## 5.0 LINEAR EQUATIONS

### 5.1 LINEAR EQUATION FORMATION

Single Element - Figure 5.1-1 shows a linearly elastic spring. It is defined by its nodal points 1 and 2, and its stiffness coefficient,  $k$  (force/displacement). Forces  $F_x$  and displacements  $u$  are assigned to the nodes as shown in the figure. It is then desired to determine the stiffness matrix for this element. To do this consider all possible displacement modes which the spring is capable of experiencing.

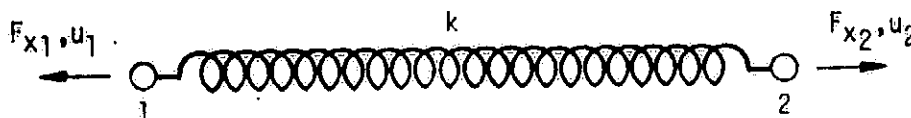


Figure 5.1-1: ELASTIC SPRING AS A FINITE ELEMENT

For example,

a.  $u_1 = 0, u_2 \neq 0$

For this case it is clear that  $F_{x2}$  must be an applied load, while  $F_{x1}$  is a reaction. Also,  $u_2$  is an unknown displacement. From the definition of  $k$ ,

$$k = \frac{F_{x2}}{u_2} \quad \text{or} \quad F_{x2} = ku_2$$

Equilibrium of forces gives,

$$F_{x1} = -F_{x2}$$

b.  $u_1 \neq 0, u_2 = 0$

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This is the second (and final) displacement state which the spring element can undergo. Conditions are now reversed from those which applied in the first case; that is,  $Fx_1$  is now an applied load and  $u_1$  are unknown displacement. The applicable equations are therefore,

$$k = \frac{Fx_1}{u_1} \quad \text{or} \quad Fx_1 = ku_1$$

and

$$Fx_2 = -Fx_1$$

Now assume the superposition of the individual displacement modes. Then  $u_1 \neq 0$  and  $u_2 \neq 0$ . From cases (a) and (b) above it then follows that,

$$Fx_1 = ku_1 - ku_2$$

$$Fx_2 = -ku_1 + ku_2$$

or in matrix form,

$$\begin{Bmatrix} Fx_1 \\ Fx_2 \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad (5.1-1a)$$

or simply,

$$\{F\} = [K] \{u\} \quad (5.1-1b)$$

In this equation  $[K]$  represents the element stiffness matrix. It is seen to relate nodal displacements and corresponding nodal forces. The most important characteristics of a finite element are represented by its stiffness matrix.

Assemblage of Spring Elements - Consider now an assemblage of spring elements as shown in Figure 5.1-2. Nodes are given as 1, 2, 3, and 4. The elements are connected to each other at the nodes. Each spring has a different stiffness as designated by the constants  $k_a$ ,  $k_b$ , and  $k_c$ . It is then desired to find the stiffness equation for this assemblage.

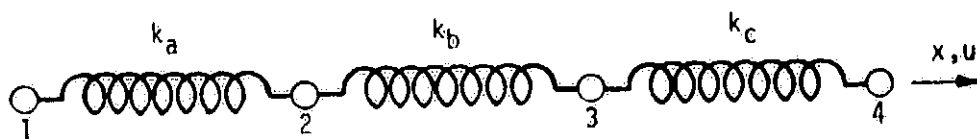


Figure 5.1-2: ASSEMBLAGE OF FINITE SPRING ELEMENTS

Nodal forces are  $Fx_i$ ,  $i = 1, 2, 3, 4$  and the corresponding displacements are simply  $u_i$ . All possible displacement configurations for this system can be obtained by superimposing the four basic states. These occur as each node is displaced in turn, the other nodes in the meantime being held fixed. As in the case of the single element these displacement states can be imposed on the assemblage. The relevant force-displacement expressions can then be written for each case as follows:

a.  $u_1 \neq 0, u_i = 0, i = 2, 3, 4$

Only spring 1-2 is strained.  $Fx_1$  can be considered an applied load and  $Fx_2$  a reaction. No forces are carried through to nodes 3 and 4. Hence,

$$k_a = \frac{Fx_1}{u_1} \quad \text{or} \quad Fx_1 = k_a u_1$$

and by equilibrium,

$$Fx_2 = -Fx_1 = -k_a u_1$$

b.  $u_2 \neq 0, u_i = 0, i = 1, 3, 4$

Both springs 1-2 and 2-3 are now strained.  $Fx_2$  is the applied load. Reactions occur at nodes 1 and 3. No load carries through to node 4. Let,

$$Fx_2 = (Fx_2)_a + (Fx_2)_b$$

so that,

$$k_a = \frac{(Fx_2)_a}{u_2}, \quad k_b = \frac{(Fx_2)_b}{u_2}$$

then,

$$Fx_2 = (k_a + k_b) u_2$$

while,

$$Fx_1 = -(Fx_2)_a = -k_a u_2$$

$$Fx_3 = -(Fx_2)_b = -k_b u_2$$

c.  $u_3 \neq 0, u_i = 0, i = 1, 2, 4$

This is similar to case (b). By a similar calculation it follows that,

$$Fx_3 = (k_b + k_c) u_3$$

$$Fx_2 = -k_b u_3 \text{ and } Fx_4 = -k_c u_3$$

d.  $u_4 \neq 0, u_i = 0, i = 1, 2, 3$

This is similar to case (a). Hence,

$$Fx_4 = k_c u_4 \text{ and } Fx_3 = -k_c u_4$$

e.  $u_i \neq 0, i = 1, 2, 3, 4$

This is the general case. It is obtained by superimposing the first four cases. Collecting terms from the previous force-displacement equations yields,

$$Fx_1 = k_a u_1 - k_a u_2$$

$$Fx_2 = -k_a u_1 + (k_a + k_b) u_2 - k_b u_3$$

$$Fx_3 = -k_b u_2 + (k_b + k_c) u_3 - k_c u_4$$

$$Fx_4 = -k_c u_3 + k_c u_4$$

In matrix form,

$$\begin{Bmatrix} Fx_1 \\ Fx_2 \\ Fx_3 \\ Fx_4 \end{Bmatrix} = \begin{bmatrix} k_a & -k_a & 0 & 0 \\ -k_a & (k_a + k_b) & -k_b & 0 \\ 0 & -k_b & (k_b + k_c) & -k_c \\ 0 & 0 & -k_c & k_c \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \quad (5.1-2a)$$

or again,

$$\{F\} = [K] \{u\} \quad (5.1-2b)$$



The assemblage stiffness matrix is shown in Equation 5.1-2a. Note that its order (4 x 4) is governed by the number of distinct nodal displacements which the assemblage can experience. Some other important features about [K] are as follows: (a) it is a square, symmetric matrix; (b) the stiffness matrix for an assemblage is only sparsely populated with non-zero elements and these can be banded along the main diagonal; (c) the sum of elements in any column (or row) is zero; (d) the stiffness matrix is singular or the determinant of [K] vanishes. These properties are important and except for (c) are generally true for any problem\*. They will be referred to from time to time in this document.

A useful form for writing a stiffness matrix is shown in Equation 5.1-3. The matrix is the same as previously given in Equation 5.1-2a.

$$[K] = \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} \\ \begin{bmatrix} k_a & -k_a & 0 & 0 \\ -k_a & (k_a + k_b) & -k_b & 0 \\ 0 & -k_b & (k_b + k_c) & -k_c \\ 0 & 0 & -k_c & k_c \end{bmatrix} \end{array} \quad (5.1-3)$$

Note that the column of displacements {u} in Equation 5.1-2a has simply been turned into a horizontal position and written above the columns making up [K]. Actually Equation 5.1-3 is sufficient for defining the total stiffness equation since the order of terms in the force column {F} must agree with that given for {u}. See Equation 5.1-2a.

\* Vector sum of "forces" are zero; however moment terms require products of moment arms and forces in the moment equilibrium equations.

Fortunately a simple, direct procedure exists for forming the assemblage stiffness matrix. In other words it is not necessary to use the procedure just described for the spring assemblage. Instead the total assemblage stiffness matrix can be formed directly from the stiffness matrices for the individual elements.

Assemblage Stiffness by Superimposing Element Stiffnesses - It can be shown rigorously and is here demonstrated for the assemblage of Figure 5.1-2 that superimposing the stiffnesses of the component elements will yield the stiffness matrix for the assemblage. It is this process which characterizes the direct stiffness method.

Prior to superposition a simple preliminary step is necessary; namely, to increase the order of each element stiffness matrix to that applying to  $[K]$  for the total assemblage. This is done by simply adding columns of zeros (plus corresponding rows) for each displacement inapplicable to the given element. For example, for spring 1-2 of Figure 5.1-2, displacements  $u_3$  and  $u_4$  are irrelevant; hence,

$$[K_a] = \begin{array}{c} \begin{array}{cc} u_1 & u_2 \end{array} \\ \begin{array}{cc} \hline k_a & -k_a \\ -k_a & k_a \\ \hline 0 & 0 \\ 0 & 0 \end{array} \end{array} \begin{array}{cc} u_3 & u_4 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array} \end{array}$$

The columns and rows of zeros are seen to apply to  $u_3$  and  $u_4$ . Likewise for the other two springs,

$$[K_b] = \begin{array}{c} \begin{array}{cc|cc} u_1 & u_2 & u_3 & u_4 \\ \hline 0 & 0 & 0 & 0 \\ 0 & k_b & -k_b & 0 \\ \hline 0 & -k_b & k_b & 0 \\ 0 & 0 & 0 & 0 \end{array} \end{array} \quad [K_c] = \begin{array}{c} \begin{array}{cc|cc} u_1 & u_2 & u_3 & u_4 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & k_c & -k_c \\ 0 & 0 & -k_c & k_c \end{array} \end{array}$$

Summing the element stiffness matrices is now seen to lead directly to  $[K]$  for the assemblage as given by Equation 5.1-2a or Equation 5.1-3. In carrying out this superposition the  $2 \times 2$  non-vanishing parts of the element stiffness matrices are seen to lie along the main diagonal as illustrated by Equation 5.1-4. As a result the tendency of the gross stiffness matrix to be sparsely populated by non-zero elements, which are banded along the main diagonal, can be appreciated. This fact is important in the numerical calculations which lead to the solution for the unknown displacements, particularly for large problems.

$$[K] = \begin{array}{c} \begin{array}{cc|cc} u_1 & u_2 & u_3 & u_4 \\ \hline \begin{array}{cc} K_a & \\ \hline & K_b \end{array} & & & \\ \hline & & \begin{array}{cc} & \\ \hline & K_c \end{array} & \end{array} \end{array} \quad (5.1-4)$$

Shaded areas in Equation 5.1-4 represent regions where element stiffnesses overlap and hence jointly contribute to the gross stiffness. Non-zero terms lie inside the three boxes labeled  $K_a$ ,  $K_b$ , and  $K_c$  respectively.

Forming the assemblage stiffness matrix may be regarded as the key to the stiffness method of solution. The procedure as illustrated above for carrying out this step is well suited to machine operation. The computer determines the individual element stiffness matrices and then combines these to form the gross stiffness matrix.

Special Equation Generation - Figure 5.1-3 shows two triangular plates connected at a single node.

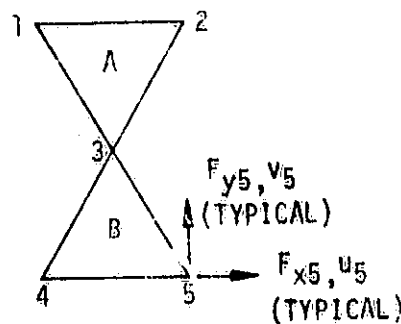


Figure 5.1-3: TWO PLATE ASSEMBLAGE

The two plates are defined by their nodal points 1, 2, and 3, and 3, 4, and 5. Forces  $F_x$  and  $F_y$ , and displacements  $u$  and  $v$ , are assigned to each node as shown for node 5 in the figure. Each plate has a 6 x 6 stiffness matrix as shown in Equation 5.1-5.

$$\begin{Bmatrix} F_{x_1} \\ F_{y_1} \\ F_{x_2} \\ F_{y_2} \\ F_{x_3} \\ F_{y_3} \end{Bmatrix} = \begin{bmatrix} a_{k_{11}}^{xx} & a_{k_{11}}^{xy} & a_{k_{12}}^{xy} & a_{k_{12}}^{xy} & a_{k_{13}}^{xx} & a_{k_{13}}^{xy} \\ & a_{k_{11}}^{yy} & a_{k_{12}}^{yx} & a_{k_{12}}^{yy} & a_{k_{13}}^{yx} & a_{k_{13}}^{yy} \\ & & a_{k_{22}}^{xx} & a_{k_{22}}^{xy} & a_{k_{23}}^{xx} & a_{k_{23}}^{xy} \\ & & & a_{k_{22}}^{yy} & a_{k_{23}}^{yy} & a_{k_{23}}^{yy} \\ & (SYM) & & & a_{k_{33}}^{xx} & a_{k_{33}}^{xy} \\ & & & & & a_{k_{33}}^{yy} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \quad (5.1-5a)$$

$$\begin{Bmatrix} F_{x_3} \\ F_{y_3} \\ F_{x_4} \\ F_{y_4} \\ F_{x_5} \\ F_{y_5} \end{Bmatrix} = \begin{bmatrix} b_{k_{33}}^{xx} & b_{k_{33}}^{xy} & b_{k_{34}}^{xx} & b_{k_{34}}^{xy} & b_{k_{35}}^{xx} & b_{k_{35}}^{xy} \\ & b_{k_{33}}^{yy} & b_{k_{34}}^{yx} & b_{k_{34}}^{yy} & b_{k_{35}}^{yx} & b_{k_{35}}^{yy} \\ & & b_{k_{44}}^{xx} & b_{k_{44}}^{xy} & b_{k_{45}}^{xx} & b_{k_{45}}^{xy} \\ & (SYM) & & b_{k_{44}}^{yy} & b_{k_{45}}^{yx} & b_{k_{45}}^{yy} \\ & & & & b_{k_{55}}^{xx} & b_{k_{55}}^{xy} \\ & & & & & b_{k_{55}}^{yy} \end{bmatrix} \begin{Bmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{Bmatrix} \quad (5.1-5b)$$

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$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \\ F_{x4} \\ F_{y4} \\ F_{x5} \\ F_{y5} \end{Bmatrix} = \begin{bmatrix} a_{k11}^{xx} & a_{k11}^{xy} & a_{k12}^{xx} & a_{k12}^{xy} & a_{k13}^{xx} & a_{k13}^{xy} & 0 & 0 & 0 & 0 \\ & a_{k11}^{yy} & a_{k12}^{yx} & a_{k12}^{yy} & a_{k13}^{yx} & a_{k13}^{yy} & 0 & 0 & 0 & 0 \\ & & a_{k22}^{xx} & a_{k12}^{xy} & a_{k23}^{xx} & a_{k23}^{xy} & 0 & 0 & 0 & 0 \\ & & & a_{k12}^{yy} & a_{k23}^{yx} & a_{k23}^{yy} & 0 & 0 & 0 & 0 \\ & & & & a_{k33}^{xx} + b_{k33}^{xx} & a_{k33}^{xy} + b_{k33}^{xy} & b_{k34}^{xx} & b_{k34}^{xy} & b_{k35}^{xx} & b_{k35}^{xy} \\ & & & & & a_{k33}^{yy} + b_{k33}^{yy} & b_{k34}^{yx} & b_{k34}^{yy} & b_{k35}^{yx} & b_{k35}^{yy} \\ & & & & & & b_{k44}^{xx} & b_{k44}^{xy} & b_{k45}^{xx} & b_{k45}^{xy} \\ & & & & & & & b_{k44}^{yy} & b_{k45}^{yx} & b_{k45}^{yy} \\ & & & & & & & & b_{k55}^{xx} & b_{k55}^{xy} \\ & & & & & & & & & b_{k55}^{44} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{Bmatrix} \quad (5.1-6)$$

(SYM)

$$\begin{Bmatrix} Fx_1 \\ Fy_1 \\ Fx_2 \\ Fy_2 \\ a_{Fx_3} \\ Fy_3 \\ b_{Fx_3} \\ Fx_4 \\ Fy_4 \\ Fx_5 \\ Fy_5 \end{Bmatrix} = \begin{bmatrix} a_{k11}^{xx} & a_{k11}^{xy} & a_{k12}^{xx} & a_{k12}^{xy} & a_{k13}^{xx} & a_{k13}^{xy} & 0 & 0 & 0 & 0 & 0 \\ & a_{k11}^{yy} & a_{k12}^{yx} & a_{k12}^{yy} & a_{k13}^{yx} & a_{k13}^{yy} & 0 & 0 & 0 & 0 & 0 \\ & & a_{k22}^{xx} & a_{k22}^{xy} & a_{k23}^{xx} & a_{k23}^{xy} & 0 & 0 & 0 & 0 & 0 \\ & & & a_{k22}^{yy} & a_{k23}^{yx} & a_{k23}^{yy} & 0 & 0 & 0 & 0 & 0 \\ & & & & a_{k23}^{xx} & a_{k23}^{xy} & 0 & 0 & 0 & 0 & 0 \\ & & & & & a_{k33}^{yy} + b_{k33}^{yy} & b_{k33}^{yx} & b_{k34}^{xx} & b_{k34}^{yy} & b_{k35}^{yx} & b_{k35}^{yy} \\ & & & & & & b_{k33}^{xx} & b_{k34}^{xx} & b_{k34}^{xy} & b_{k35}^{xx} & b_{k35}^{xy} \\ & & & & & & & b_{k44}^{xx} & b_{k44}^{xy} & b_{k45}^{xx} & b_{k45}^{xy} \\ & & & & & & & & b_{k44}^{yy} & b_{k45}^{yx} & b_{k45}^{yy} \\ & & & & & & & & & b_{k55}^{xx} & b_{k55}^{xy} \\ & & & & & & & & & & b_{k55}^{yy} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ a_{u_3} \\ v_3 \\ b_{u_3} \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{Bmatrix} \quad (5.1-7)$$

(SYM)

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Merging these matrices using superposition as described in the preceeding section yields a 10 x 10 matrix for the entire assemblage as shown in Equation 5.1-6. Note that the displacements  $u_3$  and  $v_3$  are constrained to be equal for the two elements.

Structural discontinuity is idealized by allowing one or more of the displacements at a connecting node to be unconstrained in the displacement of the attached elements, while the remaining displacements of the node are constrained to be equal.

The equation generation procedure is modified to handle this condition. In the problem of Figure 5.1-3 assume that the  $v$  displacements at node 3 of the two elements are constrained to be equal, and the  $u$  displacements are constrained. Merging the elemental stiffness matrices of Equations 5.1-5a and 5.1-5b for this condition yields an 11 x 11 matrix for the entire assemblage as shown in Equation 5.1-7.

## 5.2 SOLUTION OF EQUATIONS

Introduction - The stiffness equations can be written in matrix form as shown in Equation 5.2-1:

$$\{F\} = [K] \{Q\} \quad (5.2-1)$$

where  $\{F\}$  is the force vector

$[K]$  is the stiffness matrix

$\{Q\}$  is the displacement vector



The solution of Equation 5.2-1 generally involves a mixed set of known and unknown forces and displacements. Traditionally the approach has been to reorder the equations collecting the known and unknown forces and displacements such that Equation 5.2-1 is rewritten in partitioned matrix form as shown in Equation 5.2-2:

$$\begin{Bmatrix} F_k \\ F_u \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} Q_u \\ Q_k \end{Bmatrix} \quad (5.2-2)$$

where the subscripts are

k = known

u = unknown

While this approach is efficient and yields accurate solution, the re-ordering necessary is a tedious detail which should be bypassed when possible. A somewhat different approach is the wavefront method, which is efficient and as accurate as any other method. The approach is to decompose the  $n \times n$  stiffness matrix  $[K]$ , then solve for the unknown forces  $\{F_u\}$  and displacements  $\{Q_u\}$  by a forward and backward substitution.

The solution steps are first shown here for a system of equations involving only known forces and unknown displacements. Then the modifications are described for the general case of mixed known and unknown forces and displacements.

Solution with Known Forces - The decomposed form of Equation 5.2-1 takes the form

$$[U]^T [D]^{-1} [U] (Q_U) = (F_K) \quad (5.2-3)$$

where  $[U]$  is an upper triangular matrix, and  $[D]$  is a diagonal matrix whose elements are the diagonal elements of  $[U]$ . The solution is in three steps.

### 1. Decomposition

The elements of  $K$  are

$$K_{ij} = \sum_{k=1}^{i-1} U_{ki} D_{kk}^{-1} U_{kj} + U_{ii} D_{ii}^{-1} U_{ij} \quad (5.2-4)$$

Since  $D_{ii} = U_{ii}$ , the elements of  $U$  are obtained successively by row, as

$$U_{ij} = K_{ij} - \sum_{k=1}^{i-1} D_{kk}^{-1} U_{ki} U_{kj} \quad (5.2-5)$$

### 2. Forward Substitution

$$\text{Let } Y = D^{-1} U Q$$

then

$$F_i = \sum_{k=1}^{i-1} U_{ki} Y_k + U_{ii} Y_i \quad (5.2-6)$$

giving

$$Y_i = D_{ii}^{-1} (F_i - \sum_{k=1}^{i-1} U_{ki} Y_k) \quad (5.2-7)$$

### 3. Backward Substitution

$$Y_i = \sum_{k=i+1}^n D_{ik}^{-1} U_{ik} Q_k + D_{ii}^{-1} U_{ii} Q_i \quad (5.2-8)$$

giving

$$\bar{Q}_i = Y_i - D_{ii}^{-1} \sum_{k=i+1}^n U_{ik} Q_k \quad (5.2-9)$$

Solution of Mixed Set - For the general case where there is a mixed set of known and unknown forces and displacements, the three solution steps must be modified. The procedure as described is for the case in which a single displacement (the  $r^{\text{th}}$  displacement) is known, with the corresponding  $r^{\text{th}}$  force unknown. Additional known displacements with corresponding unknown forces would be treated in the same manner.

The modified form of the decomposed matrix is given by Equation 5.2-3:

$$\begin{bmatrix} [U_{11}] & [U_{1r}] & [U_{1n}] \\ 0 & [U_{rr}] & [U_{rn}] \\ 0 & 0 & [U_{nn}] \end{bmatrix}^T \begin{bmatrix} [D_{11}]^{-1} & & \\ & [1] & \\ & & [D_{nn}]^{-1} \end{bmatrix} \begin{bmatrix} [U_{11}] & [U_{1r}] & [U_{1n}] \\ 0 & [U_{rr}] & [U_{rn}] \\ 0 & 0 & [U_{nn}] \end{bmatrix} \begin{Bmatrix} (Q_1) \\ (\bar{Q}_r) \\ (Q_n) \end{Bmatrix} = \begin{Bmatrix} (\bar{F}_1) \\ (F_r) \\ (\bar{F}_n) \end{Bmatrix} \quad (5.2-10)$$

where  $\bar{Q}_r$ , and  $\bar{F}_1$  and  $\bar{F}_n$  denote known quantities. The elements of  $U$  are given by equation 5.2-4 except that no contribution from  $[U_{rm}]$  is distributed to the elements of  $[U_{nn}]$ . Detailed steps are:

## 1) Decomposition

First rows: Compute each row (1 to  $r-1$ ) according to Equation 5.2-4 and distribute contributions to later rows.

$r^{\text{th}}$  row: Compute the  $r^{\text{th}}$  row of  $U$  according to Equation 5.2-4 but do not distribute contributions to later rows.

Last rows: Compute each row ( $r+1$  to  $r$ ) according to Equation 5.2-4 and distribute contributions to later rows.

## 2) Forward Substitution

First rows: Compute

$$\{Y_1\} = [U_{11}]^{-1} \{F_1\} \text{ using Equation 5.2-7} \quad (5.2-11)$$

and distribute to  $\{Y_r\}$  and  $\{Y_n\}$ . This produces the vector

$$\{Y\} = \begin{cases} \{Y_1\} \\ \{Y_r\} \\ \{Y_n\} \end{cases} = \begin{cases} - [U_{1r}]^T \{Y_1\} \\ - [U_{1n}]^T \{Y_1\} \end{cases} \quad (5.2-12)$$

$r^{\text{th}}$  row:

The  $r^{\text{th}}$  row of Equation 5.2-3 gives the relationship

$$\begin{pmatrix} [U_{rr}] & [U_{rn}] \end{pmatrix} \begin{pmatrix} \{\bar{Q}_r\} \\ \{Q_n\} \end{pmatrix} = \begin{pmatrix} \{F_r\} - [U_{1r}]^T \{Y_1\} \end{pmatrix} \quad (5.2-13)$$

where  $-[U_{1r}]^T \{Y_1\}$  is from Equation 5.2-11 and replaces  $\{F_r\}$ , and  $\{Y_r\}$  is set to  $\{\bar{Q}_r\}$  and contributions are distributed forward as in Equation 5.2-6.

Last rows:

Continue forward substitution by Equation 5.2-6 to obtain

$$\{Y_2\} = [U_{nn}^T]^{-1} \left\{ \{F_n\} - [U_{rm}]^T \{\bar{Q}_n\} - [U_{ln}]^T \{Y_1\} \right\} \quad (5.2-14)$$

### 3. Backward Substitution

Last rows:

From Equation 5.2-8

$$\{Q_n\} = [U_{22}]^{-1} [D_{22}] \{Y_n\} \quad (5.2-15)$$

rth row:

From Equation 5.2-12

$$\{F_r\} = -[U_{lr}] \{Y_1\} + [U_{rr}] \{\bar{Q}_r\} + [U_{rm}] \{Q_n\} \quad (5.2-16)$$

First rows:

From Equation 5.2-8

$$\{Q_1\} = [U_{11}]^{-1} [D_{11}] \left\{ \{Y_1\} - [U_{1r}] \{\bar{Q}_r\} - [U_{1m}] \{Q_n\} \right\} \quad (5.2-17)$$

## 6.0 DEFINITIONS - THEORETICAL MANUAL

This section defines symbols used in the BOPACE Theoretical Manual. Symbols used in Section 5 (on solution of linear equations) are not included here, because that section uses somewhat different notation and is considered to be self contained.

Variables

|                |   |
|----------------|---|
| a              | Deviatoric stress center                |
| c              | Kinematic hardening slope               |
| e              | Deviatoric strain                       |
| g              | Shape function derivatives matrix       |
| p,q            | Local nodal forces, displacements       |
| r              | Isotropic hardening slope               |
| s              | Deviatoric (total - hydrostatic) stress |
| u,v            | Displacements in x, y directions        |
| x,y            | Cartesian coordinates                   |
| A              | Elasto-plastic hardening parameter      |
| B              | Strain-displacement matrix              |
| C              | Kinematic hardening matrix              |
| D              | Elasticity matrix                       |
| E              | Young's modulus; Global strain          |
| E'             | Basic surface normal vector             |
| F              | Yield surface function                  |
| F <sup>C</sup> | Creep hardening factor                  |

|                    |  |
|--------------------|--|
| $F^k$              | Kinematic hardening factor                                 |
| $G$                | Tensorial shear modulus; shape function derivatives matrix |
| $I$                | Identity matrix  |
| $J$                | Jacobian matrix  |
| $K$                | Stiffness matrix   |
| $P, Q$             | Global nodal forces, displacements                         |
| $R$                | Isotropic hardening matrix                                 |
| $T$                | Temperature  |
| $W$                | Work; Integration point weighting factor                   |
| $\alpha$           | Stress center of yield surface                             |
| $\beta$            | Strain center of yield surface                             |
| $\gamma$           | Thermal coefficient of expansion; Shear strain             |
| $\epsilon$         | Strain   |
| $\theta$           | Displacement derivatives                                   |
| $\kappa$           | Cumulative plastic hardening parameter                     |
| $\kappa^c$         | Creep hardening parameter                                  |
| $\kappa^k$         | Kinematic hardening parameter                              |
| $\lambda$          | Plastic flow parameter                                     |
| $\nu$              | Poisson's ratio  |
| $\sigma$           | Stress   |
| $\xi, \eta, \zeta$ | Parent element Cartesian coordinate system                 |

### Subscripts

$a, b, i, j, k, l, m, n$  General indices

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### Superscripts

|   |                             |
|---|-----------------------------|
| 0 | Start-of-increment quantity |
| 1 | End-of-increment quantity   |
| ° | Known test value            |
| c | Creep quantity              |
| e | Elastic quantity            |
| p | Plastic quantity            |
| t | Thermal quantity            |

### Special Symbols

|                 |  |
|-----------------|--|
| $\delta( )$     | Residual (corrective) quantity; virtual quantity |
| $\Delta( )$     | Incremental quantity                             |
| $( )^*$         | Reference Equilibrium quantity                   |
| $( )^T$         | Matrix transpose                                 |
| $( )^{-1}$      | Matrix inverse                                   |
| $(\bar{\quad})$ | Effective quantity                               |
| $(\sim)$        | Relative deviatoric quantity                     |
| $\Sigma$        | Summation  |



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THE **BOEING** COMPANY

BOPACE 3-D

PART II. USER MANUAL

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## 8.0 SUMMARY OF BOPACE 3-D INPUT DATA

A pictorial of the BOPACE 3-D input deck is shown in Figure 8.0-1. The input data consists of the following three general types:

Type C: Data on the usual card file. These are data which are needed for each start or restart.

Type I: Data on File I. These are basic structural data for a given problem, such as material properties and mesh data. They are the same for all load increments and are needed only when starting.

Type II: Data on File II. These are incremental thermal-load data which are needed for each start or restart.

The data included on each file are described below. Formats are consistent with FORTRAN IV conventions.

C-1. Start-restart code and data file numbers:

- a. "START" if new problem, or "RESTART" if restarting.
- b. If starting give unit number for file I.

# BOPACE INPUT DECK SETUP

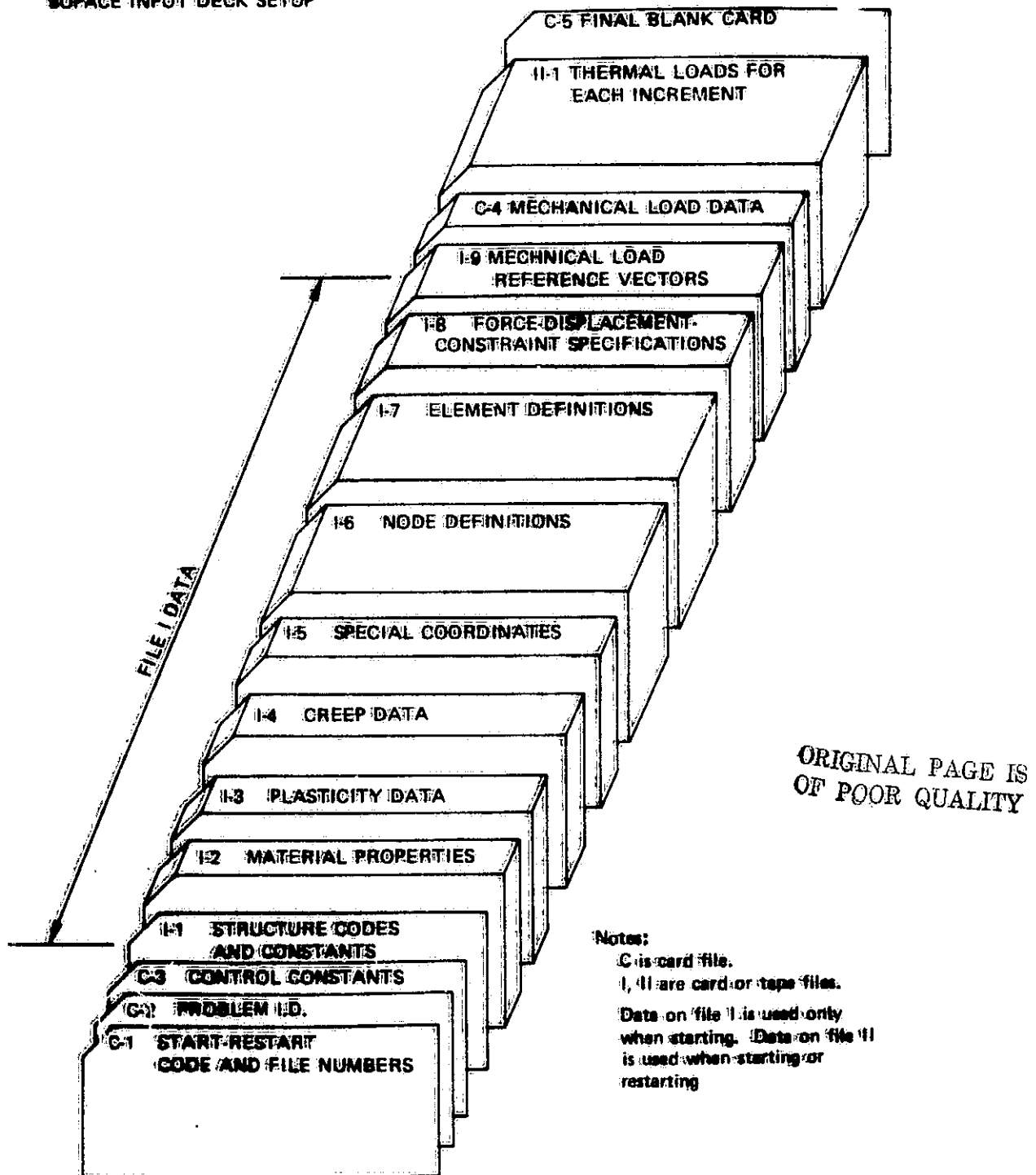


Figure B.0-1. BOPACE 3-D Input Deck Setup

- c. Unit number for file II.
- d. Unit number for output file (e.g. printer).
- e. If restarting give load increment number from the end of which a restart is to be made.
- f. If restarting give input restart-tape unit number.
- g. If data is to be saved for future restart give output restart-tape unit number.

Format (A4,6X,6I5)

C-2. Problem I.D. title.

Format (20A4).

C-3. Program control constants (any constant left blank is assigned a default value):

- a. Code for system matrix decomposition and solution. This code controls only the method of iteration and convergence, and does not affect final computed results.

Code 1 = use only elastic matrix with no updating.

2 = update elastic matrix.

3 = update plastic matrix.

4 = update total Jacobian (elastic+plastic) matrix.

5 = update both elastic and total Jacobian matrices.

The default code is 5.

- b. Maximum number of stiffness updates per load increment (in order to achieve convergence to within allowable residual norm).  
Default = 1.
- c. Maximum number of residual-force iterations per stiffness update. Default = 10.
- d. Maximum number of initial iterations using elastic matrix (to account for possible unloading). Default = 2.
- e. Maximum allowable magnitude for elastic-plastic sum code YCODE1. Default = 2.
- f. Maximum number of cuts to be performed (giving new solution with a fraction of previously used displacement corrections) if residual norm is not decreasing. Default = 0.

- g. Cutting fraction (displacement correction previous correction times cutting fraction). Default = 0.5.
- h. Maximum allowable error norm. Default = 0.001.
- i. Fraction from end of increment to evaluate stress vs. plastic-strain slope in forming plastic stiffness. Default = 0.1.

Format (6I5/3F10.0)

I-1. Number of materials and fabrication temperature.

Format (5X,I5,I0X,F10.0)

I-2. Material property data for each consecutive material.

- a. Material number

Format (I10)

- b. Three consecutive temperaturedependent property curves (thermal strain, elastic modulus, Poisson's ratio). For each curve point give temperature and value, with points in order of increasing temperature. User has option of from 1 to 4 points per card.



Format (8F10.0)

Blank card after last point of each curve.

I-3. Plasticity data for each consecutive material (see Section 2.3).

a. Material number, plasticity type, kinematic code.

Type 1 = strain hardening ( $\kappa$  = sum of increments of effective plastic strain)

2 = work hardening ( $\kappa$  = cumulative plastic work density)

Code 0 = kinematic hardening is function of one parameter  
( $\kappa^k$ )

1 = kinematic hardening is function of two parameters  
( $\kappa^k, \kappa$ )

Format (3I10)

b. Temperature-parameter-hardening data, in order of low to high temperatures. For each temperature:

Material number, temperature.

Format (I10,F10.0)

Three consecutive hardening curves; 1) cumulative hardening parameter  $\kappa$  vs. isotropic hardening = yield surface size (= average of tensile and compressive yield stresses for uniaxial case); 2) kinematic parameter  $\kappa^k$  vs. kinematic hardening shape; 3) cumulative parameter  $\kappa$  vs. kinematic hardening factor. Bauschinger hardening is computed as kinematic hardening shape times hardening factor. If kinematic code = 0, curve of kinematic factors is not given and all factors are taken as 1.0. Bauschinger hardening = yield surface translation  $\alpha$  (= tensile - compressive yield stress for uniaxial tension case). For each curve point give parameter and hardening value. User has option of from 1 to 4 points per card. First point on each curve must be at yield point (parameter 0.0).

Format (8F10.0)

Insert blank card after each input curve.

Blank card after all temperatures for a given material.

I-4. Creep data for each consecutive material (see Section 2.4).

a. Material number, creep type.

Type 1 = age hardening ( $\kappa^c$  = creep time)

2 = strain hardening ( $\kappa^c$  = sum of increments of effective creep strain)

3 = work hardening ( $\kappa^c$  = cumulative creep work density)

Format (2I10)

- b. Reference creep curve. For each point, the time and creep strain, in order of increasing time. User has the option of from 1 to 4 points per card.

Format (8F10.0)

Blank card after last point of curve.

- c. Table of creep factors, in order of low to high temperatures. BOPACE computes creep as creep factor (function of temperature and stress) times reference creep curve.

For each temperature:

Material number, temperature

Format (I10,F10.0)

For each point given at this temperature, the stress and creep factor, in order of increasing stress. User has option of from 1 to 4 points per card.

Format (8F10.0)

Blank card after all points for a given temperature.

Blank card after all temperatures for a given material.

I-5. Definition of each special Cartesian coordinate system. Each coordinate system is defined by giving three points which locate the origin, a point on the x-axis, and a point in the x-y plane. Two options are available to define each coordinate system as follows:

- a) Node points may be used to define the three points. In this case, input:

Option (=1), coordinate system ID (integer > 2), node A, node B, node C.

Format (I1,I9,3I10)

- b) Coordinates locating the points in one of the basic coordinate systems (Cartesian, cylindrical or spherical, as shown in Figure 8.0-2) may be used to define the three points. In this case, input:

Option (=2), coordinate system ID (integer > 2), reference coordinate system indicator (0=Cartesian, 1=cylindrical, or 2=spherical),  $x_A, y_A, z_A, x_B, y_B, z_B, x_C, y_C, z_C$ .

Format (I1,I9,I10,6F10.0/3F10.0)

Blank card after last coordinate system.

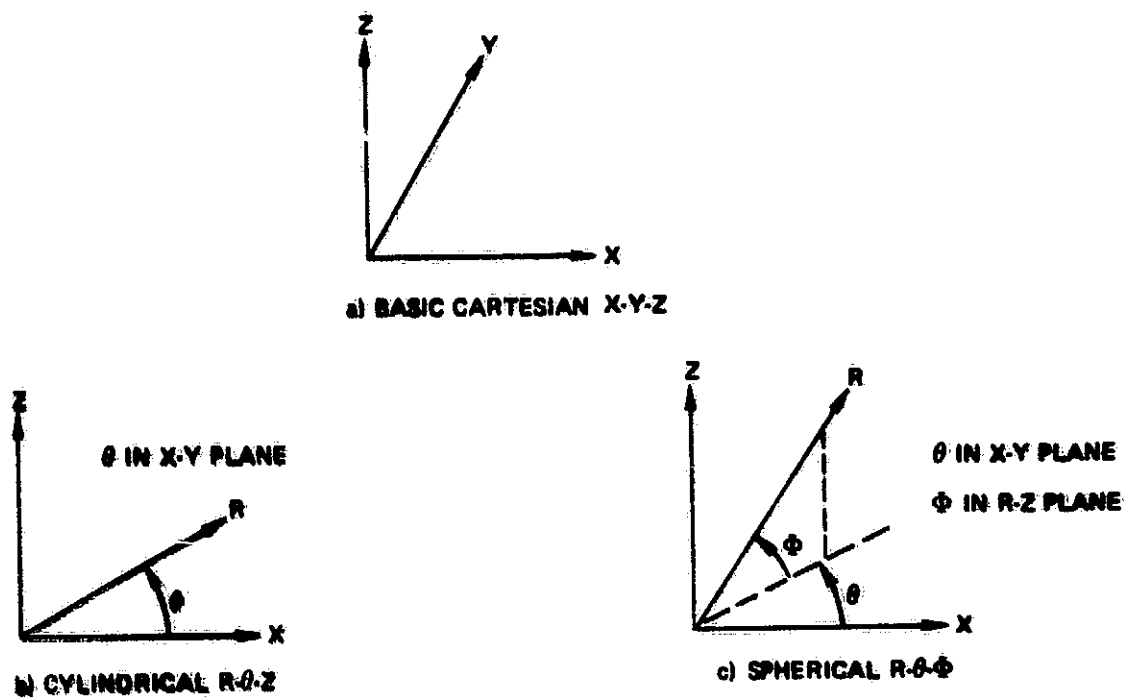


Figure 8.0-2. Coordinate Systems

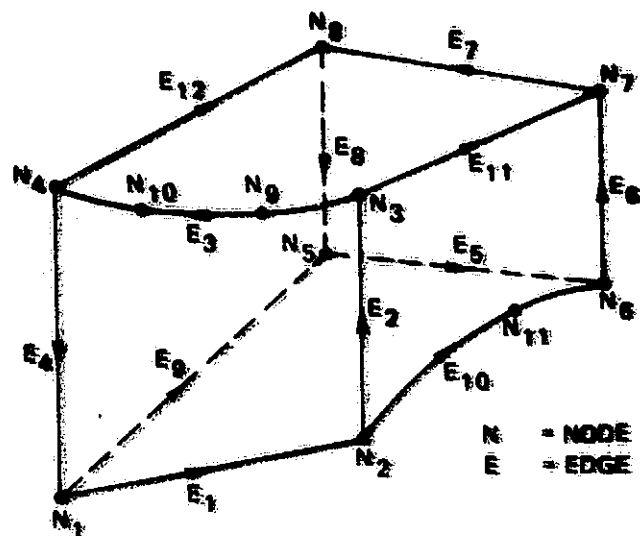


Figure 8.0-3. Isoparametric Element Example

I-6. For each node: Node I.D. number, identification number of coordinate system to define location (= 0, 1 or 2), X-Y-Z (or R- $\theta$ -Z or R- $\theta$ - $\phi$ ), and identification number of coordinate system to define displacements (= 0, 1, 2, or other). (Coordinate identification number 0 specifies the basic Cartesian system, 1 specifies the basic cylindrical system, 2 specifies the basic spherical system, and >2 specifies a special Cartesian system). Order of nodes in data deck is internal order used to form system matrix. (See Section 10 for limits on node ordering and problem size.)

Format (2I5,3F10.0,I5)

Blank card after last node.

I-7. For each element: element I.D. number, material number, the eight corner node numbers of the element in the order shown in Figure 8.0-3, and the maximum number of intermediate nodes along the edges.

Format (11I5)

If intermediate edge nodes are used, they are input next for each edge in the order shown in the figure. The same number of inputs per edge is expected (i.e., the maximum number of nodes per edge as input on the previous card), with zeros used for edges which do not

have the maximum number of intermediate nodes. Up to three intermediate nodes per edge may be used, but the total number of intermediate nodes may not exceed 12.

Format (16I5/16I5/4I5)

For example, the input for the element shown in Figure 8.0-3 would be:

|    |     |                |                 |                |                 |                |                |                |                |   |   |   |   |   |   |   |   |
|----|-----|----------------|-----------------|----------------|-----------------|----------------|----------------|----------------|----------------|---|---|---|---|---|---|---|---|
| ID | IDM | N <sub>1</sub> | N <sub>2</sub>  | N <sub>3</sub> | N <sub>4</sub>  | N <sub>5</sub> | N <sub>6</sub> | N <sub>7</sub> | N <sub>8</sub> | 2 |   |   |   |   |   |   |   |
| 0  | 0   | 0              | 0               | N <sub>9</sub> | N <sub>10</sub> | 0              | 0              | 0              | 0              | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0  | 0   | 0              | N <sub>11</sub> | 0              | 0               | 0              | 0              |                |                |   |   |   |   |   |   |   |   |

Blank card after last element.

I-8. For each degree of freedom with a specified force, displacement or constraint: give node I.D. number, component number (1, 2, or 3) and code. The code to be given is:

1. For specified force, the node I.D. number
2. For specified displacement, the negative of the node I.D. number.

3. For dependent constrained DOF, the node I.D. number of the independent DOF in constraint.

The default code is specified force. User has option of specifying from one to four DOF on each card.

Format (4(3I5,5X))

Blank card after last force-displacement-constraint DOF.

#### I-9. Mechanical load reference vectors

Number of vectors (for current program version must be 2)

Format (I10)

For each non-zero component of load vector: node I.D. number, component number (1,2,3 for X,Y,Z or R, $\theta$ ,Z or R, $\theta$ , $\phi$ ), value. (Each node in the system has three load components.) User has option of from 1 to 4 values per card.

Format (4(2I5,F10.0))

Blank card after last value of each vector.



C-4. Incremental mechanical load data.

Number of load increments

Format (I10)

For each load increment: Maximum iterations per stiffness update (if left blank, value from C-3 is used), the cumulative factors to be applied to load reference vectors (for current version of program two factors must be given), creep time increment (if left blank, no creep calculations are made).

Format (I10,3F10.0)

II-1. Incremental thermal load data.

a. Increment I.D. title

Format (20A4)

b. Nodal thermal loads. For each specified component of load: Node I.D. number, temperature at end of increment. (All nodes are assigned temperature values, from which element integration point temperatures are determined by interpolation using the element shape functions.) Non-specified nodal temperatures

are taken to be the fabrication temperature for the first increment; for later increments they are taken to be the temperature of the preceding increment. User has option of from one to four values per card.

Format (4(I10,F10.0))

Blank card after last specified value of thermal load for each load increment.

C-5. Blank card after last problem.

## 9.0 SUMMARY OF OUTPUT

A discussion of BOPACE 3-D output is conveniently divided into two parts. The first covers output which is primarily an echo check of the input data, and the second covers output results for each load increment.

### 9.1 ECHO CHECK OF INPUT DATA

Initial Output - The first page of BOPACE output for a problem is essentially an echo check of input items C-1, C-2, C-3 and I-1. An indication is given as to whether the problem is being started or restarted. If it is restarted then the previous increment number is given, from the end of which the restart is progressing. Next the problem I.D. title is printed, followed by the various control constants which are determined from their default values or from input values C-3. Finally the input data from I-1 are printed.

Material Properties - For each material the three curves (thermal strain, elastic modulus and Poisson's ratio) input in data item I-2 are printed.

Plasticity Data - For each material the plasticity data input in data item I-3 are printed. These include the plasticity hardening type (1 = strain hardening, 2 = work hardening) and the kinematic hardening code (0, 1 for one, two parameter hardening, respectively). Following are groups of two or three curves given at each temperature (surface size ( $\sigma - \alpha$ ) vs. cumulative hardening parameter  $\kappa$ , kinematic hardening curve shape vs. kinematic shape parameter  $\kappa^k$ , and kinematic curve magnitude vs.  $\kappa$  if given).

The hardening curves are input and output by order of associated low to high temperature. Abscissas of the curves in the first (lowest temperature) group are used as the basis for tabulating all curves, i.e., curve points of higher temperatures are interpolated to the low-temperature abscissas. These interpolated hardening curves are printed following printing of the input hardening curves.

Creep Data - For each material the creep data input in data item I-4 are printed. These include the creep hardening type (1, 2, 3 for age, strain, work hardening, respectively), and the reference creep curve shape of creep strain vs. time. Following are the creep factors for each combination of temperature and stress, grouped by low to high temperature. Stress values in the first (lowest temperature) group are used as the basis for tabulating all groups, i.e., creep factors at higher temperatures are interpolated to the low-temperature stress values. The interpolated hardening factors are printed following printing of the input hardening factors.

Special Coordinate Systems - These are the user-defined direction (special Cartesian) systems of input data item I-5. Quantities printed are the system I.D. number, definition option, and definition quantities. The definition quantities consist of either three node numbers, or a coordinate system indicator and coordinates of three points.

Node Definitions - The information given in input item I-6 is printed.

Values are the node number, node I.D., location coordinate system number (0 = basic Cartesian, 1 = basic cylindrical, 2 = basic spherical), the location coordinates (X-Y-Z, R- $\theta$ -Z, or R- $\theta$ - $\phi$ ), and the direction coordinate system number (0 = basic Cartesian, 1 = basic cylindrical, 2 = basic spherical, >2 = I.D. of special system).

Element Definitions - The information given in input item I-7 is printed.

Values are the element number, element I.D., material number, corner nodes, computed element straight-line volume, and intermediate edge nodes. After all elements are printed, the accumulated sum of all element straight-line volumes is given. The straight-line volumes serve as a check for erroneous node locations, missing elements, etc. The volumes are computed assuming straight edges between corner nodes. This introduces error into the accumulated sum of volumes only if curved edges occur on a boundary (i.e. edges not surrounded by elements).

Force-Displacement-Constraint Prescriptions - These are the codes given in input data item I-8. Quantities printed are the node I.D., the component number, and code for each degree of freedom with a user-specified force, displacement or constraint.

Mechanical Load Reference Vectors - For each input component of the two load vectors from input item I-9, the node I.D., component number, and load are printed.

Element Integration-Point Coordinates - This data is computed and printed for each element during the first stiffness generation and merging operation. Values given are the element I.D., location coordinate system number (always 0 = basic), and the three actual coordinates for each integration point.

Incremental Mechanical Load Data - Quantities related to input data item C-4 are printed. First is printed the number of load increments to be run. Then for each increment is given the increment number, input or default value for maximum number of iterations per increment, factors to be applied to the two load reference vectors, and the creep time increment. The increment numbers are consecutive numbers beginning at 1 (i.e. they refer to increments of the current run, even though it may be a restart based on several previous load increments).

## 9.2 RESULTS FOR EACH LOAD INCREMENT

Thermal Loads - The cumulative values of thermal load for each node, given in input item II-1, are printed. A heading gives the increment number, and is followed by the input I.D. title for the increment. Then the node I.D. and cumulative temperature are printed, by groups of ten nodes.

Residual Norm Values - A residual norm computed at the end of each iteration is printed. The residual norm is obtained by a ratio of unbalanced (residual) forces to "total" forces.

system number (always 0 = basic), and the three actual coordinates for each integration point.

Incremental Mechanical Load Data - Quantities related to input data item C-4 are printed. First is printed the number of load increments to be run. Then for each increment is given the increment number, input or default value for maximum number of iterations per increment, factors to be applied to the two load reference vectors, and the creep time increment. The increment numbers are consecutive numbers beginning at 1 (i.e. they refer to increments of the current run, even though it may be a restart based on several previous load increments).

## 9.2 RESULTS FOR EACH LOAD INCREMENT

Thermal Loads - The cumulative values of thermal load for each node, given in input item II-1, are printed. A heading gives the increment number, and is followed by the input I.D. title for the increment. Then the node I.D. and cumulative temperature are printed, by groups of ten nodes.

Residual Norm Values - A residual norm computed at the end of each iteration is printed. The residual norm is obtained by a ratio of unbalanced (residual) forces to "total" forces.

Increment Heading - The load increment number is printed, along with the increment I.D. title given in input item II-1. Following this are the mechanical load curve factors, the creep time increment, the number of elastic and plastic material integration points at the end of the increment, the number of points which have changed elastic to plastic and plastic to elastic during the increment, the maximum allowable number of Jacobian updates and the number performed during this increment, the maximum allowable number of iterations per update and the number performed since the last update, and the maximum allowable residual norm (unbalanced force error) and the residual norm actually obtained.

Cumulative Forces and Displacements - These are the cumulative internal forces (corresponding to computed integration point stresses) and displacements for each node. The node number and node I.D. are printed, followed by the u-v-w components of force and displacement. u-v-w are in the directions defined by the nodal direction coordinate system (X-Y-Z, R- $\theta$ -Z, R- $\theta$ - $\phi$ , or special coordinate system).

Thermal and Elastic Strains - These are incremental and cumulative values of the thermal and elastic strains for an element. The element number, element I.D., and integration point number are printed, followed by the thermal strains and six components of the elastic strains. All strain components are referred to the basic global (X-Y-Z) coordinate system, and are given in tensor form.



Plasticity Results - These are the incremental and cumulative values of the plastic work and plastic strains for an element. The element number, element I.D., and integration point number are printed, followed by the plastic work density and components of the plastic strains. All strains are again given in the basic global coordinate system.

Creep Results - These are printed only if the creep time increment (input in data item C-4) is greater than zero. Printed creep results correspond to those for plasticity.

Stress Results - These are cumulative stress-center and stress values for an element. The element number, element I.D., and integration point number are printed, followed by the effective value and components.

Summarized Quantities - The element number, element I.D., and integration point number are printed, followed by the elastic-plastic "code" and "sum code." The code is 0, -1 or +1, respectively, according to whether the integration point condition has remained unchanged, gone from plastic to elastic (unloaded), or gone from elastic to plastic (yielded) during the increment. The sum code gives the value for the program variable YCODE1, and is + or -, respectively, according to whether the integration point condition is plastic or elastic at the end of the increment. Its magnitude is an indication of the iterative tendency for the point to remain in that condition. Next are given the total temperature and the yield surface size ( $\sigma - \alpha$ ) at the end of the increment. Finally are printed, for both plastic and creep strains, three values of effective strain (incremental  $\Delta \bar{\epsilon}$ , sum of incremental  $\sum \Delta \bar{\epsilon}$ , and cumulative  $\bar{\epsilon}$ ).

## 10.0 SIZE LIMITATIONS

## 10.1 GENERAL LIMITATIONS

The following variables are used to specify maximum size limitations in BOPACE 3-D. The values set for these variables in the 1500-DOF and 3000-DOF program versions are given in Table 10.1-1.

|        |   |   |
|--------|---|---|
| NMAX1  | = | maximum number of materials                                   |
| NMAX2  | = | maximum number of nodes                                       |
| NMAX3  | = | maximum number of elements                                    |
| NMAX4  | = | maximum node I.D. number                                      |
| NMAX5  | = | maximum element I.D. number                                   |
| NMAX6  | = | maximum number of points in a material property curve         |
| NMAX7  | = | maximum number of temperature plasticity curves per material  |
| NMAX8A | = | maximum number of points per isotropic hardening curve        |
| NMAX8B | = | maximum number of points per kinematic hardening shape curve  |
| NMAX8C | = | maximum number of points per kinematic hardening factor curve |
| NMAX9  | = | maximum number of points in a creep reference curve           |
| NMAX10 | = | maximum number of creep-factor temperatures per material      |
| NMAX11 | = | maximum number of creep-factor stresses per temperature       |
| NMAX12 | = | maximum number of special coordinate systems                  |
| NMAX13 | = | required number of mechanical load reference vectors          |
| NMAX14 | = | maximum number of load increments per run                     |
| MNNPE  | = | maximum number of nodes per element                           |
| B      | = | maximum wavefront (in nodes)                                  |

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The maximum wavefront, B, is less than or equal to the maximum half bandwidth. A conservative estimate for B is one plus the maximum difference between node numbers on any element. For example, if an element were connected to nodes 3, 7, 26 and 31, then this element could require a value of  $B = 1 + 31 - 3 = 29$  nodes for the wavefront.

TABLE 10.1-1: MAXIMUM PROGRAM SIZE LIMITATIONS

| <u>Maximum</u> | <u>1500 DOF</u> | <u>3000 DOF</u> |
|----------------|-----------------|-----------------|
| Core Size      | 64K Words       | 128K Words      |
| NMAX1          | 5               | 5               |
| NMAX2          | 500             | 1000            |
| NMAX3          | 300             | 500             |
| NMAX4          | 2000            | 5000            |
| NMAX5          | 1000            | 2000            |
| NMAX6          | 20              | 20              |
| NMAX7          | 6               | 6               |
| NMAX8A         | 30              | 30              |
| NMAX8B         | 20              | 20              |
| NMAX8C         | 30              | 30              |
| NMAX9          | 10              | 10              |
| NMAX10         | 6               | 6               |
| NMAX11         | 10              | 10              |
| NMAX12         | 100             | 1000            |
| NMAX13         | 2               | 2               |
| NMAX14         | 60              | 60              |
| MNNPE          | 20              | 20              |
| B              | 46              | 82              |

## 10.2 LINEAR EQUATION SOLVER LIMITATIONS

The linear equations solution routines have one user controlled limitation. This is the maximum bandwidth of active nodes during the decomposition of the stiffness matrix. The bandwidth is defined as the number of nodes following the node being processed which have non-zero terms associated with the node being processed. Melosh and Bamford [1] discuss this in some detail as the wavefront analysis concept. Whetstone [2] also discusses this concept and gives rules and procedures which can be used to keep the bandwidth as small as possible by proper numbering of the nodes. (See also the previous Section 10.1.)

BOPACE uses the maximum active bandwidth (wavefront) to determine core storage requirements during decomposition.

Let

K = the words of storage available to equation solver (K = 22,000 for the 1500 DOF version, K = 67,000 for the 3000 DOF version)

N = the number of DOF per node (a constant for each node in the analysis, with N = 3 for BOPACE 3-D)

B = the maximum active bandwidth (see Section 10.1)

$$T = B + B * N^2 + 4$$

$$M = B * (B+1)/2$$

Then the following two equations must be satisfied.

$$T \leq 1000 \quad (10.2-1)$$

$$K \geq B + M + 2*N^2*M + 6*N^2 + T \quad (10.2-2)$$

These definitions are provided in case larger problems were to be run, and the additional core requirements were available. To run problems within the current program size capabilities, the user needs to be concerned only with the limitations defined in Table 10.1-1.

## 11.0 PROGRAM FLOW AND RESTART OPTIONS

The major steps accomplished during a BOPACE run are shown in the program flow summary of Figure 11.0-1.

Steps 1-7 accomplish the initialization of basic variables (program control constants, material data, mesh, and load vectors) and incremental variables, and the formation of merged and decomposed elastic stiffness matrices. These steps include reading of input data in the case where a new problem is being started, or reading of the input restart tape in the case of a restart. If data is to be saved for a future restart, steps 9-10 write the basic variables and elastic merged and decomposed matrices onto the output restart tape. In step 11 the incremental mechanical load data, including load factors and creep time increment for each load increment to be run, are read.

The remaining steps involve the incremental and iterative calculations. Updating of the Jacobian matrices in step 23 is performed only when convergence slows down, and in general this occurs only once per increment or once every several increments.

In step 26 the computed incremental variables are written onto the output restart tape, if a future restart is provided for. This allows the user to later request a restart from the end of any such load increment.

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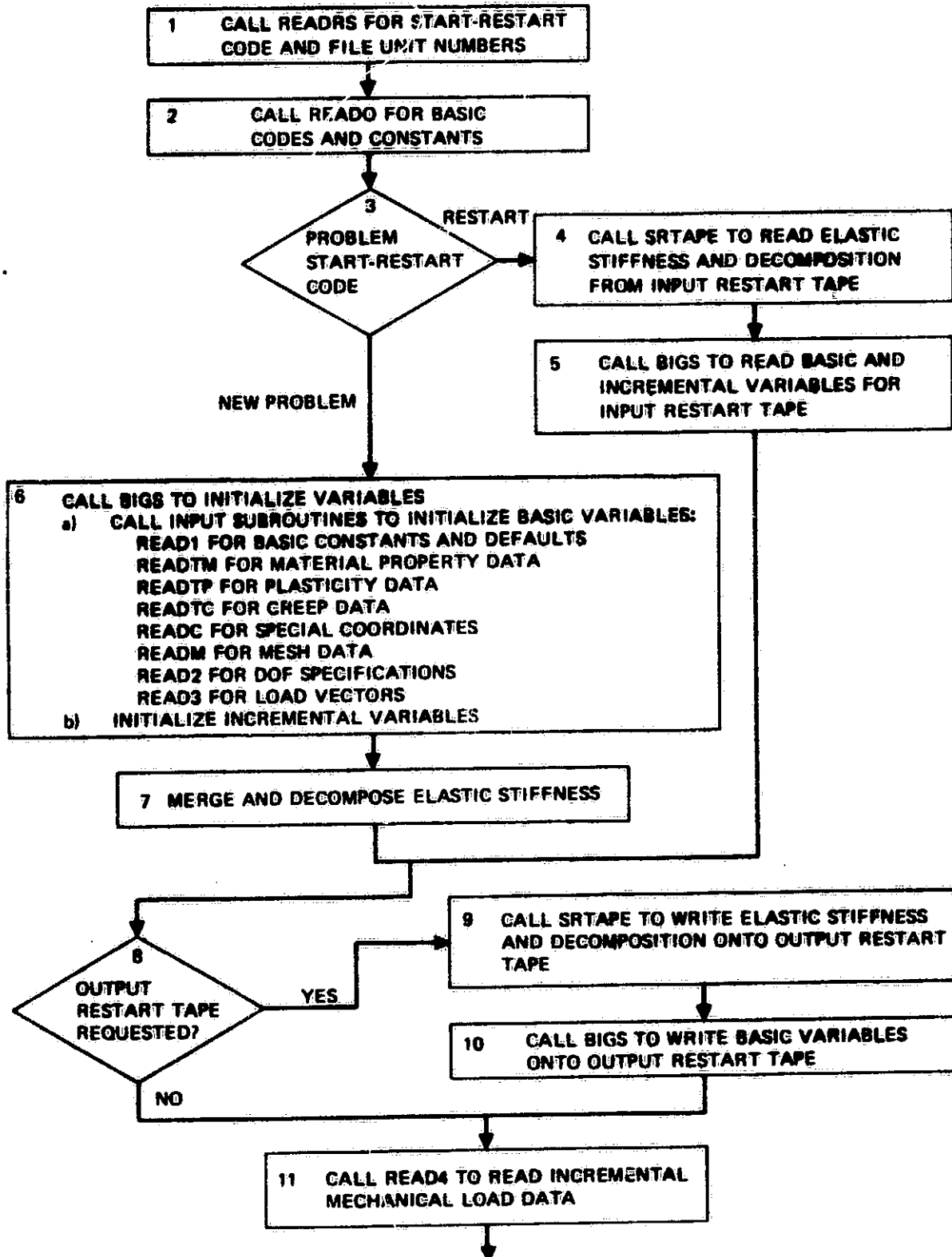


Figure 11.0-1. Program Flow

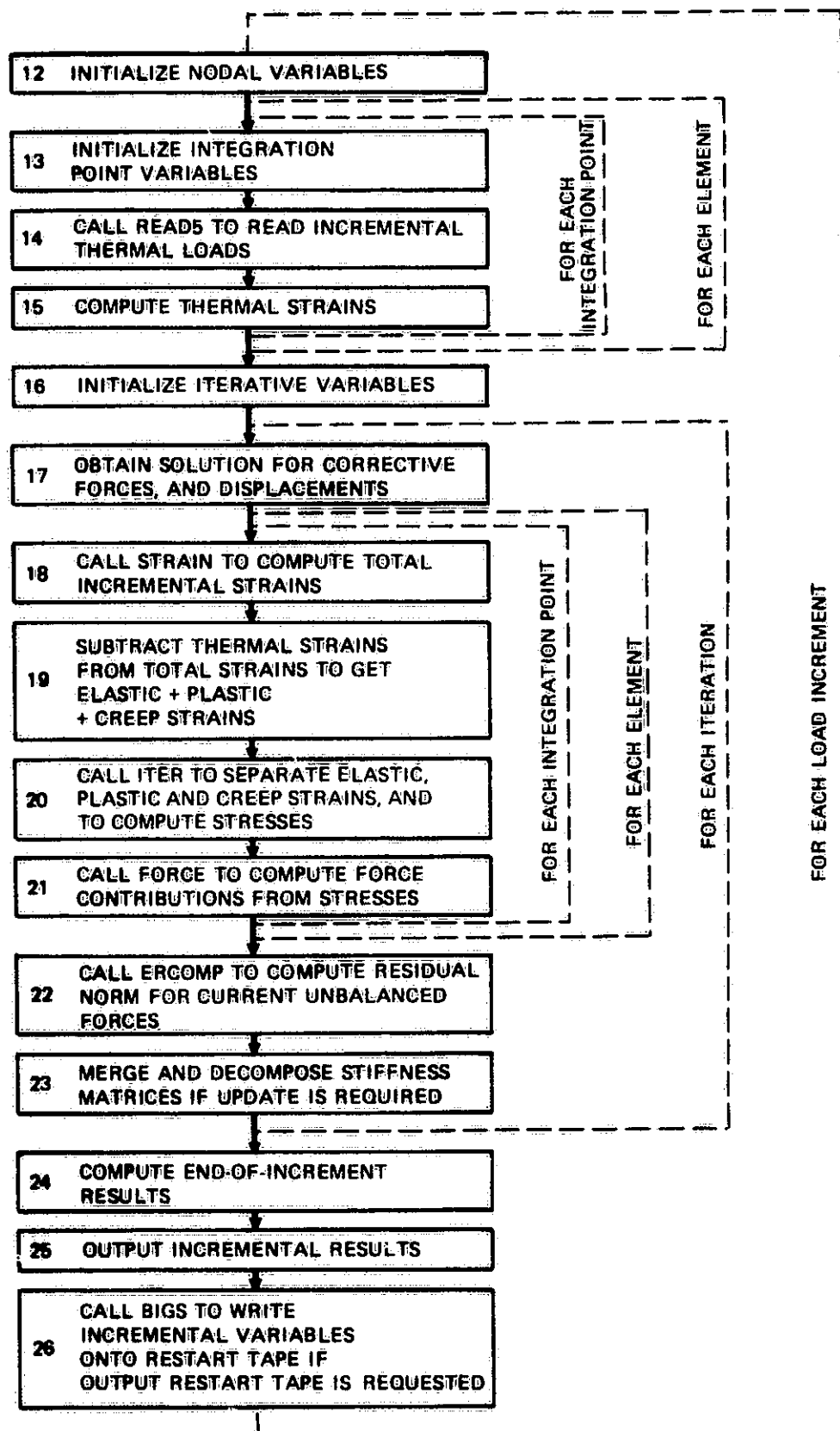


Figure 11.0-1. Program Flow (Continued)



## 12.0 BOPACE 3-D ERROR MESSAGES

BOPACE 3-D uses the FORTRAN STOP codes described in this section to indicate error conditions which may occur during execution of the program. Certain errors also generate a printed error message, in order to aid the user in locating the source of the error.

Errors fall into two categories, those due to the problem definition or user input data, and those caused by a program or machine malfunction. Errors due to a machine malfunction rarely occur and in these cases a rerun of the problem will usually eliminate the error. If an error recurs and help is needed in correcting the problem, contact a BOPACE programmer for aid. Have available a listing of the input data, the printouts of the runs which failed, the input data deck, and a description of the problem.

The following are explanations of the error STOP codes, listed by subroutine in which they occur.

### READRS

9999 Normal program exit caused by reading final blank card after all problems are run.

- 101 Non-positive value input for a required file unit number.

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READ1

202        Illegal value input for number of materials.

READTM

301        Wrong material number input on material property card.

302        Number of points input for material property curve exceeds  
            maximum

303        Non-positive value input for modulus of elasticity.

304        Value input for Poisson's ratio is less than  $-.99$  or  
            greater than  $+.49$ .

305        No points input for a required material property curve.

READTP

401        Wrong material number input on plasticity type-code card.

402        Illegal plasticity type input.

403        Illegal kinematic hardening code input.

- 404 Wrong material number input on plasticity temperature card.
- 405 Plasticity temperatures not in ascending order.
- 406 Number of temperatures for a material exceeds maximum.
- 407 Hardening point defines non-positive yield-surface size, or negative kinematic value.
- 408 Number of points input for a curve exceeds maximum.
- 409 No points input for a required curve.
- 410 First point input on a curve has non-zero abscissa.
- 411 No curves input for a required hardening description of a material.

READTC

- 501 Wrong material number input on creep type card.
- 502 Illegal creep type input.

- 503      Number of points input for a creep reference curve exceeds maximum.
- 504      No points input on the creep reference curve for a material.
- 505      Wrong material number input on creep temperature card.
- 506      Creep temperatures not in ascending order.
- 507      Number of creep temperature factors for a material exceeds maximum.
- 508      Number of creep stress factors at a temperature exceeds maximum.
- 509      No creep stress factors input at a temperature.
- 510      No creep temperatures input for a material.

READC

- 601      I.D. of special coordinate system exceeds maximum.

READM

- 701      Mesh node I.D. exceeds maximum.
- 702      I.D. of a node location coordinate system not equal to 0,  
          1 or 2.
- 703      I.D. of a node displacement coordinate system exceeds  
          maximum.
- 704      Number of input nodes exceeds maximum.
- 705      No nodes input.
- 711      Element I.D. exceeds maximum.
- 712      Illegal value input for element material number.
- 713      Illegal node I.D. on an element.
- 714      Number of input elements exceeds maximum.
- 715      No elements input.

- 716 The next element references an illegal node number.
- 717 Number of nodes per element exceeds the maximum (20),  
on the next element.

READ2

- 801 Illegal node I.D. for a force-displacement-constraint  
specification.
- 802 Illegal component number (not equal to 1, 2 or 3).
- 803 Illegal force-displacement-constraint code.
- 804 Constraint specified with dependent (already constrained)  
DOF.
- 805 Constraint specified between DOF on same element.

READ3

- 901 Number of input mechanical load reference curves not  
equal to 2.
- 902 Illegal node I.D. on a mechanical load.

903        Illegal component number (not equal to 1, 2 or 3) on a mechanical load.

904        Load input on dependent DOF constrained to DOF with specified displacement.

READ4

1001       Number of load increments exceeds maximum per run.

READ5

1101       Illegal node I.D. on a thermal load.

Linear Equation-Solver Routines

5011       I/O Error during merge. Program or machine malfunction during the merging of the elemental stiffness matrices to form the global stiffness matrix.

5021       Bandwidth too large for decomposition save array. The bandwidth is too large for the amount of core storage allocated (see Equation 10.2-1). Corrective action: remember the nodes to reduce the bandwidth.

- 5023 No decomposition partitions available. Program or machine malfunction during decomposition.
  
- 5024 Decomposition mode not in active node array. Program or machine malfunction during decomposition.
  
- 5031 Scratch array too small for solution work. Program or machine malfunction during forward and backward substitution.
  
- 5041 Illegal save tape I/O operation command. Program or machine malfunction in reading or writing the checkpoint tape.
  
- 5042 Illegal matrix type. Program or machine malfunction in reading or writing the checkpoint tape.
  
- 5043 Illegal save tape defined for save operation. Program or machine malfunction in reading or writing the checkpoint tape.
  
- 5051 Large decomposition not available. The bandwidth is too large to solve the problem using in-core decomposition. See Equation 10.2-2. Corrective action: Reduce the bandwidth by renumbering the nodes or reducing the problem size.

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13.0 REFERENCES - USER MANUAL

1. R. J. Melosh and R. M. Bamford, "Efficient Solution of Load-Deflection Equations," Journal of the Structural Division, ASCE, April 1969.
2. W. D. Whetstone, "Computer Analysis of Large Linear Frames," Journal of the Structural Division, ASCE, November 1969.

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## 14.0 EXAMPLE PROBLEMS

The example problems provided here are intended to aid the user in understanding the BOPACE 3-D results and in setting up his problem input data. The problems in Section 14.1 serve to demonstrate and check out some of the general BOPACE 3-D capabilities, including temperature-dependent elasticity, and plasticity and creep. One of these problems is again used in Section 14.2, but is analyzed using an 8-element mesh which includes midside nodes and curved boundaries. Section 14.3 provides a simple problem with a distributed loading which results in non uniform stress and strain within the element.

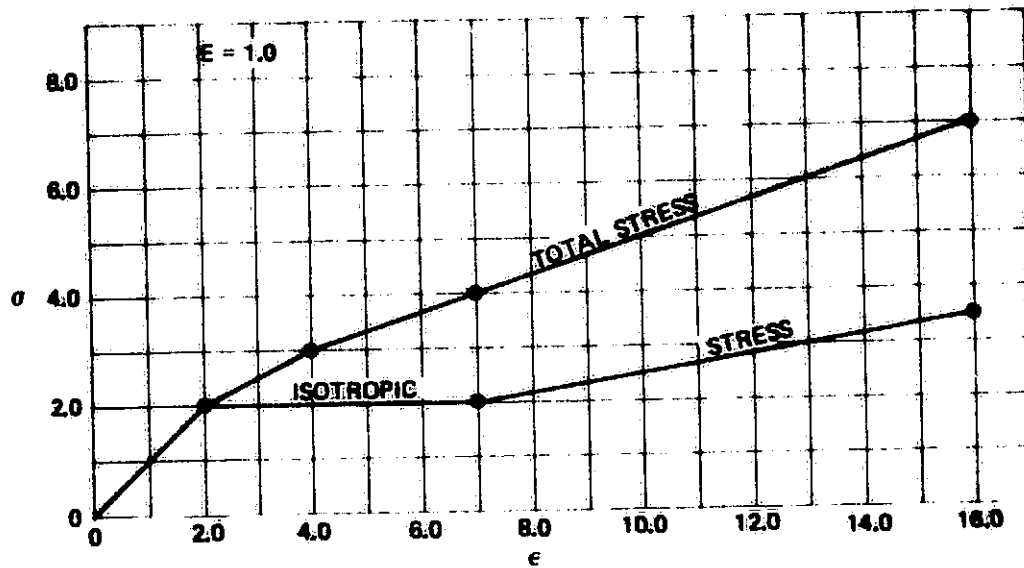
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## 14.1 GENERAL CHECKOUT PROBLEMS

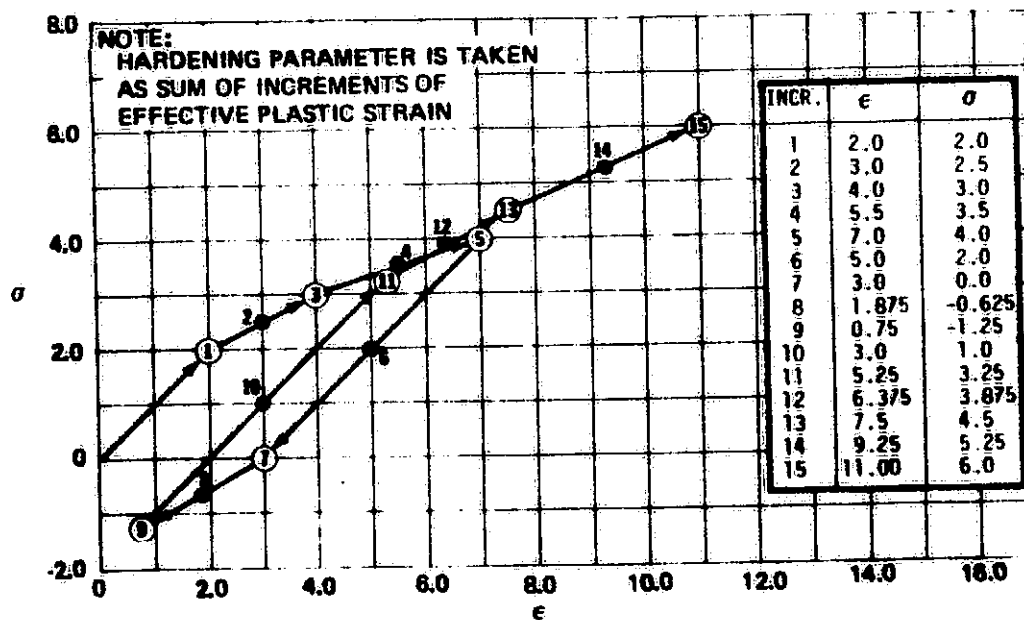
Plane Stress with Combined Hardening - The basic characteristics of BOPACE combined hardening are shown in Figure 14.1-1 for a uniaxial (special case of a plane-stress or 3-D) problem. Figure 14.1-1A gives the assumed monotonic stress-strain hardening curves. The size of the yield surface is defined by the isotropic stress (= average of tensile and compressive yield stresses), while the Bauschinger kinematic hardening is defined by the difference between the total stress and the isotropic stress. Thus the hardening is completely kinematic out to a strain value of 7.0 (elastic strain =  $\sigma/E = 4.0$ , plastic strain = 3.0), after which there are equal amounts of isotropic and kinematic hardening. For an actual material, these curves would have been determined from cyclic test data.

A resulting cyclic stress-strain curve is given in Figure 14.1-1B. The 15 load increments were chosen so as to result in the exact  $\sigma$ - $\epsilon$  points given in the figure insert table. Note that the hardening parameters ( $\kappa$  and  $\kappa^k$ ) in this example were based on effective plastic strain rather than on plastic work, because it makes the relationship between the monotonic and cyclic curves more readily apparent.

This problem may be used as a test problem for BOPACE 3-D by applying appropriate boundary conditions to a 3-D brick element, so that it provides a uniaxial or plane-stress problem.



(A) MATERIAL STRESS-STRAIN CURVES



(B) STRESS-STRAIN PATH UNDER LOADING

Figure 14.1-1. Uniaxial Test Problem for Cyclic Combined Hardening

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Plane-Strain with Additional Options - The plane-stress problem just described was altered to illustrate the use of several additional BOPACE 3-D options, including temperature-dependent elasticity, creep and use of two prescribed load vectors.

A BOPACE 3-D plane-strain checkout analysis was performed using the 3-D brick element and loading given in Figure 14.1-2. A listing of the input data and the printed output results are included at the end of this section.

A summary of the problem is provided by Table 14.1-1. The 15 increments correspond to those of the previous plane-stress problem. The values of incremental plastic strain, stress, effective stress center, and yield-surface size given in columns 2-5 of Table 14.1-1 were kept the same as those of the plane-stress problem. The stress is equal to the product of the temperature-dependent elastic modulus (column 6) and the elastic strain (column 7).

The creep strain listed in column 9 results from the material creep definition of Figure 14.1-3. There the reference creep curve for a strain-hardening material is assumed as shown in (A), while (B) defines the creep factor  $F^C$  as a function of average stress level during the increment. The creep strain may be determined using the average stress level (column 10 of Table 14.1-1), the creep factor (column 11) and the specified creep time increment (column 12).

In addition, the Z-load strains given in column 13 and thermal strains in column 14 were imposed. (The Z-load strains were imposed by applying appropriate factors to the displacement loading #2.) In order to keep the results simple and exact (all numbers in Table 14.1-1 are given exactly), the Z-load and thermal-strain values were selected so as to give zero normal stress in each increment. For example, in increment 11:

$$\begin{aligned}\epsilon_{ZZ}^e &= -0.3 \\ \epsilon_{ZZ}^p &= -1.0 \\ \epsilon_{ZZ}^c &= -0.5 \\ \epsilon_{ZZ}^t &= 1.5 \\ \hline \Sigma \epsilon_{ZZ} &= -0.3\end{aligned}$$

Because the imposed Z-load strain also equals -0.3, a zero value results for the normal stress  $\sigma_{ZZ}$ . Thus it may be noted that this example can be used for either a plane-stress or a plane-strain checkout run.

The prescribed displacements shown in column 15 were determined from the various components of the total strain. For example, again in increment 11:

$$\begin{aligned}\epsilon_{YY}^e &= 1.0 \\ \epsilon_{YY}^p &= 2.0 \\ \epsilon_{YY}^c &= 1.0 \\ \epsilon_{YY}^t &= 1.5 \\ \hline Q_{YY} = \Sigma \epsilon_{YY} &= 5.5\end{aligned}$$

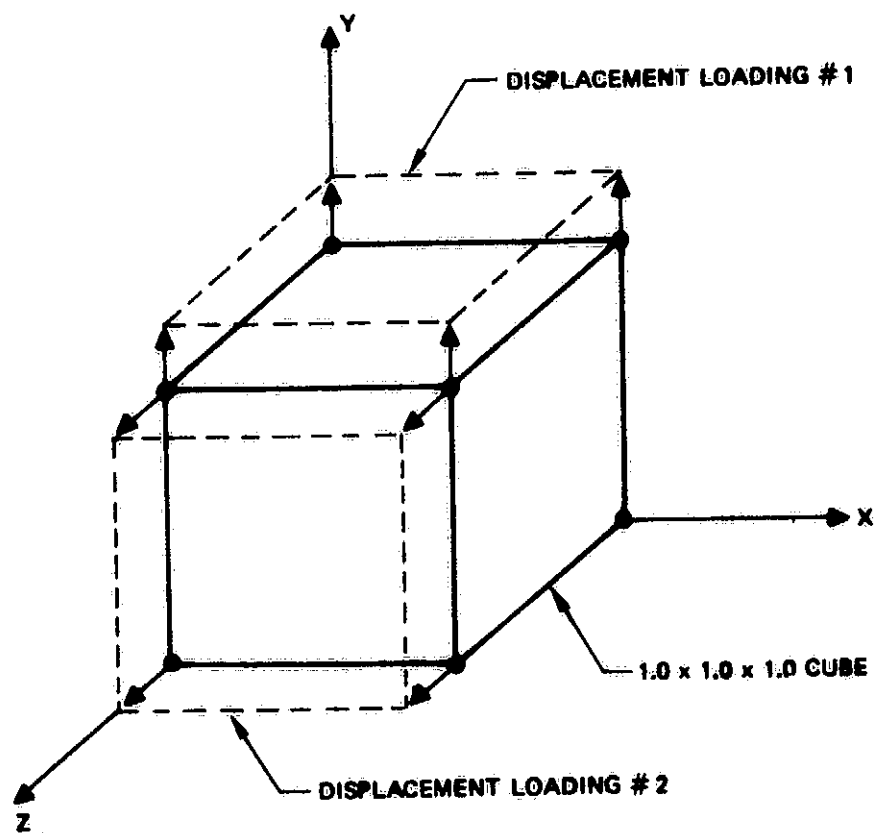
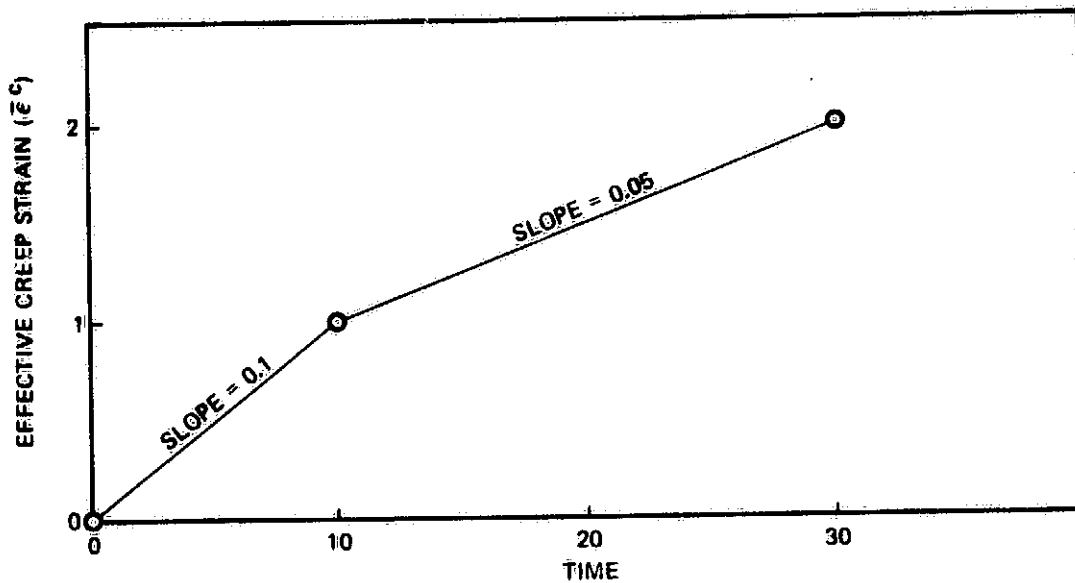


Figure 14.1-2. Plane-Strain Checkout Problem

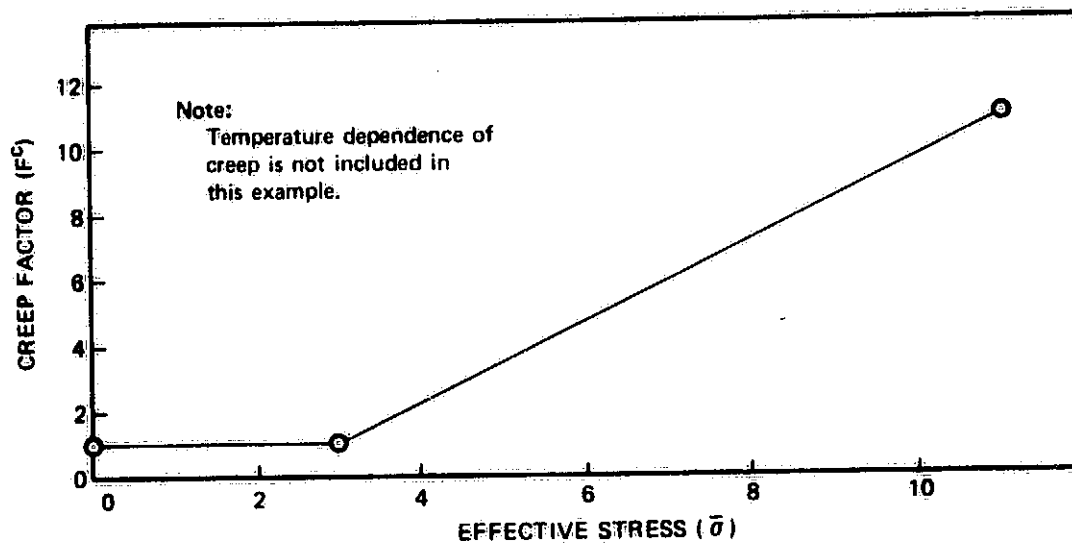
Table 14.1-1. Results for Plane Strain with Z-Load and Creep

| (1)   | (2)                      | (3)           | (4)                         | (5)                 | (6)   | (7)               | (8)               | (9)               | (10)                 | (11)   | (12)         | (13)          | (14)         | (15)          | (16)  |
|-------|--------------------------|---------------|-----------------------------|---------------------|-------|-------------------|-------------------|-------------------|----------------------|--------|--------------|---------------|--------------|---------------|-------|
| Incr. | $\Delta \epsilon_{YY}^D$ | $\sigma_{YY}$ | $\frac{3}{2} \epsilon_{YY}$ | $(\sigma - \alpha)$ | $E^I$ | $\epsilon_{YY}^e$ | $\epsilon_{YY}^D$ | $\epsilon_{YY}^C$ | $\bar{\sigma}_{ave}$ | $P^C$  | $\Delta t^C$ | Z-load strain | $\epsilon^t$ | $\sigma_{YY}$ | Temp. |
| 0     | -                        | 0             | 0                           | 2.0                 | 1.0   | 0                 | 0                 | 0                 | -                    | -      | -            | 0             | 0            | 0             | 1.0   |
| 1     | 0                        | 2.0           | 0                           | 2.0                 | 2.0   | 1.0               | 0                 | 0                 | 1.0                  | 1.0    | 0            | 0             | 0.3          | 1.3           | 2.0   |
| 2     | 0.5                      | 2.5           | 0.5                         | 2.0                 | 2.5   | 1.0               | 0.5               | 0.5               | 2.25                 | 1.0    | 5.0          | 0             | 0.8          | 2.8           | 3.0   |
| 3     | 0.5                      | 3.0           | 1.0                         | 2.0                 | 3.0   | 1.0               | 1.0               | 0.5               | 2.75                 | 1.0    | 0            | 0             | 1.05         | 3.55          | 4.0   |
| 4     | 1.0                      | 3.5           | 1.5                         | 2.0                 | 3.5   | 1.0               | 2.0               | 1.0               | 3.25                 | 1.25   | 4.0          | 0             | 1.8          | 5.8           | 5.0   |
| 5     | 1.0                      | 4.0           | 2.0                         | 2.0                 | 4.0   | 1.0               | 3.0               | 1.0               | 3.75                 | 1.75   | 0            | 0             | 2.3          | 7.3           | 6.0   |
| 6     | 0                        | 2.0           | 2.0                         | 2.0                 | 2.0   | 1.0               | 3.0               | 1.0               | 3.0                  | 1.0    | 0            | 0             | 2.3          | 7.3           | 7.0   |
| 7     | 0                        | 0             | 2.0                         | 2.0                 | 1.0   | 0                 | 3.0               | 1.0               | 1.0                  | 1.0    | 0            | 0             | 2.0          | 6.0           | 5.4   |
| 8     | -0.5                     | -0.625        | 1.5                         | 2.125               | 1.25  | -0.5              | 2.5               | 1.0               | 0.3125               | 1.0    | 0            | -0.6          | 1.0          | 4.0           | 8.0   |
| 9     | -0.5                     | -1.25         | 1.0                         | 2.25                | 1.25  | -1.0              | 2.0               | 0                 | 0.9375               | 1.0    | 10.0         | -0.2          | 0.5          | 1.5           | 9.0   |
| 10    | 0                        | 1.0           | 1.0                         | 2.25                | 2.0   | 0.5               | 2.0               | 0                 | 0.125                | 1.0    | 0            | 0             | 1.15         | 3.05          | 10.0  |
| 11    | 0                        | 3.25          | 1.0                         | 2.25                | 3.25  | 1.0               | 2.0               | 1.0               | 2.125                | 1.0    | 10.0         | -0.3          | 1.5          | 5.5           | 4.5   |
| 12    | 0.5                      | 3.875         | 1.5                         | 2.375               | 3.875 | 1.0               | 2.5               | 1.0               | 3.5625               | 1.5625 | 0            | 0             | 2.05         | 6.55          | 12.0  |
| 13    | 0.5                      | 4.5           | 2.0                         | 2.5                 | 4.5   | 1.0               | 3.0               | 1.0               | 4.1875               | 2.1875 | 0            | 0.2           | 2.5          | 7.5           | 13.0  |
| 14    | 1.0                      | 5.25          | 2.5                         | 2.75                | 5.25  | 1.0               | 4.0               | 1.0               | 4.875                | 2.875  | 0            | 0             | 2.8          | 8.8           | 14.0  |
| 15    | 1.0                      | 6.0           | 3.0                         | 3.0                 | 3.0   | 2.0               | 5.0               | 4.125             | 5.625                | 3.625  | 10.0         | 0             | 5.1875       | 16.2875       | 15.0  |





(A) ASSUMED SHAPE FOR REFERENCE CREEP CURVE



(B) ASSUMED DEPENDENCE OF CREEP FACTOR ON STRESS

Figure 14.1-3. Creep Definition for Plane-Strain Checkout Problem

START 5 5 6 12/05/74  
 BOPACE 3-C CHECK (PLANE STRAIN WITH Z-LOADS, ELASTIC-PLASTIC-CREEP)

| 5    | 3 | 5       | 6    | 1    | 0.8  | 2.3    | 1.15 | 5.1625 | 4.0  | 7.0 | 12.0 | 1.05  | 2.3 | 2.05 |
|------|---|---------|------|------|------|--------|------|--------|------|-----|------|-------|-----|------|
| 0.5  | 1 | 0.00001 | 1.0  |      |      |        |      |        |      |     |      |       |     |      |
| 1.0  | 1 | 0.0     | 2.0  | 0.3  | 3.0  | 0.8    |      |        | 4.0  | 7.0 |      | 1.05  |     |      |
| 4.5  |   | 1.5     | 5.0  | 1.8  | 6.0  | 2.3    |      |        |      |     |      |       | 2.3 |      |
| 8.0  |   | 1.0     | 9.0  | 0.5  | 10.0 | 1.15   |      |        |      |     |      |       |     | 2.05 |
| 13.0 |   | 2.5     | 14.0 | 2.8  | 15.0 | 5.1625 |      |        |      |     |      |       |     |      |
| 1.0  |   | 1.0     | 2.0  | 2.0  | 3.0  | 2.5    |      |        | 4.0  | 7.0 |      | 3.0   |     |      |
| 5.0  |   | 3.5     | 5.4  | 1.0  | 6.0  | 4.0    |      |        |      |     |      | 2.0   |     |      |
| 8.0  |   | 1.25    | 9.0  | 1.25 | 10.0 | 2.0    |      |        | 12.0 |     |      | 3.875 |     |      |
| 13.0 |   | 4.5     | 14.0 | 5.25 | 15.0 | 3.0    |      |        |      |     |      |       |     |      |
| 0.0  |   | 0.3     |      |      |      |        |      |        |      |     |      |       |     |      |

| 5   | 3 | 5   | 6    | 1   | 0.8  | 2.3 | 1.15 | 5.1625 | 4.0 | 7.0 | 12.0 | 1.05 | 2.3 | 2.05 |
|-----|---|-----|------|-----|------|-----|------|--------|-----|-----|------|------|-----|------|
| C.C | 1 | 0.0 | 3.0  | 2.0 | 9.0  | 3.5 |      |        |     |     |      |      |     |      |
| 0.0 |   | 0.0 | 1.0  | 1.0 | 3.0  | 2.0 |      |        | 9.0 |     |      | 3.5  |     |      |
| 0.0 | 1 | 0.0 | 10.0 | 1.0 | 30.0 | 2.0 |      |        |     |     |      |      |     |      |
| C.C | 1 | 0.0 | 3.0  | 1.0 | 11.0 | 9.0 |      |        |     |     |      |      |     |      |

| 5 | 3 | 5   | 6   | 7   |
|---|---|-----|-----|-----|
| 1 | 1 | 0.0 | 0.0 | 1.0 |
| 2 | 1 | 1.0 | 0.0 | 1.0 |
| 3 | 1 | 1.0 | 1.0 | 1.0 |
| 4 | 1 | 0.0 | 1.0 | 1.0 |
| 5 | 1 | 0.0 | 0.0 | 0.0 |
| 6 | 1 | 1.0 | 0.0 | 0.0 |
| 7 | 1 | 1.0 | 1.0 | 0.0 |



|           |                  |    |                  |  |                  |  |                  |
|-----------|------------------|----|------------------|--|------------------|--|------------------|
|           | 1 4.0<br>5 4.0   |    | 2 4.0<br>6 4.0   |  | 3 4.0<br>7 4.0   |  | 4 4.0<br>8 4.0   |
| INCREMENT |                  | 4  |                  |  |                  |  |                  |
|           | 1 5.0<br>5 5.0   |    | 2 5.0<br>6 5.0   |  | 3 5.0<br>7 5.0   |  | 4 5.0<br>8 5.0   |
| INCREMENT |                  | 5  |                  |  |                  |  |                  |
|           | 1 6.0<br>5 6.0   |    | 2 6.0<br>6 6.0   |  | 3 6.0<br>7 6.0   |  | 4 6.0<br>8 6.0   |
| INCREMENT |                  | 6  |                  |  |                  |  |                  |
|           | 1 7.0<br>5 7.0   |    | 2 7.0<br>6 7.0   |  | 3 7.0<br>7 7.0   |  | 4 7.0<br>8 7.0   |
| INCREMENT |                  | 7  |                  |  |                  |  |                  |
|           | 1 5.4<br>5 5.4   |    | 2 5.4<br>6 5.4   |  | 3 5.4<br>7 5.4   |  | 4 5.4<br>8 5.4   |
| INCREMENT |                  | 8  |                  |  |                  |  |                  |
|           | 1 8.0<br>5 8.0   |    | 2 8.0<br>6 8.0   |  | 3 8.0<br>7 8.0   |  | 4 8.0<br>8 8.0   |
| INCREMENT |                  | 9  |                  |  |                  |  |                  |
|           | 1 9.0<br>5 9.0   |    | 2 9.0<br>6 9.0   |  | 3 9.0<br>7 9.0   |  | 4 9.0<br>8 9.0   |
| INCREMENT |                  | 10 |                  |  |                  |  |                  |
|           | 1 10.0<br>5 10.0 |    | 2 10.0<br>6 10.0 |  | 3 10.0<br>7 10.0 |  | 4 10.0<br>8 10.0 |
| INCREMENT |                  | 11 |                  |  |                  |  |                  |
|           | 1 4.5<br>5 4.5   |    | 2 4.5<br>6 4.5   |  | 3 4.5<br>7 4.5   |  | 4 4.5<br>8 4.5   |
| INCREMENT |                  | 12 |                  |  |                  |  |                  |
|           | 1 12.0<br>5 12.0 |    | 2 12.0<br>6 12.0 |  | 3 12.0<br>7 12.0 |  | 4 12.0<br>8 12.0 |
| INCREMENT |                  | 13 |                  |  |                  |  |                  |



|           |  |        |  |        |  |        |  |
|-----------|--|--------|--|--------|--|--------|--|
| 1 13.0    |  | 2 13.0 |  | 3 13.0 |  | 4 13.0 |  |
| 5 13.0    |  | 6 13.0 |  | 7 13.0 |  | 8 13.0 |  |
| INCREMENT |  | 14     |  | 3 14.0 |  | 4 14.0 |  |
| 1 14.0    |  | 2 14.0 |  | 7 14.0 |  | 8 14.0 |  |
| 5 14.0    |  | 6 14.0 |  |        |  |        |  |
| INCREMENT |  | 15     |  | 3 15.0 |  | 4 15.0 |  |
| 1 15.0    |  | 2 15.0 |  | 7 15.0 |  | 8 15.0 |  |
| 5 15.0    |  | 6 15.0 |  |        |  |        |  |

## STARTING PROBLEM

ROPAGE 3-D CHECK (PLANE STRAIN WITH 2-LOADS, ELASTIC-PLASTIC-CREEP)

12/05/74

SOLUTION METHOD CODE = 5  
 MAXIMUM NO. STIFFNESS UPDATES PER INCREMENT = 3  
 MAXIMUM TOTAL ITERATIONS PER INCREMENT = 10  
 MAXIMUM ELASTIC ITERATIONS PER INCREMENT = 2  
 MAXIMUM MAGNITUDE FOR ELASTIC-PLASTIC SUM CODE = 2  
 MAXIMUM REDUCTIONS = 1  
 CONVERGENCE REDUCTION FACTOR = 0.50000E 00  
 MAXIMUM SPECIFIED ERROR NORM = 0.10000E-04  
 FRACTION FROM END OF INCREMENT TO EVALUATE SLOPE = 0.10000E 00

NO. OF MATERIALS = 1  
 FABRICATION TEMPERATURE = 0.10000E 01

## MATERIAL NO. 1 TEMPERATURE DEPENDENT PROPERTIES

## TEMPERATURE THERMAL STRAIN

0.1000E 01 0.0  
 0.20 00E 01 0.3000E 00  
 0.30 00E 01 0.3000E 00  
 0.40 00E 01 0.1050E 01  
 0.45 00E 01 0.1500E 01  
 0.50 00E 01 0.1800E 01  
 0.60 00E 01 0.2300E 01  
 0.70 00E 01 0.2300E 01  
 0.80 00E 01 0.1000E 01  
 0.90 00E 01 0.5000E 00  
 0.10 00E 02 0.1150E 01  
 0.12 00E 02 0.2050E 01  
 0.13 00E 02 0.2500E 01  
 0.14 00E 02 0.2400E 01  
 0.15 00E 02 0.5162E 01

## TEMPERATURE ELASTIC MOD.

0.1000E 01 0.1000E 01  
 0.20 00E 01 0.2000E 01  
 0.30 00E 01 0.2500E 01  
 0.40 00E 01 0.3000E 01  
 0.50 00E 01 0.3000E 01  
 0.55 00E 01 0.1000E 01  
 0.60 00E 01 0.4000E 01  
 0.70 00E 01 0.2000E 01  
 0.90 00E 01 0.1250E 01  
 0.90 00E 01 0.1250E 01  
 0.10 00E 02 0.2000E 01  
 0.12 00E 02 0.3875E 01  
 0.13 00E 02 0.4500E 01  
 0.14 00E 02 0.5250E 01  
 0.15 00E 02 0.3000E 01

## TEMPERATURE POISSONS RATIO

0.0 0.3000E 00

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MATERIAL NO. 1, PLASTICITY TYPE 1, KINEMATIC CODE 0

MATERIAL NO. 1, TEMPERATURE = 0.0

PARAMETER ISOTROPIC STRESS

0.0 0.20000E 01  
0.30000E 01 0.20000E 01  
0.90000E 01 0.35000E 01

PARAMETER KINEMATIC SHAPE

0.0 0.0  
0.10000E 01 0.10000E 01  
0.30000E 01 0.20000E 01  
0.90000E 01 0.35000E 01

TEMPERATURE = 0.0

PARAMETER ISOTROPIC STRESS

0.0 0.20000E 01  
0.30000E 01 0.20000E 01  
0.90000E 01 0.35000E 01

PARAMETER KINEMATIC SHAPE

0.0 0.0  
0.10000E 01 0.10000E 01  
0.30000E 01 0.20000E 01  
0.90000E 01 0.35000E 01

MATERIAL NO. 1, CREEP TYPE 2

TIME CREEP STRAIN

0.0 0.0  
0.1000E 02 0.1000E 01  
0.3000E 02 0.2000E 01

MATERIAL NO. 1, TEMPERATURE = 0.0

STRESS CREEP FACTOR

0.0 0.1000E 01  
0.3000E 01 0.1000E 01  
0.1100E 02 0.9000E 01

MATERIAL NO. 1, TEMPERATURE = 0.0

STRESS CREEP FACTOR

0.0 0.1000E 01  
0.3000E 01 0.1000E 01  
0.1100E 02 0.9000E 01

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# CARTESIAN COORDINATE SYSTEMS DEFINED

| NUMBER | OPTION | REFERENCE<br>COORD SYS | .....ORIGIN..... | POINT ON<br>.....X-AXIS..... | POINT IN<br>.....XY-PLANE..... |
|--------|--------|------------------------|------------------|------------------------------|--------------------------------|
|        |        |                        | NODE 5           | NODE 6                       | NODE 7                         |
| 3      | 1      | N/A                    |                  |                              |                                |

## \*\* NODE \*\*

| NO. | I.D. | LOCATE | X1          | X2          | X3          | DISPLACE |
|-----|------|--------|-------------|-------------|-------------|----------|
| 1   | 1    | 0      | 0.0         | 0.0         | 0.100000 01 | 0        |
| 2   | 2    | 0      | 0.100000 01 | 0.0         | 0.100000 01 | 0        |
| 3   | 3    | 0      | 0.100000 01 | 0.100000 01 | 0.100000 01 | 0        |
| 4   | 4    | 0      | 0.0         | 0.100000 01 | 0.100000 01 | 0        |
| 5   | 5    | 0      | 0.0         | 0.0         | 0.0         | 0        |
| 6   | 6    | 0      | 0.100000 01 | 0.0         | 0.0         | 0        |
| 7   | 7    | 0      | 0.100000 01 | 0.100000 01 | 0.0         | 0        |
| 8   | 8    | 0      | 0.0         | 0.100000 01 | 0.0         | 0        |

| ELEMENT |      |      | CORNER NODES |    |    |    |    |    |    |    | VOLUME      | INTERMEDIATE EDGE NODES |
|---------|------|------|--------------|----|----|----|----|----|----|----|-------------|-------------------------|
| NO.     | I.D. | MATL | N1           | N2 | N3 | N4 | N5 | N6 | N7 | N8 | (ST. LINE)  |                         |
| 1       | 3    | 1    | 1            | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 0.100000 01 |                         |

SUM OF ELEMENT VOLUMES = 0.100000 01

## SPECIFIED FORCE-DISPLACEMENT-CONSTRAINT DCF

| NODE | I.D. | COMPONENT | CODE |
|------|------|-----------|------|
| 1    | 2    | -1        |      |
| 2    | 2    | -2        |      |
| 5    | 2    | -5        |      |
| 6    | 2    | -6        |      |
| 3    | 2    | -3        |      |
| 4    | 2    | -4        |      |
| 7    | 2    | -7        |      |
| 8    | 2    | -8        |      |
| 1    | 3    | -1        |      |
| 2    | 3    | -2        |      |
| 3    | 3    | -3        |      |
| 4    | 3    | -4        |      |
| 5    | 3    | -5        |      |
| 6    | 3    | -6        |      |
| 7    | 3    | -7        |      |
| 8    | 3    | -8        |      |
| 5    | 1    | -5        |      |
| 5    | 3    | -5        |      |
| 6    | 3    | -6        |      |

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NO. OF LOAD REFERENCE CURVES = 2

LOAD REFERENCE CURVE NO. 1

| NODE | COMPONENT | LOAD        |
|------|-----------|-------------|
| 3    | 2         | 0.10000E 01 |
| 4    | 2         | 0.10000E 01 |
| 7    | 2         | 0.10000E 01 |
| 8    | 2         | 0.10000E 01 |

LOAD REFERENCE CURVE NO. 2

| NODE | COMPONENT | LOAD        |
|------|-----------|-------------|
| 1    | 3         | 0.10000E 01 |
| 2    | 3         | 0.10000E 01 |
| 3    | 3         | 0.10000E 01 |
| 4    | 3         | 0.10000E 01 |

ELEMENT INTEGRATION

| ELEM. | LOCATE | POINT | X1        | X2        | X3        |
|-------|--------|-------|-----------|-----------|-----------|
| 3     | 0      | 1     | 2.113E-01 | 2.113E-01 | 2.113E-01 |
|       |        | 2     | 2.113E-01 | 7.887E-01 | 2.113E-01 |
|       |        | 3     | 7.887E-01 | 2.113E-01 | 2.113E-01 |
|       |        | 4     | 7.887E-01 | 7.887E-01 | 2.113E-01 |
|       |        | 5     | 2.113E-01 | 2.113E-01 | 7.887E-01 |
|       |        | 6     | 2.113E-01 | 7.887E-01 | 7.887E-01 |
|       |        | 7     | 7.887E-01 | 2.113E-01 | 7.887E-01 |
|       |        | 8     | 7.887E-01 | 7.887E-01 | 7.887E-01 |

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NO. OF LOAD INCREMENTS = 15

| INCREMENT | MAX. ITERATIONS | MECHANICAL CURVE FACTORS | CREEP TIME   |
|-----------|-----------------|--------------------------|--------------|
| 1         | 10              | 0.13000E 01              | 0.0          |
| 2         | 10              | 0.28000E 01              | 0.0          |
| 3         | 10              | 0.35500E 01              | 0.0          |
| 4         | 10              | 0.58000E 01              | 0.0          |
| 5         | 10              | 0.73000E 01              | 0.0          |
| 6         | 10              | 0.73000E 01              | 0.0          |
| 7         | 10              | 0.60000E 01              | 0.0          |
| 8         | 10              | 0.40000E 01              | -0.60000E 00 |
| 9         | 10              | 0.15000E 01              | -0.20000E 00 |
| 10        | 10              | 0.36500E 01              | 0.0          |
| 11        | 10              | 0.55000E 01              | -0.37000E 00 |
| 12        | 10              | 0.65500E 01              | 0.0          |
| 13        | 10              | 0.75000E 01              | 0.20000E 00  |
| 14        | 10              | 0.88000E 01              | 0.0          |
| 15        | 10              | 0.16287E 02              | 0.0          |

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# CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 1

INCREMENT 1

| NODE I.D. | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| TEMP.     | 0.20000E-01 | 0.20000E-01 | 0.20000E-01 | 0.20000E-01 | 0.20000E-01 | 0.20000E-01 | 0.20000E-01 | 0.20000E-01 |

RESIDUAL URM = 0.37500E-00  
 RESIDUAL VORM = 0.27425E-00  
 RESIDUAL WORM = 0.37500E-00  
 RESIDUAL UORV = 0.14269E-06

END OF LOAD INCREMENT 1

INCREMENT 1

MECHANICAL LOAD CURVE FACTORS = 0.1300E 01, 0.0

CREEP TIME INCREMENT = 0.0

NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 0

0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 0

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 4

SPECIFIED MAX. UNBALANCED-FORCE ERROR = 0.1000E-04, ACTUAL ERROR = 0.1427E-06

## CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** |      | ***** FORCES ***** |                |                | ***** DISPLACEMENTS ***** |                |     |
|------------|------|--------------------|----------------|----------------|---------------------------|----------------|-----|
| NO.        | I.D. | U                  | V              | W              | U                         | V              | W   |
| 1          | 1    | 0.1144691E-06      | -0.4999995E 00 | -0.1450820E-06 | 0.7105145E-06             | 0.0            | 0.0 |
| 2          | 2    | 0.2382088E-07      | -0.4999993E 00 | 0.5528082E-07  | 0.1072884E-05             | 0.0            | 0.0 |
| 3          | 3    | 0.7610460E-07      | 0.4999995E 00  | -0.4602698E-07 | 0.1013274E-05             | -0.1299999E 01 | 0.0 |
| 4          | 4    | 0.1008448E-06      | 0.4999992E 00  | -0.7024994E-07 | 0.6556511E-06             | 0.1299999E 01  | 0.0 |
| 5          | 5    | -0.1253808E-06     | -0.4999994E 00 | -0.5528081E-07 | 0.0                       | 0.0            | 0.0 |
| 6          | 6    | -0.2224407E-06     | -0.4999991E 00 | -0.1456820E-06 | 0.2930232E-06             | 0.0            | 0.0 |
| 7          | 7    | 0.5899743E-08      | 0.4999994E 00  | 0.7024988E-07  | 0.9536743E-06             | 0.1299999E 01  | 0.0 |
| 8          | 8    | 0.2519116E-07      | 0.4999990E 00  | -0.4602698E-07 | 0.5509828E-06             | 0.1299999E 01  | 0.0 |

# THERMAL AND ELASTIC STRAINS

| ELEMENT NO. | INTEGR. POINT | INTEGRAL THERMAL STRAINS | XX         | YY        | ZZ         | XY        | XZ        | YZ        |
|-------------|---------------|--------------------------|------------|-----------|------------|-----------|-----------|-----------|
| 1           | 3             |                          |            |           |            |           |           |           |
| 1           | 1             | 0.300E 00                | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.220E-06 | 0.295E-06 | 0.138E-16 |
|             | 2             | 0.300E 00                | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.220E-06 | 0.295E-06 | 0.138E-16 |
|             | 3             | 0.300E 00                | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.243E-06 | 0.307E-06 | 0.390E-17 |
|             | 4             | 0.300E 00                | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.243E-06 | 0.307E-06 | 0.390E-17 |
|             | 5             | 0.300E 00                | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.385E-07 | 0.295E-06 | 0.693E-17 |
|             | 6             | 0.300E 00                | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.385E-07 | 0.114E-06 | 0.551E-16 |
|             | 7             | 0.300E 00                | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.438E-07 | 0.307E-06 | 0.346E-17 |
|             | 8             | 0.300E 00                | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.438E-07 | 0.107E-06 | 0.624E-16 |

| INTEGR. POINT | CUMULATIVE THERMAL STRAINS | XX         | YY        | ZZ         | XY        | XZ        | YZ        |
|---------------|----------------------------|------------|-----------|------------|-----------|-----------|-----------|
| 1             | 0.300E 00                  | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.220E-06 | 0.295E-06 | 0.138E-16 |
| 2             | 0.300E 00                  | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.220E-06 | 0.114E-06 | 0.551E-16 |
| 3             | 0.300E 00                  | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.243E-06 | 0.307E-06 | 0.390E-17 |
| 4             | 0.300E 00                  | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.243E-06 | 0.107E-06 | 0.624E-16 |
| 5             | 0.300E 00                  | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.385E-07 | 0.295E-06 | 0.693E-17 |
| 6             | 0.300E 00                  | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.385E-07 | 0.114E-06 | 0.551E-16 |
| 7             | 0.300E 00                  | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.438E-07 | 0.307E-06 | 0.346E-17 |
| 8             | 0.300E 00                  | -0.300E 00 | 0.100E 01 | -0.300E 00 | 0.438E-07 | 0.107E-06 | 0.624E-16 |

# PLASTIC WORK AND STRAINS

| ELEMENT NO. | INTEGR. POINT | INTEGRAL PLASTIC WORK | XX  | YY  | ZZ  | XY  | XZ  | YZ  |
|-------------|---------------|-----------------------|-----|-----|-----|-----|-----|-----|
| 1           | 3             |                       |     |     |     |     |     |     |
| 1           | 1             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 2             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 3             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 4             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 5             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 6             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 7             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 8             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| INTEGR. POINT | CUMULATIVE PLASTIC WORK | XX  | YY  | ZZ  | XY  | XZ  | YZ  |
|---------------|-------------------------|-----|-----|-----|-----|-----|-----|
| 1             | 0.0                     | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2             | 0.0                     | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 3             | 0.0                     | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 4             | 0.0                     | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 5             | 0.0                     | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 6             | 0.0                     | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 7             | 0.0                     | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 8             | 0.0                     | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

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# CUMULATIVE STRESS QUANTITIES

| ELEMENT NO. | I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | *****CUMULATIVE STRESS CENTER***** |            |             |            |            |             |
|-------------|------|---------------|-------------------------|------------------------------------|------------|-------------|------------|------------|-------------|
|             |      |               |                         | XX                                 | YY         | ZZ          | XY         | XZ         | YZ          |
| 1           | 3    | 1             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0        | 0.0        | 0.0         |
|             |      | 2             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0        | 0.0        |             |
|             |      | 3             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0        | 0.0        |             |
|             |      | 4             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0        | 0.0        |             |
|             |      | 5             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0        | 0.0        |             |
|             |      | 6             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0        | 0.0        |             |
|             |      | 7             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0        | 0.0        |             |
|             |      | 8             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0        | 0.0        |             |
|             |      |               |                         | ***** CUMULATIVE STRESSES *****    |            |             |            |            |             |
|             |      | INTEGR. POINT | EFFECTIVE STRESS        | XX                                 | YY         | ZZ          | XY         | XZ         | YZ          |
|             |      | 1             | 0.2000E 01              | -0.2292E-06                        | 0.2000E 01 | -0.2292E-06 | 0.3386E-06 | 0.4548E-06 | 0.2135E-16  |
|             |      | 2             | 0.2000E 01              | 0.0                                | 0.2000E 01 | 0.0         | 0.3386E-06 | 0.1754E-06 | 0.8540E-16  |
|             |      | 3             | 0.2000E 01              | -0.2292E-06                        | 0.2000E 01 | -0.2292E-06 | 0.3748E-06 | 0.4731E-06 | 0.5005E-17  |
|             |      | 4             | 0.2000E 01              | 0.0                                | 0.2000E 01 | 0.0         | 0.3748E-06 | 0.1657E-06 | -0.9608E-16 |
|             |      | 5             | 0.2000E 01              | -0.2292E-06                        | 0.2000E 01 | -0.2292E-06 | 0.5927E-07 | 0.4548E-06 | 0.1068E-16  |
|             |      | 6             | 0.2000E 01              | 0.0                                | 0.2000E 01 | 0.0         | 0.5927E-07 | 0.1754E-06 | -0.8540E-16 |
|             |      | 7             | 0.2000E 01              | -0.2292E-06                        | 0.2000E 01 | -0.2292E-06 | 0.6743E-07 | 0.4731E-06 | 0.5338E-17  |
|             |      | 8             | 0.2000E 01              | 0.0                                | 0.2000E 01 | 0.0         | 0.6743E-07 | 0.1657E-06 | 0.9608E-16  |

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## PLASTIC AND CREEP STRAINS

| ELEMENT NO. = 1 ID = 3 |      |      |             | *** EFFECTIVE PLASTIC STRAINS *** |            |             |           |            | *** EFFECTIVE CREEP STRAINS *** |           |            |
|------------------------|------|------|-------------|-----------------------------------|------------|-------------|-----------|------------|---------------------------------|-----------|------------|
| INT                    | E-P  | SUM  | INCREMENTAL | TOTAL                             | SURFACE    | INCREMENTAL | SUM INCR. | CUMULATIVE | INCREMENTAL                     | SUM INCR. | CUMULATIVE |
| PNT                    | CODE | CODE | TEMPERATURE | TEMPERATURE                       | YIELD SIZE |             |           |            |                                 |           |            |
| 1                      | 0    | -2   | 0.1000E 01  | 0.2000E 01                        | 0.2000E 01 | 0.0         | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 2                      | 0    | -2   | 0.1000E 01  | 0.2000E 01                        | 0.2000E 01 | 0.0         | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 3                      | 0    | -2   | 0.1000E 01  | 0.2000E 01                        | 0.2000E 01 | 0.0         | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 4                      | 0    | -2   | 0.1000E 01  | 0.2000E 01                        | 0.2000E 01 | 0.0         | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 5                      | 0    | -2   | 0.1000E 01  | 0.2000E 01                        | 0.2000E 01 | 0.0         | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 6                      | 0    | -2   | 0.1000E 01  | 0.2000E 01                        | 0.2000E 01 | 0.0         | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 7                      | 0    | -2   | 0.1000E 01  | 0.2000E 01                        | 0.2000E 01 | 0.0         | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 8                      | 0    | -2   | 0.1000E 01  | 0.2000E 01                        | 0.2000E 01 | 0.0         | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |

# CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 2

INCREMENT 2

NODE 1.0. 1 2 3 4 5 6 7 8  
TEMP. 0.30000E 01 0.30000E 01 0.30000E 01 0.30000E 01 0.30000E 01 0.30000E 01 0.30000E 01 0.30000E 01

RESIDUAL URM = 0.50446E 00  
RESIDUAL NURM = 0.38127E 00  
RESIDUAL NJRM = 0.38299E 00  
RESIDUAL NJRM = 0.36912E 00  
RESIDUAL NJRM = 0.26263E 00  
RESIDUAL NJRM = 0.45319E 00  
RESIDUAL NJRM = 0.34348E 01  
RESIDUAL NJRM = 0.46428E 01  
RESIDUAL NJRM = 0.15608E 01  
RESIDUAL NJRM = 0.17045E 01  
RESIDUAL NJRM = 0.26358E 02  
RESIDUAL NJRM = 0.61001E 04  
RESIDUAL NJRM = 0.14768E 04  
RESIDUAL NJRM = 0.50025E 06

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## END OF LOAD INCREMENT 2

INCREMENT 2

MECHANICAL LOAD CURVE FACTORS = 0.2800E 01, 0.0

CREEP TIME INCREMENT = 0.5000E 01

NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 8

8 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 1

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 4

SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.1000E 04, ACTUAL ERROR = 0.5003E 06

## CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** |      | ***** FORCES ***** |                |                | ***** DISPLACEMENTS ***** |               |     |
|------------|------|--------------------|----------------|----------------|---------------------------|---------------|-----|
| NO.        | 1.0. | U                  | V              | W              | U                         | V             | W   |
| 1          | 1    | -0.7168171E-07     | -0.6249995E 00 | -0.7880753E-06 | 0.4231051E-06             | 0.0           | 0.0 |
| 2          | 2    | 0.7177510E-07      | -0.6249995E 00 | -0.7880072E-06 | 0.2517809E-05             | 0.0           | 0.0 |
| 3          | 3    | 0.7036764E-07      | 0.6249996E 00  | -0.7880594E-06 | 0.2480459E-05             | 0.2799999E 01 | 0.0 |
| 4          | 4    | -0.7107801E-07     | 0.6249994E 00  | -0.7880236E-06 | 0.3928208E-06             | 0.2799999E 01 | 0.0 |
| 5          | 5    | -0.7236690E-07     | -0.6249993E 00 | -0.7880072E-06 | 0.0                       | 0.0           | 0.0 |
| 6          | 6    | 0.7271979E-07      | -0.6249993E 00 | 0.7880754E-06  | 0.2082109E-05             | 0.0           | 0.0 |
| 7          | 7    | 0.7139793E-07      | 0.6249993E 00  | 0.7880234E-06  | 0.2444451E-05             | 0.2799999E 01 | 0.0 |
| 8          | 8    | -0.7093282E-07     | -0.6249992E 00 | -0.7880594E-06 | 0.3348783E-06             | 0.2799999E 01 | 0.0 |

## THERMAL AND ELASTIC STRAINS

| ELEMENT       |      |               | INCREMENTAL ELASTIC STRAINS |             |            |             |             |             |             |
|---------------|------|---------------|-----------------------------|-------------|------------|-------------|-------------|-------------|-------------|
| NO.           | I.D. | INTEGR. POINT | INCREMENTAL THERMAL STRAINS | XX          | YY         | ZZ          | XY          | XZ          | YZ          |
| 1             | 1    | 1             | 0.5000E 00                  | 0.1788E-06  | 0.2563E-05 | -0.1192E-05 | -0.2169E-06 | -0.2937E-06 | 0.3046E-17  |
|               |      | 2             | 0.5000E-00                  | 0.1192E-06  | 0.2563E-05 | -0.1192E-05 | -0.2169E-06 | -0.1139E-06 | -0.1219E-16 |
|               |      | 3             | 0.5000E 00                  | 0.1788E-06  | 0.2563E-05 | -0.1192E-05 | -0.2465E-06 | -0.3092E-06 | 0.5945E-17  |
|               |      | 4             | 0.5000E 00                  | 0.1192E-06  | 0.2563E-05 | -0.1192E-05 | -0.2465E-06 | -0.1080E-06 | -0.1591E-16 |
|               |      | 5             | 0.5000E-00                  | 0.1788E-06  | 0.2563E-05 | -0.1192E-05 | -0.3715E-07 | -0.2937E-06 | -0.8124E-17 |
|               |      | 6             | 0.5000E 00                  | 0.1192E-06  | 0.2563E-05 | -0.1192E-05 | -0.3715E-07 | -0.1139E-06 | -0.1219E-16 |
|               |      | 7             | 0.5000E 00                  | 0.1788E-06  | 0.2563E-05 | -0.1192E-05 | -0.4534E-07 | -0.3092E-06 | 0.5262E-17  |
|               |      | 8             | 0.5000E 00                  | 0.1192E-06  | 0.2563E-05 | -0.1192E-05 | -0.4534E-07 | -0.1080E-06 | -0.1591E-16 |
| INTEGR. POINT |      |               | CUMULATIVE THERMAL STRAINS  | XX          | YY         | ZZ          | XY          | XZ          | YZ          |
|               |      | 1             | 0.8000E 00                  | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | 0.3156E-08  | 0.1906E-08  | 0.1692E-16  |
|               |      | 2             | 0.8000E 00                  | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | 0.3156E-08  | 0.1226E-09  | -0.5770E-16 |
|               |      | 3             | 0.8000E-00                  | -0.3000E-00 | 0.1000E-01 | -0.3000E-00 | 0.2512E-08  | -0.1684E-08 | -0.9443E-17 |
|               |      | 4             | 0.8000E 00                  | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.2012E-08 | -0.2766E-09 | -0.7836E-16 |
|               |      | 5             | 0.8000E 00                  | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | 0.1373E-06  | 0.1406E-08  | 0.1596E-16  |
|               |      | 6             | 0.8000E 00                  | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.1373E-08 | -0.1226E-09 | -0.6770E-16 |
|               |      | 7             | 0.8000E 00                  | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.1505E-08 | -0.1684E-08 | 0.9731E-17  |
|               |      | 8             | 0.8000E 00                  | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.1505E-08 | -0.2766E-09 | -0.7836E-16 |

## PLASTIC WORK AND STRAINS

| ELEMENT       |      |               | INCREMENTAL PLASTIC STRAINS |             |            |             |            |            |             |
|---------------|------|---------------|-----------------------------|-------------|------------|-------------|------------|------------|-------------|
| NO.           | I.D. | INTEGR. POINT | INCREMENTAL PLASTIC WORK    | XX          | YY         | ZZ          | XY         | XZ         | YZ          |
| 1             | 1    | 1             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.6393E-07 | 0.8578E-07 | 0.3885E-17  |
|               |      | 2             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.6393E-07 | 0.3293E-07 | -0.3554E-16 |
|               |      | 3             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.6990E-07 | 0.8824E-07 | 0.3967E-17  |
|               |      | 4             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.6990E-07 | 0.3099E-07 | -0.4062E-16 |
|               |      | 5             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.1108E-07 | 0.8578E-07 | 0.5347E-17  |
|               |      | 6             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.1108E-07 | 0.3293E-07 | -0.3554E-16 |
|               |      | 7             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.1264E-07 | 0.8824E-07 | 0.5308E-17  |
|               |      | 8             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.1264E-07 | 0.3099E-07 | -0.4062E-16 |
| INTEGR. POINT |      |               | CUMULATIVE PLASTIC WORK     | XX          | YY         | ZZ          | XY         | XZ         | YZ          |
|               |      | 1             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.6393E-07 | 0.8578E-07 | 0.3885E-17  |
|               |      | 2             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.6393E-07 | 0.3293E-07 | -0.3554E-16 |
|               |      | 3             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.6990E-07 | 0.8824E-07 | 0.3967E-17  |
|               |      | 4             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.6990E-07 | 0.3099E-07 | -0.4062E-16 |
|               |      | 5             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.1108E-07 | 0.8578E-07 | 0.5347E-17  |
|               |      | 6             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.1108E-07 | 0.3293E-07 | -0.3554E-16 |
|               |      | 7             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.1264E-07 | 0.8824E-07 | 0.5308E-17  |
|               |      | 8             | 0.1125E 01                  | -0.2500E 00 | 0.5000E 00 | -0.2500E 00 | 0.1264E-07 | 0.3099E-07 | -0.4062E-16 |

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# CUMULATIVE STRESS QUANTITIES

| ELEMENT NO. | I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | *****CUMULATIVE STRESS CENTER***** |            |             |             |             |              |
|-------------|------|---------------|-------------------------|------------------------------------|------------|-------------|-------------|-------------|--------------|
|             |      |               |                         | XX                                 | YY         | ZZ          | XY          | XZ          | YZ           |
| 1           | 3    | 1             | 0.5000E 00              | 0.1667E 00                         | 0.3333E 00 | 0.1667E 00  | 0.4262E -07 | 0.5718E -07 | 0.5923E -17  |
|             |      | 2             | 0.5000E 00              | -0.1667E 00                        | 0.3333E 00 | -0.1667E 00 | 0.4262E -07 | 0.2195E -07 | -0.2369E -16 |
|             |      | 3             | 0.5000E 00              | -0.1667E 00                        | 0.3333E 00 | -0.1667E 00 | 0.4660E -07 | 0.5883E -07 | 0.2644E -17  |
|             |      | 4             | 0.5000E 00              | -0.1667E 00                        | 0.3333E 00 | -0.1667E 00 | 0.4660E -07 | 0.2066E -07 | 0.2708E -16  |
|             |      | 5             | 0.5000E 00              | -0.1667E 00                        | 0.3333E 00 | -0.1667E 00 | 0.7387E -08 | 0.5718E -07 | 0.4231E -17  |
|             |      | 6             | 0.5000E 00              | -0.1667E 00                        | 0.3333E 00 | -0.1667E 00 | 0.7387E -08 | 0.2195E -07 | -0.2369E -16 |
|             |      | 7             | 0.5000E 00              | -0.1667E 00                        | 0.3333E 00 | -0.1667E 00 | 0.8430E -08 | 0.5883E -07 | 0.2539E -17  |
|             |      | 8             | 0.5000E 00              | -0.1667E 00                        | 0.3333E 00 | -0.1667E 00 | 0.8430E -08 | 0.2066E -07 | -0.2708E -16 |

| INTEGR. POINT | EFFECTIVE STRESS | *****CUMULATIVE STRESSES***** |            |              |              |              |              |
|---------------|------------------|-------------------------------|------------|--------------|--------------|--------------|--------------|
|               |                  | XX                            | YY         | ZZ           | XY           | XZ           | YZ           |
| 1             | 0.2500E 01       | 0.2866E -06                   | 0.2500E 01 | -0.3152E -05 | 0.6069E -08  | 0.3665E -08  | 0.1255E -16  |
| 2             | 0.2500E 01       | 0.2866E -06                   | 0.2500E 01 | -0.3152E -05 | 0.6069E -08  | 0.2956E -09  | -0.1302E -15 |
| 3             | 0.2500E 01       | 0.2866E -06                   | 0.2500E 01 | -0.3152E -05 | -0.5600E -08 | -0.3239E -08 | 0.1194E -16  |
| 4             | 0.2500E 01       | 0.2866E -06                   | 0.2500E 01 | -0.3152E -05 | -0.5600E -08 | -0.5320E -09 | -0.1507E -15 |
| 5             | 0.2500E 01       | 0.2866E -06                   | 0.2500E 01 | -0.3152E -05 | 0.2640E -08  | 0.3665E -08  | -0.2697E -16 |
| 6             | 0.2500E 01       | 0.2866E -06                   | 0.2500E 01 | -0.3152E -05 | 0.2640E -08  | 0.2356E -09  | -0.1302E -15 |
| 7             | 0.2500E 01       | 0.2866E -06                   | 0.2500E 01 | -0.3152E -05 | -0.2893E -08 | -0.3239E -08 | 0.1194E -16  |
| 8             | 0.2500E 01       | 0.2866E -06                   | 0.2500E 01 | -0.3152E -05 | -0.2893E -08 | 0.5320E -09  | 0.1507E -15  |

## CREEP WORK AND STRAINS

| ELEMENT NO. | I.D. | INTEGR. POINT | INCREMENTAL CREEP WORK | *****INCREMENTAL CREEP STRAINS***** |            |             |             |             |              |
|-------------|------|---------------|------------------------|-------------------------------------|------------|-------------|-------------|-------------|--------------|
|             |      |               |                        | XX                                  | YY         | ZZ          | XY          | XZ          | YZ           |
| 1           | 3    | 1             | 0.1125E 01             | -0.2500E 00                         | 0.5000E 00 | -0.2500E 00 | 0.6393E -07 | 0.8578E -07 | 0.8885E -17  |
|             |      | 2             | 0.1125E 01             | -0.2500E 00                         | 0.5000E 00 | -0.2500E 00 | 0.6393E -07 | 0.3293E -07 | -0.3554E -16 |
|             |      | 3             | 0.1125E 01             | -0.2500E 00                         | 0.5000E 00 | -0.2500E 00 | 0.6990E -07 | 0.8825E -07 | 0.3967E -17  |
|             |      | 4             | 0.1125E 01             | -0.2500E 00                         | 0.5000E 00 | -0.2500E 00 | 0.6990E -07 | 0.3099E -07 | -0.4062E -16 |
|             |      | 5             | 0.1125E 01             | -0.2500E 00                         | 0.5000E 00 | -0.2500E 00 | 0.1108E -07 | 0.8578E -07 | 0.6347E -17  |
|             |      | 6             | 0.1125E 01             | -0.2500E 00                         | 0.5000E 00 | -0.2500E 00 | 0.1108E -07 | 0.3293E -07 | -0.3554E -16 |
|             |      | 7             | 0.1125E 01             | -0.2500E 00                         | 0.5000E 00 | -0.2500E 00 | 0.1264E -07 | 0.8825E -07 | 0.3008E -17  |
|             |      | 8             | 0.1125E 01             | -0.2500E 00                         | 0.5000E 00 | -0.2500E 00 | 0.1264E -07 | 0.3099E -07 | -0.4062E -16 |

| INTEGR. POINT | CUMULATIVE CREEP WORK | *****CUMULATIVE CREEP STRAINS***** |            |             |             |             |              |
|---------------|-----------------------|------------------------------------|------------|-------------|-------------|-------------|--------------|
|               |                       | XX                                 | YY         | ZZ          | XY          | XZ          | YZ           |
| 1             | 0.1125E 01            | -0.2500E 00                        | 0.5000E 00 | -0.2500E 00 | 0.6393E -07 | 0.8578E -07 | 0.8885E -17  |
| 2             | 0.1125E 01            | -0.2500E 00                        | 0.5000E 00 | -0.2500E 00 | 0.6393E -07 | 0.3293E -07 | -0.3554E -16 |
| 3             | 0.1125E 01            | -0.2500E 00                        | 0.5000E 00 | -0.2500E 00 | 0.6990E -07 | 0.8825E -07 | 0.3967E -17  |
| 4             | 0.1125E 01            | -0.2500E 00                        | 0.5000E 00 | -0.2500E 00 | 0.6990E -07 | 0.3099E -07 | -0.4062E -16 |
| 5             | 0.1125E 01            | -0.2500E 00                        | 0.5000E 00 | -0.2500E 00 | 0.1108E -07 | 0.8578E -07 | 0.6347E -17  |
| 6             | 0.1125E 01            | -0.2500E 00                        | 0.5000E 00 | -0.2500E 00 | 0.1108E -07 | 0.3293E -07 | -0.3554E -16 |
| 7             | 0.1125E 01            | -0.2500E 00                        | 0.5000E 00 | -0.2500E 00 | 0.1264E -07 | 0.8825E -07 | 0.3008E -17  |
| 8             | 0.1125E 01            | -0.2500E 00                        | 0.5000E 00 | -0.2500E 00 | 0.1264E -07 | 0.3099E -07 | -0.4062E -16 |

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# PLASTIC AND CREEP STRAINS

| ELEMENT NO. = 1 ID = 3 |      |      |             |             |            |                                   |            |            |                                 |            |            |  |
|------------------------|------|------|-------------|-------------|------------|-----------------------------------|------------|------------|---------------------------------|------------|------------|--|
| INT                    | E-P  | SUM  | INCREMENTAL | TOTAL       | SURFACE    | *** EFFECTIVE PLASTIC STRAINS *** |            |            | *** EFFECTIVE CREEP STRAINS *** |            |            |  |
| PNT                    | CODE | CODE | TEMPERATURE | TEMPERATURE | YIELD SIZE | INCREMENTAL                       | SUM INCR.  | CUMULATIVE | INCREMENTAL                     | SUM INCR.  | CUMULATIVE |  |
| 1                      | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00                        | 0.5000E 00 | 0.5000E 00 | 0.5000E 00                      | 0.5000E 00 | 0.5000E 00 |  |
| 2                      | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00                        | 0.5000E 00 | 0.5000E 00 | 0.5000E 00                      | 0.5000E 00 | 0.5000E 00 |  |
| 3                      | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00                        | 0.5000E 00 | 0.5000E 00 | 0.5000E 00                      | 0.5000E 00 | 0.5000E 00 |  |
| 4                      | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00                        | 0.5000E 00 | 0.5000E 00 | 0.5000E 00                      | 0.5000E 00 | 0.5000E 00 |  |
| 5                      | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00                        | 0.5000E 00 | 0.5000E 00 | 0.5000E 00                      | 0.5000E 00 | 0.5000E 00 |  |
| 6                      | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00                        | 0.5000E 00 | 0.5000E 00 | 0.5000E 00                      | 0.5000E 00 | 0.5000E 00 |  |
| 7                      | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00                        | 0.5000E 00 | 0.5000E 00 | 0.5000E 00                      | 0.5000E 00 | 0.5000E 00 |  |
| 8                      | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00                        | 0.5000E 00 | 0.5000E 00 | 0.5000E 00                      | 0.5000E 00 | 0.5000E 00 |  |

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OF POOR QUALITY



# CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 3

INCREMENT 3

| NODE I.D. | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| TEMP.     | 0.40000E-01 | 0.40000E-01 | 0.40000E-01 | 0.40000E-01 | 0.40000E-01 | 0.40000E-01 | 0.40000E-01 | 0.40000E-01 |

RESIDUAL NORM = 0.39559E-00  
 RESIDUAL NORM = 0.36563E-00  
 RESIDUAL NORM = 0.28344E-00  
 RESIDUAL NORM = 0.11238E-01  
 RESIDUAL NORM = 0.19959E-02  
 RESIDUAL NORM = 0.64149E-03  
 RESIDUAL NORM = 0.72958E-04  
 RESIDUAL NORM = 0.87527E-05

END OF LOAD INCREMENT 3

INCREMENT 3

MECHANICAL LOAD CURVE FACTORS = 0.3550E 01, 0.0

CREEP TIME INCREMENT = 0.0

NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 8

0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 0

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 8

SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.1000E-04, ACTUAL ERROR = 0.8753E-05

## CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** | NO. I.D. | U              | V              | W              | U              | V             | W   |
|------------|----------|----------------|----------------|----------------|----------------|---------------|-----|
| 1          | 1        | 0.1275306E-05  | -0.7499990E-00 | 0.1375493E-05  | 0.4272662E-06  | 0.0           | 0.0 |
| 2          | 2        | -0.1275311E-05 | -0.7499989E-00 | 0.1375486E-05  | -0.6008136E-05 | 0.0           | 0.0 |
| 3          | 3        | 0.1271124E-05  | -0.7499990E-00 | 0.1375490E-05  | -0.4997051E-05 | 0.3549999E-01 | 0.0 |
| 4          | 4        | 0.1271125E-05  | 0.7499987E-00  | 0.1375488E-05  | 0.3571586E-06  | 0.3549999E-01 | 0.0 |
| 5          | 5        | 0.1219421E-05  | -0.7499986E-00 | -0.1375435E-05 | 0.0            | 0.0           | 0.0 |
| 6          | 6        | 0.1219412E-05  | -0.7499986E-00 | -0.1375442E-05 | -0.5342135E-05 | 0.0           | 0.0 |
| 7          | 7        | -0.1221301E-05 | 0.7499988E-00  | -0.1375439E-05 | -0.5029721E-05 | 0.3549999E-01 | 0.0 |
| 8          | 8        | 0.1220296E-05  | 0.7499986E-00  | -0.1375490E-05 | 0.3001215E-06  | 0.3549999E-01 | 0.0 |

## THERMAL AND ELASTIC STRAINS

| ELEMENT<br>NO. | I.D. | INTEGR.<br>POINT | INCREMENTAL<br>THERMAL STRAINS | INCREMENTAL ELASTIC STRAINS |             |            |             |             |             |
|----------------|------|------------------|--------------------------------|-----------------------------|-------------|------------|-------------|-------------|-------------|
|                |      |                  |                                | XX                          | YY          | ZZ         | XY          | XZ          | YZ          |
| 1              | 3    | 1                | 0.2500E 00                     | -0.2205E-05                 | -0.2623E-05 | 0.3636E-05 | -0.2348E-08 | 0.6971E-08  | 0.2894E-17  |
|                |      | 2                | 0.2500E 00                     | -0.2205E-05                 | -0.2623E-05 | 0.3636E-05 | -0.2348E-08 | 0.2176E-08  | -0.3457E-16 |
|                |      | 3                | 0.2500E 00                     | -0.2205E-05                 | -0.2623E-05 | 0.3636E-05 | 0.2093E-08  | -0.7203E-08 | -0.6372E-18 |
|                |      | 4                | 0.2500E 00                     | -0.2205E-05                 | -0.2623E-05 | 0.3636E-05 | 0.2093E-08  | -0.7019E-08 | 0.3417E-16  |
|                |      | 5                | 0.2500E 00                     | -0.2205E-05                 | -0.2623E-05 | 0.3636E-05 | -0.7142E-08 | 0.6471E-08  | -0.6610E-17 |
|                |      | 6                | 0.2500E 00                     | -0.2265E-05                 | -0.2623E-05 | 0.3636E-05 | 0.7142E-08  | 0.2176E-08  | 0.2432E-16  |
|                |      | 7                | 0.2500E 00                     | -0.2265E-05                 | -0.2623E-05 | 0.3636E-05 | 0.7278E-08  | -0.7213E-08 | -0.5237E-18 |
|                |      | 8                | 0.2500E 00                     | -0.2265E-05                 | -0.2623E-05 | 0.3636E-05 | -0.7278E-08 | 0.2019E-08  | 0.3417E-16  |

| INTEGR.<br>POINT | CUMULATIVE<br>THERMAL STRAINS | CUMULATIVE ELASTIC STRAINS |            |             |             |             |             |
|------------------|-------------------------------|----------------------------|------------|-------------|-------------|-------------|-------------|
|                  |                               | XX                         | YY         | ZZ          | XY          | XZ          | YZ          |
| 1                | 0.1050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | 0.8083E-09  | 0.8876E-08  | 0.1982E-16  |
| 2                | 0.1050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | 0.8083E-09  | 0.2299E-08  | -0.3312E-16 |
| 3                | 0.1050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | -0.8188E-09 | 0.8888E-08  | -0.7211E-17 |
| 4                | 0.1050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | -0.8188E-09 | -0.2295E-08 | -0.4419E-16 |
| 5                | 0.1050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | -0.5709E-08 | 0.8876E-08  | 0.4252E-17  |
| 6                | 0.1050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | 0.5769E-08  | 0.2299E-08  | -0.4338E-16 |
| 7                | 0.1050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | 0.5773E-08  | -0.8888E-08 | 0.6003E-17  |
| 8                | 0.1050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | 0.5773E-08  | -0.2295E-08 | -0.4419E-16 |

## PLASTIC WORK AND STRAINS

| ELEMENT<br>NO. | I.D. | INTEGR.<br>POINT | INCREMENTAL<br>PLASTIC WORK | INCREMENTAL PLASTIC STRAINS |            |             |             |             |             |
|----------------|------|------------------|-----------------------------|-----------------------------|------------|-------------|-------------|-------------|-------------|
|                |      |                  |                             | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
| 1              | 3    | 1                | 0.1375E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.1301E-07 | -0.1473E-07 | -0.1093E-16 |
|                |      | 2                | 0.1375E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.1301E-07 | -0.6505E-08 | -0.2763E-16 |
|                |      | 3                | 0.1375E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.1695E-07 | -0.2426E-07 | 0.5841E-17  |
|                |      | 4                | 0.1375E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.1695E-07 | -0.7898E-08 | 0.5417E-16  |
|                |      | 5                | 0.1375E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.4705E-08 | -0.1473E-07 | 0.5810E-17  |
|                |      | 6                | 0.1375E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.4705E-08 | -0.6505E-08 | -0.3126E-16 |
|                |      | 7                | 0.1375E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.5881E-09 | -0.2426E-07 | 0.5554E-17  |
|                |      | 8                | 0.1375E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.5881E-09 | -0.7898E-08 | -0.3417E-16 |

| INTEGR.<br>POINT | CUMULATIVE<br>PLASTIC WORK | CUMULATIVE PLASTIC STRAINS |            |             |            |             |             |
|------------------|----------------------------|----------------------------|------------|-------------|------------|-------------|-------------|
|                  |                            | XX                         | YY         | ZZ          | XY         | XZ          | YZ          |
| 1                | 0.2500E 01                 | -0.5000E 00                | 0.1000E 01 | -0.5000E 00 | 0.5051E-07 | 0.7104E-07  | 0.1937E-16  |
| 2                | 0.2500E 01                 | -0.5000E 00                | 0.1000E 01 | -0.5000E 00 | 0.5041E-07 | 0.2643E-07  | -0.6317E-16 |
| 3                | 0.2500E 01                 | -0.5000E 00                | 0.1000E 01 | -0.5000E 00 | 0.5295E-07 | 0.6198E-07  | 0.9308E-17  |
| 4                | 0.2500E 01                 | -0.5000E 00                | 0.1000E 01 | -0.5000E 00 | 0.5295E-07 | 0.2309E-07  | -0.7479E-16 |
| 5                | 0.2500E 01                 | -0.5000E 00                | 0.1000E 01 | -0.5000E 00 | 0.6295E-08 | 0.7104E-07  | 0.1316E-16  |
| 6                | 0.2500E 01                 | -0.5000E 00                | 0.1000E 01 | -0.5000E 00 | 0.6295E-08 | 0.2643E-07  | -0.6680E-16 |
| 7                | 0.2500E 01                 | -0.5000E 00                | 0.1000E 01 | -0.5000E 00 | 0.1206E-07 | 0.6398E-07  | 0.9702E-17  |
| 8                | 0.2500E 01                 | -0.5000E 00                | 0.1000E 01 | -0.5000E 00 | 0.1206E-07 | -0.2309E-07 | -0.7479E-16 |

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# CUMULATIVE STRESS QUANTITIES

| ELEMENT NO.   | I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | *****CUMULATIVE STRESS CENTER***** |            |             |            |            |             |
|---------------|------|---------------|-------------------------|------------------------------------|------------|-------------|------------|------------|-------------|
|               |      |               |                         | XX                                 | YY         | ZZ          | XY         | XZ         | YZ          |
| 1             | 3    | 1             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.3334E 07 | 0.4730E 07 | -0.1525E 16 |
|               |      | 2             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.3334E 07 | 0.1762E 07 | -0.4212E 16 |
|               |      | 3             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.3530E 07 | 0.4266E 07 | 0.6539E 17  |
|               |      | 4             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.3530E 07 | 0.1539E 07 | -0.4780E 16 |
|               |      | 5             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.4197E 08 | 0.4736E 07 | 0.3771E 17  |
|               |      | 6             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.4197E 08 | 0.1762E 07 | -0.4453E 16 |
|               |      | 7             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.4038E 08 | 0.4266E 07 | 0.6534E 17  |
|               |      | 8             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.4038E 08 | 0.1539E 07 | -0.4780E 16 |
| INTEGR. POINT |      |               |                         | *****CUMULATIVE STRESSES*****      |            |             |            |            |             |
|               |      |               |                         | XX                                 | YY         | ZZ          | XY         | XZ         | YZ          |
|               |      | 1             | 0.3000E 01              | -0.4814E 05                        | 0.3000E 01 | 0.5502E 05  | 0.1805E 08 | 0.2048E 07 | 0.4573E 16  |
|               |      | 2             | 0.3000E 01              | -0.4814E 05                        | 0.3000E 01 | 0.5502E 05  | 0.1805E 08 | 0.5365E 08 | 0.7044E 16  |
|               |      | 3             | 0.3000E 01              | -0.4814E 05                        | 0.3000E 01 | 0.5502E 05  | 0.1805E 08 | 0.2051E 07 | 0.2120E 16  |
|               |      | 4             | 0.3000E 01              | -0.4814E 05                        | 0.3000E 01 | 0.5502E 05  | 0.1390E 08 | 0.5247E 08 | 0.1020E 15  |
|               |      | 5             | 0.3000E 01              | -0.5158E 05                        | 0.3000E 01 | 0.5502E 05  | 0.1351E 07 | 0.2148E 07 | 0.1504E 16  |
|               |      | 6             | 0.3000E 01              | -0.5158E 05                        | 0.3000E 01 | 0.5502E 05  | 0.1331E 07 | 0.5305E 08 | 0.1001E 15  |
|               |      | 7             | 0.3000E 01              | -0.5158E 05                        | 0.3000E 01 | 0.5502E 05  | 0.1352E 07 | 0.2051E 07 | 0.2033E 16  |
|               |      | 8             | 0.3000E 01              | -0.5158E 05                        | 0.3000E 01 | 0.5502E 05  | 0.1352E 07 | 0.5247E 08 | 0.1020E 15  |

# PLASTIC AND CREEP STRAINS

ELEMENT NO. = 1 ID = 3

| INT | P | SUM | INCREMENTAL | TOTAL      | SURFACE    | *** EFFECTIVE PLASTIC STRAINS *** |            |            | *** EFFECTIVE CREEP STRAINS *** |            |            |
|-----|---|-----|-------------|------------|------------|-----------------------------------|------------|------------|---------------------------------|------------|------------|
|     |   |     |             |            |            | INCREMENTAL                       | SUM INCR.  | CUMULATIVE | INCREMENTAL                     | SUM INCR.  | CUMULATIVE |
| 1   | 0 | 2   | 0.1000E 01  | 0.4000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 | 0.0                             | 0.5000E 00 | 0.5000E 00 |
| 2   | 0 | 2   | 0.1000E 01  | 0.4000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 | 0.0                             | 0.5000E 00 | 0.5000E 00 |
| 3   | 0 | 2   | 0.1000E 01  | 0.4000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 | 0.0                             | 0.5000E 00 | 0.5000E 00 |
| 4   | 0 | 2   | 0.1000E 01  | 0.4000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 | 0.0                             | 0.5000E 00 | 0.5000E 00 |
| 5   | 0 | 2   | 0.1000E 01  | 0.4000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 | 0.0                             | 0.5000E 00 | 0.5000E 00 |
| 6   | 0 | 2   | 0.1000E 01  | 0.4000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 | 0.0                             | 0.5000E 00 | 0.5000E 00 |
| 7   | 0 | 2   | 0.1000E 01  | 0.4000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 | 0.0                             | 0.5000E 00 | 0.5000E 00 |
| 8   | 0 | 2   | 0.1000E 01  | 0.4000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 | 0.0                             | 0.5000E 00 | 0.5000E 00 |

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OF FOUR QUART.

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# CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 4

INCREMENT 4

NODE I.D. 1 2 3 4 5 6 7 8  
TEMP. 0.50000E 01 0.50000E 01 0.50000E 01 0.50000E 01 0.50000E 01 0.50000E 01 0.50000E 01

RESIDUAL URM = 0.43866E-00  
RESIDUAL NDRM = 0.47146E 00  
RESIDUAL NJRM = 0.38391E 00  
RESIDUAL URM = 0.44392E 00  
RESIDUAL NDRM = 0.32790E 00  
RESIDUAL NJRM = 0.27423E 00  
RESIDUAL URM = 0.54350E-01  
RESIDUAL NDRM = 0.10343E-01  
RESIDUAL NJRM = 0.52907E-02  
RESIDUAL URM = 0.35815E-03  
RESIDUAL NDRM = 0.31078E-03  
RESIDUAL NJRM = 0.34125E-04  
RESIDUAL URM = 0.39270E-06

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END OF LOAD INCREMENT 4

INCREMENT 4  
MECHANICAL LOAD CURVE FACTORS = 0.5800E 01, 0.0  
CREEP TIME INCREMENT = 0.4000E 01  
NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 8  
0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT  
SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 1  
SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 3  
SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.1000E-04, ACTUAL ERROR = 0.3927E-06

## CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** | ** I.D. ** | FORCES         |                |                | DISPLACEMENTS  |               |     |
|------------|------------|----------------|----------------|----------------|----------------|---------------|-----|
| NO.        | I.D.       | U              | V              | W              | U              | V             | W   |
| 1          | 1          | 0.7067592E-07  | -0.8750004E 00 | -0.1060865E-05 | 0.1852214E-06  | 0.0           | 0.0 |
| 2          | 2          | -0.7078688E-07 | -0.8750004E 00 | -0.1060870E-05 | -0.1810161E-05 | 0.0           | 0.0 |
| 3          | 3          | -0.8620318E-07 | 0.8750005E 00  | -0.9450323E-06 | -0.1421430E-05 | 0.5799999E 01 | 0.0 |
| 4          | 4          | 0.3636653E-07  | 0.8750002E 00  | -0.9450796E-06 | -0.2411602E-06 | 0.5799999E 01 | 0.0 |
| 5          | 5          | -0.3477180E-07 | -0.8749996E 00 | 0.1060871E-05  | 0.0            | 0.0           | 0.0 |
| 6          | 6          | -0.3471268E-07 | -0.8749996E 00 | 0.1060867E-05  | -0.2102155E-05 | 0.0           | 0.0 |
| 7          | 7          | 0.2193376E-07  | 0.8749998E 00  | 0.9450314E-06  | -0.2023570E-05 | 0.5799999E 01 | 0.0 |
| 8          | 8          | -0.2197430E-07 | 0.8749994E 00  | 0.9450314E-06  | 0.2982190E-06  | 0.5799999E 01 | 0.0 |

## THERMAL AND ELASTIC STRAINS

| ELEMENT |      | INTEGR. | INCREMENTAL     | INCREMENTAL ELASTIC STRAINS |            |             |             |             |             |
|---------|------|---------|-----------------|-----------------------------|------------|-------------|-------------|-------------|-------------|
| NO.     | I.D. | POINT   | THERMAL STRAINS | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
| 1       | 3    | 1       | 0.7500E 00      | 0.1252E-05                  | 0.2742E-05 | -0.3695E-05 | -0.2743E-08 | -0.4058E-07 | 0.2359E-17  |
|         |      | 2       | 0.7500E 00      | 0.1252E-05                  | 0.2742E-05 | -0.3695E-05 | -0.2743E-08 | -0.6808E-07 | 0.1800E-16  |
|         |      | 3       | 0.7500E 00      | 0.1252E-05                  | 0.2742E-05 | -0.3695E-05 | 0.2818E-08  | 0.4056E-07  | 0.4998E-17  |
|         |      | 4       | 0.7500E 00      | 0.1252E-05                  | 0.2742E-05 | -0.3695E-05 | 0.2818E-08  | 0.6817E-07  | 0.2376E-16  |
|         |      | 5       | 0.7500E 00      | 0.1132E-05                  | 0.3278E-05 | -0.3934E-05 | -0.3024E-07 | -0.4058E-07 | 0.7904E-17  |
|         |      | 6       | 0.7500E 00      | 0.1788E-05                  | 0.2394E-05 | -0.3815E-05 | -0.3024E-07 | -0.6808E-07 | -0.2095E-16 |
|         |      | 7       | 0.7500E 00      | 0.1132E-05                  | 0.3278E-05 | -0.3934E-05 | 0.3043E-07  | 0.4056E-07  | 0.5355E-17  |
|         |      | 8       | 0.7500E 00      | 0.1788E-05                  | 0.2384E-05 | -0.3815E-05 | 0.3043E-07  | 0.6817E-07  | 0.2376E-16  |
|         |      | INTEGR. | CUMULATIVE      | CUMULATIVE ELASTIC STRAINS  |            |             |             |             |             |
|         |      | POINT   | THERMAL STRAINS | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
|         |      | 1       | 0.1800E 01      | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.1934E-08 | -0.3170E-07 | 0.2218E-16  |
|         |      | 2       | 0.1800E 01      | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.1934E-08 | -0.6578E-07 | -0.1513E-16 |
|         |      | 3       | 0.1800E 01      | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.1934E-08 | 0.3167E-07  | 0.1421E-16  |
|         |      | 4       | 0.1800E 01      | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | 0.1999E-08  | 0.6587E-07  | -0.2143E-16 |
|         |      | 5       | 0.1800E 01      | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.3601E-07 | -0.3170E-07 | 0.1532E-16  |
|         |      | 6       | 0.1800E 01      | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.3601E-07 | -0.6578E-07 | -0.5433E-16 |
|         |      | 7       | 0.1800E 01      | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | 0.3620E-07  | 0.3167E-07  | 0.1416E-16  |
|         |      | 8       | 0.1800E 01      | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | 0.3620E-07  | 0.6587E-07  | -0.2143E-16 |

## PLASTIC WORK AND STRAINS

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT           | INCREMENTAL<br>PLASTIC WORK | INCREMENTAL PLASTIC STRAINS |             |             |             |             |             |
|---------------------|----------------------------|-----------------------------|-----------------------------|-------------|-------------|-------------|-------------|-------------|
|                     |                            |                             | XX                          | YY          | ZZ          | XY          | XZ          | YZ          |
| 1 3                 | 1                          | 0.3250E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.2380E-07 | -0.4662E-07 | 0.2618E-16  |
|                     | 2                          | 0.3250E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.2380E-07 | -0.5918E-07 | -0.1200E-16 |
|                     | 3                          | 0.3250E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.2272E-07 | -0.1394E-07 | 0.1517E-16  |
|                     | 4                          | 0.3250E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.2272E-07 | 0.3706E-07  | -0.2047E-16 |
|                     | 5                          | 0.3250E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.3036E-07 | -0.4662E-07 | 0.1579E-16  |
|                     | 6                          | 0.3250E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.3036E-07 | -0.5918E-07 | -0.6005E-16 |
|                     | 7                          | 0.3250E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.2829E-07 | -0.1394E-07 | 0.1493E-16  |
|                     | 8                          | 0.3250E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.2829E-07 | 0.3706E-07  | -0.2047E-16 |
| INTEGR.<br>POINT    | CUMULATIVE<br>PLASTIC WORK | CUMULATIVE PLASTIC STRAINS  |                             |             |             |             |             |             |
|                     |                            | XX                          | YY                          | ZZ          | XY          | XZ          | YZ          |             |
| 1                   | 0.5750E 01                 | -0.1000E 01                 | 0.2000E 01                  | -0.1000E 01 | 0.2711E-07  | 0.2443E-07  | 0.4605E-16  |             |
| 2                   | 0.5750E 01                 | -0.1000E 01                 | 0.2000E 01                  | -0.1000E 01 | 0.2711E-07  | -0.3275E-07 | -0.7517E-16 |             |
| 3                   | 0.5750E 01                 | -0.1000E 01                 | 0.2000E 01                  | -0.1000E 01 | 0.3023E-07  | 0.5004E-07  | 0.2498E-16  |             |
| 4                   | 0.5750E 01                 | -0.1000E 01                 | 0.2000E 01                  | -0.1000E 01 | 0.3023E-07  | 0.6015E-07  | -0.4525E-16 |             |
| 5                   | 0.5750E 01                 | -0.1000E 01                 | 0.2000E 01                  | -0.1000E 01 | -0.3007E-07 | -0.2443E-07 | 0.2695E-16  |             |
| 6                   | 0.5750E 01                 | -0.1000E 01                 | 0.2000E 01                  | -0.1000E 01 | -0.3007E-07 | -0.3275E-07 | -0.1268E-15 |             |
| 7                   | 0.5750E 01                 | -0.1000E 01                 | 0.2000E 01                  | -0.1000E 01 | 0.4034E-07  | 0.5004E-07  | 0.2444E-16  |             |
| 8                   | 0.5750E 01                 | -0.1000E 01                 | 0.2000E 01                  | -0.1000E 01 | 0.4034E-07  | 0.6015E-07  | -0.4525E-16 |             |

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# CUMULATIVE STRESS QUANTITIES

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | EFFECTIVE<br>STRESS CENTER | ***** CUMULATIVE STRESS CENTER ***** |             |             |             |             |             |
|---------------------|------------------|----------------------------|--------------------------------------|-------------|-------------|-------------|-------------|-------------|
|                     |                  |                            | XX                                   | YY          | ZZ          | XY          | XZ          | YZ          |
| 1 3                 | 1                | 0.1500E 01                 | -0.5000E 00                          | -0.1000E 01 | -0.5000E 00 | 0.2601E-07  | -0.3182E-07 | -0.2109E-08 |
|                     | 2                | 0.1500E 01                 | -0.5000E 00                          | 0.1000E 01  | -0.5000E 00 | 0.2601E-07  | -0.2109E-08 | -0.4612E-16 |
|                     | 3                | 0.1500E 01                 | -0.5000E 00                          | 0.1000E 01  | -0.5000E 00 | 0.2773E-07  | 0.3801E-07  | 0.1160E-16  |
|                     | 4                | 0.1500E 01                 | -0.5000E 00                          | -0.1000E 01 | -0.5000E 00 | -0.2773E-07 | -0.2775E-07 | -0.5608E-16 |
|                     | 5                | 0.1500E 01                 | -0.5000E 00                          | 0.1000E 01  | -0.5000E 00 | -0.7925E-08 | 0.3182E-07  | 0.1337E-16  |
|                     | 6                | 0.1500E 01                 | -0.5000E 00                          | 0.1000E 01  | -0.5000E 00 | -0.7925E-08 | -0.2109E-08 | -0.6455E-16 |
|                     | 7                | 0.1500E 01                 | -0.5000E 00                          | -0.1000E 01 | -0.5000E 00 | 0.1747E-07  | 0.3801E-07  | -0.1131E-16 |
|                     | 8                | 0.1500E 01                 | -0.5000E 00                          | 0.1000E 01  | -0.5000E 00 | 0.1747E-07  | 0.2775E-07  | -0.5608E-16 |
|                     |                  |                            | ***** CUMULATIVE STRESSES *****      |             |             |             |             |             |
|                     |                  |                            | XX                                   | YY          | ZZ          | XY          | XZ          | YZ          |
|                     | 1                | 0.3500E 01                 | 0.4012E-06                           | 0.3500E 01  | -0.4814E-05 | -0.5208E-08 | -0.8535E-07 | 0.5971E-16  |
|                     | 2                | 0.3500E 01                 | 0.4112E-06                           | 0.3500E 01  | -0.4314E-05 | -0.5208E-08 | -0.1771E-06 | -0.4073E-16 |
|                     | 3                | 0.3500E 01                 | 0.4012E-06                           | 0.3500E 01  | -0.4814E-05 | 0.5382E-08  | 0.8527E-07  | 1.3826E-16  |
|                     | 4                | 0.3500E 01                 | 0.4012E-06                           | 0.3500E 01  | -0.4814E-05 | 0.5382E-08  | 0.1774E-06  | -0.5500E-16 |
|                     | 5                | 0.3500E 01                 | 0.4012E-06                           | 0.3500E 01  | -0.4814E-05 | -0.9695E-07 | -0.8535E-07 | -0.4124E-16 |
|                     | 6                | 0.3500E 01                 | -0.8024E-06                          | 0.3500E 01  | -0.2407E-05 | -0.9695E-07 | -0.1771E-06 | -0.1732E-16 |
|                     | 7                | 0.3500E 01                 | -0.4012E-06                          | 0.3500E 01  | -0.4012E-05 | 0.9746E-07  | 0.8527E-07  | 0.3813E-16  |
|                     | 8                | 0.3500E 01                 | -0.8024E-06                          | 0.3500E 01  | -0.2407E-05 | 0.9746E-07  | -0.1774E-06 | -0.5500E-16 |

## CREEP WORK AND STRAINS

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | INCREMENTAL<br>CREEP WORK | ***** INCREMENTAL CREEP STRAINS ***** |            |             |             |             |             |
|---------------------|------------------|---------------------------|---------------------------------------|------------|-------------|-------------|-------------|-------------|
|                     |                  |                           | XX                                    | YY         | ZZ          | XY          | XZ          | YZ          |
| 1 3                 | 1                | 0.1625E 01                | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | -0.1190E-07 | -0.2331E-07 | 0.1309E-16  |
|                     | 2                | 0.1625E 01                | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | -0.1190E-07 | -0.2459E-07 | -0.5999E-17 |
|                     | 3                | 0.1625E 01                | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | -0.1136E-07 | -0.6971E-08 | -0.7586E-17 |
|                     | 4                | 0.1625E 01                | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | -0.1136E-07 | 0.1853E-07  | -0.1023E-16 |
|                     | 5                | 0.1625E 01                | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | -0.1818E-07 | -0.2331E-07 | -0.5897E-17 |
|                     | 6                | 0.1625E 01                | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | -0.1818E-07 | -0.2459E-07 | -0.3003E-16 |
|                     | 7                | 0.1625E 01                | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | 0.1414E-07  | -0.6971E-08 | 0.7467E-17  |
|                     | 8                | 0.1625E 01                | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | 0.1414E-07  | 0.1853E-07  | -0.1023E-16 |
|                     |                  |                           | ***** CUMULATIVE CREEP STRAINS *****  |            |             |             |             |             |
|                     |                  |                           | XX                                    | YY         | ZZ          | XY          | XZ          | YZ          |
|                     | 1                | 0.2750E 01                | -0.5000E 00                           | 0.1000E 01 | -0.5000E 00 | 0.5203E-07  | -0.6247E-07 | -0.2198E-16 |
|                     | 2                | 0.2750E 01                | -0.5000E 00                           | 0.1000E 01 | -0.5000E 00 | 0.5203E-07  | 0.3342E-08  | -0.4154E-16 |
|                     | 3                | 0.2750E 01                | -0.5000E 00                           | 0.1000E 01 | -0.5000E 00 | 0.5854E-07  | 0.8127E-07  | 0.1155E-16  |
|                     | 4                | 0.2750E 01                | -0.5000E 00                           | 0.1000E 01 | -0.5000E 00 | -0.5854E-07 | -0.4952E-07 | -0.5085E-16 |
|                     | 5                | 0.2750E 01                | -0.5000E 00                           | 0.1000E 01 | -0.5000E 00 | -0.7101E-08 | 0.6247E-07  | 0.1324E-16  |
|                     | 6                | 0.2750E 01                | -0.5000E 00                           | 0.1000E 01 | -0.5000E 00 | -0.7101E-08 | 0.3342E-08  | -0.6557E-16 |
|                     | 7                | 0.2750E 01                | -0.5000E 00                           | 0.1000E 01 | -0.5000E 00 | 0.2679E-07  | -0.8127E-07 | -0.1127E-16 |
|                     | 8                | 0.2750E 01                | -0.5000E 00                           | 0.1000E 01 | -0.5000E 00 | 0.2679E-07  | 0.4952E-07  | -0.5085E-16 |

# PLASTIC AND CREEP STRAINS

ELEMENT NO. = 1 ID = 3

| INT | E-P | SUM | INCREMENTAL |             | TOTAL       |             | SURFACE    |            | *** EFFECTIVE PLASTIC STRAINS *** |            |            | **** EFFECTIVE CREEP STRAINS **** |            |            |
|-----|-----|-----|-------------|-------------|-------------|-------------|------------|------------|-----------------------------------|------------|------------|-----------------------------------|------------|------------|
|     |     |     | TEMPERATURE | TEMPERATURE | TEMPERATURE | TEMPERATURE | YIELD SIZE | SIZE       | INCREMENTAL                       | SUM INCR.  | CUMULATIVE | INCREMENTAL                       | SUM INCR.  | CUMULATIVE |
| 1   | 0   | 2   | 0.1000E 01  | 0.5000E 01  | 0.5000E 01  | 0.5000E 01  | 0.2000E 01 | 0.2000E 01 | 0.1000E 01                        | 0.2000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 |
| 2   | 0   | 2   | 0.1000E 01  | 0.5000E 01  | 0.5000E 01  | 0.5000E 01  | 0.2000E 01 | 0.2000E 01 | 0.1000E 01                        | 0.2000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 |
| 3   | 0   | 2   | 0.1000E 01  | 0.5000E 01  | 0.5000E 01  | 0.5000E 01  | 0.2000E 01 | 0.2000E 01 | 0.1000E 01                        | 0.2000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 |
| 4   | 0   | 2   | 0.1000E 01  | 0.5000E 01  | 0.5000E 01  | 0.5000E 01  | 0.2000E 01 | 0.2000E 01 | 0.1000E 01                        | 0.2000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 |
| 5   | 0   | 2   | 0.1000E 01  | 0.5000E 01  | 0.5000E 01  | 0.5000E 01  | 0.2000E 01 | 0.2000E 01 | 0.1000E 01                        | 0.2000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 |
| 6   | 0   | 2   | 0.1000E 01  | 0.5000E 01  | 0.5000E 01  | 0.5000E 01  | 0.2000E 01 | 0.2000E 01 | 0.1000E 01                        | 0.2000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 |
| 7   | 0   | 2   | 0.1000E 01  | 0.5000E 01  | 0.5000E 01  | 0.5000E 01  | 0.2000E 01 | 0.2000E 01 | 0.1000E 01                        | 0.2000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 |
| 8   | 0   | 2   | 0.1000E 01  | 0.5000E 01  | 0.5000E 01  | 0.5000E 01  | 0.2000E 01 | 0.2000E 01 | 0.1000E 01                        | 0.2000E 01 | 0.2000E 01 | 0.5000E 00                        | 0.1000E 01 | 0.1000E 01 |

14.1-29

ORIGINAL  
OF FOUR COPIES

## INCREMENT

ACDE I.D. 1 2 3 4 5 6 7 8  
 TEMP. 0.60000E-01 0.60000E-01 0.60000E-01 0.60000E-01 0.60000E-01 0.60000E-01 0.60000E-01 0.60000E-01

RESIDUAL NORM = 0.45251E-00  
 RESIDUAL NORM = 0.35218E-00  
 RESIDUAL NORM = 0.88450E-01  
 RESIDUAL NORM = 0.43692E-01  
 RESIDUAL NORM = 0.16476E-02  
 RESIDUAL NORM = 0.62756E-03  
 RESIDUAL NORM = 0.22189E-04  
 RESIDUAL NORM = 0.11591E-04  
 RESIDUAL NORM = 0.10046E-05

ORIGINAL PAGE IS  
 OF POOR QUALITY

END OF LOAD INCREMENT 5

## INCREMENT 5

MECHANICAL LOAD CURVE FACTORS = 0.7300E-01, 0.0

CREEP TIME INCREMENT = 0.0

NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 8

0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 0

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 9

SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.1000E-04, ACTUAL ERROR = 0.1005E-05

## CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** | I.D. | FORCES         |                |                | DISPLACEMENTS  |               |     |
|------------|------|----------------|----------------|----------------|----------------|---------------|-----|
| NO.        | I.D. | U              | V              | W              | U              | V             | W   |
| 1          | 1    | -0.3647755E-06 | -0.1000000E-01 | 0.4555022E-06  | -0.2384703E-06 | 0.0           | 0.0 |
| 2          | 2    | 0.3642311E-06  | -0.1000001E-01 | 0.4549975E-06  | -0.2737907E-05 | 0.0           | 0.0 |
| 3          | 2    | 0.2636528E-07  | -0.1000001E-01 | 0.2896955E-06  | -0.1650114E-05 | 0.7299999E-01 | 0.0 |
| 4          | 4    | -0.2703433E-07 | 0.1000000E-01  | 0.2894096E-06  | -0.8533784E-06 | 0.7299999E-01 | 0.0 |
| 5          | 5    | -0.1416241E-06 | -0.1000000E-01 | -0.4549966E-06 | 0.0            | 0.0           | 0.0 |
| 6          | 6    | 0.1423775E-06  | -0.1000000E-01 | -0.4555022E-06 | -0.2494345E-05 | 0.0           | 0.0 |
| 7          | 7    | 0.3564880E-07  | 0.1000000E-01  | -0.2899087E-06 | -0.2886681E-05 | 0.7299999E-01 | 0.0 |
| 8          | 8    | -0.3563636E-07 | 0.1000000E-01  | -0.2897004E-06 | 0.3329068E-06  | 0.7299999E-01 | 0.0 |



## THERMAL AND ELASTIC STRAINS

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | INCREMENTAL<br>THERMAL STRAINS | INCREMENTAL ELASTIC STRAINS |            |             |             |             |             |
|---------------------|------------------|--------------------------------|-----------------------------|------------|-------------|-------------|-------------|-------------|
|                     |                  |                                | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
| 1 3                 | 1                | 0.5000E 00                     | 0.1788E-06                  | 0.0        | 0.1192E-05  | 0.4268E-08  | -0.3382E-07 | -0.3666E-17 |
|                     | 2                | 0.5000E 00                     | 0.1788E-06                  | 0.0        | 0.1132E-05  | 0.4268E-08  | 0.3305E-07  | -0.4330E-16 |
|                     | 3                | 0.5000E 00                     | 0.1788E-06                  | 0.0        | 0.1192E-05  | -0.4796E-08 | 0.3305E-07  | -0.2922E-17 |
|                     | 4                | 0.5000E 00                     | 0.1788E-06                  | 0.0        | 0.1132E-05  | -0.4796E-08 | 0.3305E-07  | -0.4877E-16 |
|                     | 5                | 0.5000E 00                     | 0.5960E-06                  | 0.0        | 0.1252E-05  | 0.7241E-08  | -0.3382E-07 | -0.1293E-18 |
|                     | 6                | 0.5000E 00                     | -0.1013E-05                 | 0.2086E-05 | 0.8941E-06  | 0.7241E-08  | -0.3382E-07 | -0.1296E-17 |
|                     | 7                | 0.5000E 00                     | 0.5960E-06                  | 0.0        | 0.1252E-05  | -0.7237E-08 | 0.3305E-07  | -0.2774E-17 |
|                     | 8                | 0.5000E 00                     | -0.1013E-05                 | 0.2086E-05 | 0.8941E-06  | -0.7237E-08 | 0.3305E-07  | -0.4877E-16 |
|                     | INTEGR.<br>POINT | CUMULATIVE<br>THERMAL STRAINS  | CUMULATIVE ELASTIC STRAINS  |            |             |             |             |             |
|                     |                  |                                | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
|                     | 1                | 0.2300E 01                     | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | 0.2334E-08  | -0.6552E-07 | 0.1351E-16  |
|                     | 2                | 0.2300E 01                     | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | 0.2334E-08  | -0.6662E-07 | -0.5843E-16 |
|                     | 3                | 0.2300E 01                     | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | 0.2747E-08  | 0.6473E-07  | -0.1127E-16 |
|                     | 4                | 0.2300E 01                     | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.2757E-08 | 0.9649E-07  | -0.5920E-16 |
|                     | 5                | 0.2300E 01                     | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.2877E-07 | -0.6552E-07 | 0.1545E-16  |
|                     | 6                | 0.2300E 01                     | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.2977E-07 | -0.6662E-07 | -0.5562E-16 |
|                     | 7                | 0.2300E 01                     | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | 0.2036E-07  | 0.6473E-07  | 0.1119E-16  |
|                     | 8                | 0.2300E 01                     | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | 0.2896E-07  | 0.9649E-07  | -0.5920E-16 |

## PLASTIC WORK AND STRAINS

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | INCREMENTAL<br>PLASTIC WORK | INCREMENTAL PLASTIC STRAINS |            |             |             |             |             |
|---------------------|------------------|-----------------------------|-----------------------------|------------|-------------|-------------|-------------|-------------|
|                     |                  |                             | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
| 1 3                 | 1                | 0.3750E 01                  | -0.5000E 00                 | 0.1000E 01 | -0.5000E 00 | -0.1568E-07 | -0.1123E-06 | -0.2448E-16 |
|                     | 2                | 0.3750E 01                  | -0.5000E 00                 | 0.1000E 01 | -0.5000E 00 | -0.1568E-07 | -0.1460E-06 | -0.3497E-16 |
|                     | 3                | 0.3750E 01                  | -0.5000E 00                 | 0.1000E 01 | -0.5000E 00 | -0.2073E-07 | 0.6475E-07  | 0.1680E-16  |
|                     | 4                | 0.3750E 01                  | -0.5000E 00                 | 0.1000E 01 | -0.5000E 00 | -0.2073E-07 | -0.1248E-06 | -0.4837E-16 |
|                     | 5                | 0.3750E 01                  | -0.5000E 00                 | 0.1000E 01 | -0.5000E 00 | -0.4934E-07 | -0.1123E-06 | 0.2069E-16  |
|                     | 6                | 0.3750E 01                  | -0.5000E 00                 | 0.1000E 01 | -0.5000E 00 | -0.4934E-07 | -0.1460E-06 | -0.8197E-16 |
|                     | 7                | 0.3750E 01                  | -0.5000E 00                 | 0.1000E 01 | -0.5000E 00 | -0.4326E-07 | 0.6475E-07  | -0.1685E-16 |
|                     | 8                | 0.3750E 01                  | -0.5000E 00                 | 0.1000E 01 | -0.5000E 00 | 0.4326E-07  | 0.1248E-06  | -0.4837E-16 |
|                     | INTEGR.<br>POINT | CUMULATIVE<br>PLASTIC WORK  | CUMULATIVE PLASTIC STRAINS  |            |             |             |             |             |
|                     |                  |                             | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
|                     | 1                | 0.9500E 01                  | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | 0.1143E-07  | -0.8790E-07 | 0.7053E-16  |
|                     | 2                | 0.9500E 01                  | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | 0.1143E-07  | -0.1787E-06 | -0.1151E-15 |
|                     | 3                | 0.9500E 01                  | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | 0.9505E-08  | 0.1148E-06  | 0.4178E-16  |
|                     | 4                | 0.9500E 01                  | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | 0.9505E-08  | 0.1188E-06  | -0.1436E-15 |
|                     | 5                | 0.9500E 01                  | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | -0.7541E-07 | -0.8790E-07 | -0.4764E-16 |
|                     | 6                | 0.9500E 01                  | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | -0.7941E-07 | -0.1787E-06 | -0.2088E-15 |
|                     | 7                | 0.9500E 01                  | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | 0.8362E-07  | 0.1148E-06  | 0.4125E-16  |
|                     | 8                | 0.9500E 01                  | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | -0.8362E-07 | -0.1188E-06 | -0.1436E-15 |

CUMULATIVE STRESS QUANTITIES

| ELEMENT NO.                     | I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | CUMULATIVE STRESS CENTER |             |             |             |             |             |
|---------------------------------|------|---------------|-------------------------|--------------------------|-------------|-------------|-------------|-------------|-------------|
|                                 |      |               |                         | XX                       | YY          | ZZ          | XY          | XZ          | YZ          |
| 1                               | 3    | 1             | 0.2000E 01              | -0.6667E 00              | -0.1333E 01 | -0.6667E 00 | 0.2078E 07  | -0.5620E 03 | -0.2013E 16 |
|                                 |      | 2             | 0.2000E 01              | -0.6667E 00              | 0.1333E 01  | -0.6667E 00 | 0.2078E 07  | -0.5077E 07 | -0.5944E 16 |
|                                 |      | 3             | 0.2000E 01              | -0.6667E 00              | 0.1333E 01  | -0.6667E 00 | 0.2082E 07  | 0.5959E 07  | 0.1720E 16  |
|                                 |      | 4             | 0.2000E 01              | -0.6667E 00              | 0.1333E 01  | -0.6667E 00 | 0.2082E 07  | -0.7067E 07 | -0.7230E 16 |
|                                 |      | 5             | 0.2000E 01              | -0.6667E 00              | 0.1333E 01  | -0.6667E 00 | -0.2437E 07 | -0.5620E 08 | -0.2027E 16 |
|                                 |      | 6             | 0.2000E 01              | -0.6667E 00              | 0.1333E 01  | -0.6667E 00 | -0.2437E 07 | -0.5077E 07 | -0.9187E 16 |
|                                 |      | 7             | 0.2000E 01              | -0.6667E 00              | 0.1333E 01  | -0.6667E 00 | 0.3189E 07  | -0.5959E 07 | -0.1693E 16 |
|                                 |      | 8             | 0.2000E 01              | -0.6667E 00              | 0.1333E 01  | -0.6667E 00 | 0.3189E 07  | 0.7307E 07  | -0.1280E 16 |
| ***** CUMULATIVE STRESSES ***** |      |               |                         |                          |             |             |             |             |             |
|                                 |      | INTEGR. POINT | EFFECTIVE STRESS        | XX                       | YY          | ZZ          | XY          | XZ          | YZ          |
| 1                               | 2    | 1             | 0.4000E 01              | 0.4585E 06               | 0.4000E 01  | 0.1834E 05  | 0.7180E 08  | -0.2216E 06 | 0.5690E 16  |
|                                 |      | 2             | 0.4000E 01              | 0.4585E 06               | 0.4000E 01  | 0.1375E 05  | 0.7180E 08  | -0.2473E 06 | 0.1769E 15  |
|                                 |      | 3             | 0.4000E 01              | 0.4585E 06               | 0.4000E 01  | 0.1834E 05  | -0.8607E 08 | 0.1942E 06  | 0.3473E 16  |
|                                 |      | 4             | 0.4000E 01              | 0.4585E 06               | 0.4000E 01  | 0.1375E 05  | -0.8607E 08 | 0.2569E 06  | -0.2129E 15 |
|                                 |      | 5             | 0.4000E 01              | 0.2292E 05               | 0.4000E 01  | 0.2292E 05  | -0.8652E 07 | 0.2417E 06  | -0.4752E 16 |
|                                 |      | 6             | 0.4000E 01              | -0.9170E 06              | 0.4000E 01  | 0.4585E 06  | -0.8652E 07 | -0.2973E 06 | -0.2919E 15 |
|                                 |      | 7             | 0.4000E 01              | 0.2292E 05               | 0.4000E 01  | 0.2292E 05  | 0.8912E 07  | 0.1992E 06  | 0.3443E 16  |
|                                 |      | 8             | 0.4000E 01              | -0.9170E 06              | 0.4000E 01  | 0.4585E 06  | 0.8912E 07  | 0.2969E 06  | -0.2129E 15 |

## PLASTIC AND CREEP STRAINS

|                 |      |        |             |                                   |            |             |            |            |             |             |            |                                 |            |            |  |
|-----------------|------|--------|-------------|-----------------------------------|------------|-------------|------------|------------|-------------|-------------|------------|---------------------------------|------------|------------|--|
| ELEMENT NO. = 1 |      | ID = 3 |             | *** EFFECTIVE PLASTIC STRAINS *** |            |             |            |            |             |             |            | *** EFFECTIVE CREEP STRAINS *** |            |            |  |
| INT             |      | E-P    |             | SUM INCREMENTAL                   |            | TOTAL       |            | SURFACE    |             | INCREMENTAL |            | SUM INCR.                       |            | CUMULATIVE |  |
| PNT             | CODE | CODE   | TEMPERATURE | TEMPERATURE                       | YIELD SIZE | INCREMENTAL | SUM INCR.  | CUMULATIVE | INCREMENTAL | SUM INCR.   | CUMULATIVE | INCREMENTAL                     | SUM INCR.  | CUMULATIVE |  |
| 1               | 0    | 2      | 0.1000E 01  | 0.6000E 01                        | 0.2000E 01 | 0.1000E 01  | 0.3000E 01 | 0.3000E 01 | 0.0         | 0.1000E 01  | 0.1000E 01 | 0.0                             | 0.1000E 01 | 0.1000E 01 |  |
| 2               | 0    | 2      | 0.1000E 01  | 0.6000E 01                        | 0.2000E 01 | 0.1000E 01  | 0.3000E 01 | 0.3000E 01 | 0.0         | 0.1000E 01  | 0.1000E 01 | 0.0                             | 0.1000E 01 | 0.1000E 01 |  |
| 3               | 0    | 2      | 0.1000E 01  | 0.6000E 01                        | 0.2000E 01 | 0.1000E 01  | 0.3000E 01 | 0.3000E 01 | 0.0         | 0.1000E 01  | 0.1000E 01 | 0.0                             | 0.1000E 01 | 0.1000E 01 |  |
| 4               | 0    | 2      | 0.1000E 01  | 0.6000E 01                        | 0.2000E 01 | 0.1000E 01  | 0.3000E 01 | 0.3000E 01 | 0.0         | 0.1000E 01  | 0.1000E 01 | 0.0                             | 0.1000E 01 | 0.1000E 01 |  |
| 5               | 0    | 2      | 0.1000E 01  | 0.6000E 01                        | 0.2000E 01 | 0.1000E 01  | 0.3000E 01 | 0.3000E 01 | 0.0         | 0.1000E 01  | 0.1000E 01 | 0.0                             | 0.1000E 01 | 0.1000E 01 |  |
| 6               | 0    | 2      | 0.1000E 01  | 0.6000E 01                        | 0.2000E 01 | 0.1000E 01  | 0.3000E 01 | 0.3000E 01 | 0.0         | 0.1000E 01  | 0.1000E 01 | 0.0                             | 0.1000E 01 | 0.1000E 01 |  |
| 7               | 0    | 2      | 0.1000E 01  | 0.6000E 01                        | 0.2000E 01 | 0.1000E 01  | 0.3000E 01 | 0.3000E 01 | 0.0         | 0.1000E 01  | 0.1000E 01 | 0.0                             | 0.1000E 01 | 0.1000E 01 |  |
| 8               | 0    | 2      | 0.1000E 01  | 0.6000E 01                        | 0.2000E 01 | 0.1000E 01  | 0.3000E 01 | 0.3000E 01 | 0.0         | 0.1000E 01  | 0.1000E 01 | 0.0                             | 0.1000E 01 | 0.1000E 01 |  |

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OF POOR QUALITY

# CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 6

INCREMENT 6

NO. 1.0. 1 2 3 4 5 6 7 8  
TEMP. 0.70000E-01 0.70000E-01 0.70000E-01 0.70000E-01 0.70000E-01 0.70000E-01 0.70000E-01 0.70000E-01

RESIDUAL TERM = 0.20130E-05

END OF LOAD INCREMENT 6

INCREMENT 6

MECHANICAL LOAD CURVE FACTORS = 0.73000E 01, 0.0

CREEP TIME INCREMENT = 0.0

NO. ELASTIC INTEGRATION POINTS = 8, NO. PLASTIC INTEGRATION POINTS = 0

0 1 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 8 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 0

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 1

SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.10000E-04, ACTUAL ERROR = 0.2013E-05

## CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** |      | ***** FORCES ***** |                |                | ***** DISPLACEMENTS ***** |               |     |
|------------|------|--------------------|----------------|----------------|---------------------------|---------------|-----|
| NO.        | 1.0. | U                  | V              | W              | U                         | V             | W   |
| 1          | 1    | 0.1208186E-05      | -0.4999993E 00 | -0.8778679E-06 | 0.1525287E-06             | 0.0           | 0.0 |
| 2          | 2    | -0.1208147E-05     | -0.5000002E 00 | -0.8778343E-06 | -0.2576220E-05            | 0.0           | 0.0 |
| 3          | 3    | 0.1160998E-05      | -0.4999992E 00 | -0.9274763E-06 | -0.1485144E-05            | 0.7299999E 01 | 0.0 |
| 4          | 4    | 0.1161010E-05      | 0.4999993E 00  | -0.9274918E-06 | -0.9696514E-06            | 0.7299999E 01 | 0.0 |
| 5          | 5    | 0.1113060E-05      | -0.4999995E 00 | 0.8778343E-06  | 0.0                       | 0.0           | 0.0 |
| 6          | 6    | -0.1119113E-05     | -0.4999994E 00 | -0.8778679E-06 | -0.2846014E-05            | 0.0           | 0.0 |
| 7          | 7    | -0.1037709E-05     | 0.4999994E 00  | 0.9274936E-06  | -0.2903628E-05            | 0.7299999E 01 | 0.0 |
| 8          | 8    | 0.1037113E-05      | 0.4999995E 00  | 0.9274754E-06  | 0.4017872E-06             | 0.7299999E 01 | 0.0 |

04.1.133

## THERMAL AND ELASTIC STRAINS

| ELEMENT NO. | INTEGR. POINT | INTEGRAL THERMAL STRAINS | XX          | YY          | ZZ          | XY          | XZ          | YZ  |
|-------------|---------------|--------------------------|-------------|-------------|-------------|-------------|-------------|-----|
| 1           | 3             | 0.9537E-06               | -0.1046E-05 | -0.9537E-06 | -0.9537E-06 | -0.1497E-07 | 0.7810E-07  | 0.0 |
|             | 1             | 0.9537E-06               | -0.9849E-06 | -0.9537E-06 | -0.9537E-06 | -0.1497E-07 | -0.1759E-07 | 0.0 |
|             | 2             | 0.9537E-06               | -0.1046E-05 | -0.9537E-06 | -0.9537E-06 | 0.1550E-07  | -0.7719E-07 | 0.0 |
|             | 3             | 0.9537E-06               | -0.9849E-06 | -0.9537E-06 | -0.9537E-06 | 0.1550E-07  | 0.1774E-07  | 0.0 |
|             | 4             | 0.9537E-06               | -0.1046E-05 | -0.9537E-06 | -0.9537E-06 | -0.1107E-06 | -0.7810E-07 | 0.0 |
|             | 5             | 0.9537E-06               | -0.1356E-05 | -0.9537E-06 | -0.9537E-06 | -0.1107E-06 | -0.1759E-07 | 0.0 |
|             | 6             | 0.9537E-06               | -0.9142E-06 | -0.9537E-06 | -0.9537E-06 | 0.1104E-06  | -0.7719E-07 | 0.0 |
|             | 7             | 0.9537E-06               | -0.1356E-05 | -0.9537E-06 | -0.9537E-06 | 0.1104E-06  | -0.1774E-07 | 0.0 |
|             | 8             | 0.9537E-06               | -0.9142E-06 | -0.9537E-06 | -0.9537E-06 | 0.1104E-06  | -0.1774E-07 | 0.0 |

| INTEGR. POINT | CUMULATIVE THERMAL STRAINS | XX          | YY         | ZZ          | XY          | XZ          | YZ          |
|---------------|----------------------------|-------------|------------|-------------|-------------|-------------|-------------|
| 1             | 0.2300E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.1264E-07 | 0.1250E-07  | 0.1851E-16  |
| 2             | 0.2300E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.1264E-07 | -0.1142E-06 | -0.5843E-16 |
| 3             | 0.2300E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.1264E-07 | -0.1247E-07 | -0.1129E-16 |
| 4             | 0.2300E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.1264E-07 | 0.1142E-06  | -0.6920E-16 |
| 5             | 0.2300E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.1264E-07 | 0.1250E-07  | 0.1543E-16  |
| 6             | 0.2300E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.1264E-07 | -0.1142E-06 | -0.6502E-16 |
| 7             | 0.2300E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.1264E-07 | -0.1247E-07 | -0.1119E-16 |
| 8             | 0.2300E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | -0.1264E-07 | 0.1142E-06  | -0.6920E-16 |

14.1-34

## PLASTIC WORK AND STRAINS

| ELEMENT NO. | INTEGR. POINT | INTEGRAL PLASTIC WORK | XX  | YY  | ZZ  | XY  | XZ  | YZ  |
|-------------|---------------|-----------------------|-----|-----|-----|-----|-----|-----|
| 1           | 4             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 2             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 3             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 4             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 5             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 6             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 7             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             | 8             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| INTEGR. POINT | CUMULATIVE PLASTIC WORK | XX          | YY         | ZZ          | XY          | XZ          | YZ          |
|---------------|-------------------------|-------------|------------|-------------|-------------|-------------|-------------|
| 1             | 0.9500E 01              | -0.1500E 01 | 0.3000E 01 | -0.1500E 01 | 0.1143E-07  | -0.8790E-07 | 0.7053E-16  |
| 2             | 0.9500E 01              | -0.1500E 01 | 0.3000E 01 | -0.1500E 01 | 0.1143E-07  | -0.1763E-06 | -0.1151E-15 |
| 3             | 0.9500E 01              | -0.1500E 01 | 0.3000E 01 | -0.1500E 01 | 0.9505E-08  | 0.1148E-06  | 0.4178E-16  |
| 4             | 0.9500E 01              | -0.1500E 01 | 0.3000E 01 | -0.1500E 01 | 0.9505E-08  | 0.1885E-06  | -0.1435E-15 |
| 5             | 0.9500E 01              | -0.1500E 01 | 0.3000E 01 | -0.1500E 01 | 0.7941E-07  | -0.8750E-07 | 0.7047E-16  |
| 6             | 0.9500E 01              | -0.1500E 01 | 0.3000E 01 | -0.1500E 01 | -0.7941E-07 | -0.1787E-06 | -0.2000E-15 |
| 7             | 0.9500E 01              | -0.1500E 01 | 0.3000E 01 | -0.1500E 01 | 0.8362E-07  | 0.1148E-06  | 0.1129E-16  |
| 8             | 0.9500E 01              | -0.1500E 01 | 0.3000E 01 | -0.1500E 01 | 0.8362E-07  | -0.1844E-06 | -0.1436E-15 |

# CUMULATIVE STRESS QUANTITIES

| ELEMENT NO. | I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | ***** CUMULATIVE STRESS CENTER ***** |            |             |             |             |             |
|-------------|------|---------------|-------------------------|--------------------------------------|------------|-------------|-------------|-------------|-------------|
|             |      |               |                         | XX                                   | YY         | ZZ          | XY          | XZ          | YZ          |
| 1           | 1    | 1             | 0.2000E-01              | -0.6667E-00                          | 0.1333E-01 | -0.6667E-00 | 0.2078E-07  | -0.5620E-08 | -0.3013E-16 |
|             |      | 2             | 0.2000E-01              | -0.6667E-00                          | 0.1333E-01 | -0.6667E-00 | 0.2078E-07  | -0.5077E-07 | -0.5644E-16 |
|             |      | 3             | 0.2000E-01              | -0.6667E-00                          | 0.1333E-01 | -0.6667E-00 | 0.2082E-07  | -0.5459E-07 | -0.1720E-16 |
|             |      | 4             | 0.2000E-01              | -0.6667E-00                          | 0.1333E-01 | -0.6667E-00 | -0.2082E-07 | -0.7067E-07 | -0.7280E-16 |
|             |      | 5             | 0.2000E-01              | -0.6667E-00                          | 0.1333E-01 | -0.6667E-00 | -0.2437E-07 | -0.5620E-08 | 0.2027E-16  |
|             |      | 6             | 0.2000E-01              | -0.6667E-00                          | 0.1333E-01 | -0.6667E-00 | -0.2437E-07 | -0.5077E-07 | -0.4187E-16 |
|             |      | 7             | 0.2000E-01              | -0.6667E-00                          | 0.1333E-01 | -0.6667E-00 | -0.3189E-07 | -0.5959E-07 | -0.1693E-16 |
|             |      | 8             | 0.2000E-01              | -0.6667E-00                          | 0.1333E-01 | -0.6667E-00 | 0.3189E-07  | 0.7067E-07  | -0.7280E-16 |

| INTEGR. POINT | EFFECTIVE STRESS | ***** CUMULATIVE STRESSES ***** |            |             |             |             |             |
|---------------|------------------|---------------------------------|------------|-------------|-------------|-------------|-------------|
|               |                  | XX                              | YY         | ZZ          | XY          | XZ          | YZ          |
| 1             | 0.2000E-01       | -0.4356E-05                     | 0.2000E-01 | -0.3209E-05 | -0.1944E-07 | 0.1935E-07  | 0.2848E-16  |
| 2             | 0.2000E-01       | -0.4126E-05                     | 0.2000E-01 | -0.3209E-05 | -0.1944E-07 | -0.1757E-06 | -0.8989E-16 |
| 3             | 0.2000E-01       | -0.4356E-05                     | 0.2000E-01 | -0.3209E-05 | 0.1954E-07  | -0.1918E-07 | -0.1756E-16 |
| 4             | 0.2000E-01       | -0.4126E-05                     | 0.2000E-01 | -0.3209E-05 | 0.1954E-07  | 0.1757E-06  | -0.1065E-15 |
| 5             | 0.2000E-01       | -0.4314E-05                     | 0.2000E-01 | -0.3668E-05 | -0.2145E-06 | -0.1935E-07 | -0.2376E-16 |
| 6             | 0.2000E-01       | -0.5043E-05                     | 0.2000E-01 | -0.4356E-05 | -0.2145E-06 | -0.1757E-06 | -0.1010E-15 |
| 7             | 0.2000E-01       | -0.4814E-05                     | 0.2000E-01 | -0.3668E-05 | 0.2145E-06  | -0.1918E-07 | 0.1721E-16  |
| 8             | 0.2000E-01       | -0.5043E-05                     | 0.2000E-01 | -0.4356E-05 | -0.2145E-06 | 0.1757E-06  | -0.1065E-15 |

## PLASTIC AND CREEP STRAINS

| ELEMENT NO. = 1 ID = 3 |      |      |                 |             |            |             |             |            |             |
|------------------------|------|------|-----------------|-------------|------------|-------------|-------------|------------|-------------|
| INT                    | E    | P    | SUM INCREMENTAL | TOTAL       | SURFACE    | YIELD SIZE  | INCREMENTAL | SUM INCR.  | CUMULATIVE  |
| PNT                    | CODE | CODE | TEMPERATURE     | TEMPERATURE | YIELD SIZE | INCREMENTAL | SUM INCR.   | CUMULATIVE | INCREMENTAL |
| 1                      | -1   | -1   | 0.1000E-01      | 0.7000E-01  | 0.2000E-01 | 0.0         | 0.3000E-01  | 0.3000E-01 | 0.0         |
| 2                      | -1   | -1   | 0.1000E-01      | 0.7000E-01  | 0.2000E-01 | 0.0         | 0.3000E-01  | 0.3000E-01 | 0.0         |
| 3                      | -1   | -1   | 0.1000E-01      | 0.7000E-01  | 0.2000E-01 | 0.0         | 0.3000E-01  | 0.3000E-01 | 0.0         |
| 4                      | -1   | -1   | 0.1000E-01      | 0.7000E-01  | 0.2000E-01 | 0.0         | 0.3000E-01  | 0.3000E-01 | 0.0         |
| 5                      | -1   | -1   | 0.1000E-01      | 0.7000E-01  | 0.2000E-01 | 0.0         | 0.3000E-01  | 0.3000E-01 | 0.0         |
| 6                      | -1   | -1   | 0.1000E-01      | 0.7000E-01  | 0.2000E-01 | 0.0         | 0.3000E-01  | 0.3000E-01 | 0.0         |
| 7                      | -1   | -1   | 0.1000E-01      | 0.7000E-01  | 0.2000E-01 | 0.0         | 0.3000E-01  | 0.3000E-01 | 0.0         |
| 8                      | -1   | -1   | 0.1000E-01      | 0.7000E-01  | 0.2000E-01 | 0.0         | 0.3000E-01  | 0.3000E-01 | 0.0         |

INCREMENT 7 CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 7

| NODE I.D. | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| TEMP.     | 0.54000E 01 | 0.54000E 01 | 0.54000E 01 | 0.54000E 01 | 0.54000E 01 | 0.54000E 01 | 0.54000E 01 | 0.54000E 01 |

RESIDUAL URM = 0.23864E 00  
 RESIDUAL URM = 0.18486E 00  
 RESIDUAL URM = 0.13613E 00  
 RESIDUAL URM = 0.98414E 01  
 RESIDUAL URM = 0.70155E 01  
 RESIDUAL URM = 0.47494E 01  
 RESIDUAL URM = 0.34655E 01  
 RESIDUAL URM = 0.24135E 01  
 RESIDUAL URM = 0.16745E 01  
 RESIDUAL URM = 0.11587E 01  
 RESIDUAL URM = 0.04333E 08

END OF LOAD INCREMENT 7

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INCREMENT 7  
 MECHANICAL LOAD CURVE FACTORS = 0.6000E 01, 0.0  
 CREEP TIME INCREMENT = 0.0  
 NO. ELASTIC INTEGRATION POINTS = 8, NO. PLASTIC INTEGRATION POINTS = 0  
 0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT  
 SPECIFIC MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 1  
 SPECIFIC MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 1  
 SPECIFIC MAX. UNBALANCED FORCE ERROR = 0.1000E 04, ACTUAL ERROR = 0.6433E 08

# CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** | ** I.D. ** | U              | V              | W              | U              | V             | W   |
|------------|------------|----------------|----------------|----------------|----------------|---------------|-----|
| 1          | 1          | 0.6502887E-09  | 0.3576279E-06  | 0.4988806E-08  | 0.1426809E-06  | 0.0           | 0.0 |
| 2          | 2          | 0.2592969E-08  | -0.4768372E-06 | 0.1127285E-07  | -0.2244669E-05 | 0.0           | 0.0 |
| 3          | 3          | 0.3237828E-08  | -0.4768372E-06 | -0.1326157E-09 | -0.1138652E-05 | 0.6000000E 01 | 0.0 |
| 4          | 4          | 0.1415083E-08  | -0.3576279E-06 | -0.1803187E-08 | -0.1010630E-05 | 0.6000000E 01 | 0.0 |
| 5          | 5          | -0.2193703E-07 | 0.1788139E-06  | -0.1127285E-07 | 0.0            | 0.0           | 0.0 |
| 6          | 6          | -0.8041752E-08 | -0.2334186E-06 | -0.4588806E-08 | -0.2635398E-05 | 0.0           | 0.0 |
| 7          | 7          | 0.1186254E-07  | -0.2334186E-06 | 0.1804892E-08  | -0.2684036E-05 | 0.6000000E 01 | 0.0 |
| 8          | 8          | -0.5871698E-08 | -0.1788139E-06 | 0.1317630E-09  | 0.5180750E-06  | 0.6000000E 01 | 0.0 |

14.1-36

~~THERMAL AND ELASTIC STRAINS~~

| ELEMENT |      | INTEGR. | INCREMENTAL ELASTIC STRAINS |            |             |            |            |             |             |
|---------|------|---------|-----------------------------|------------|-------------|------------|------------|-------------|-------------|
| NO.     | I.D. | POINT   | THERMAL STRAINS             | XX         | YY          | ZZ         | XY         | XZ          | YZ          |
|         |      | 1       | -0.3000E 00                 | 0.3000E 00 | -0.1000E 01 | 0.3000E 00 | 0.3484E-07 | -0.3095E-08 | -0.1388E-16 |
|         |      | 2       | -0.3000E 00                 | 0.3000E 00 | -0.1000E 01 | 0.3000E 00 | 0.3484E-07 | -0.3095E-08 | -0.5551E-16 |
|         |      | 3       | -0.3000E 00                 | 0.3000E 00 | -0.1000E 01 | 0.3000E 00 | 0.1371E-07 | 0.4446E-07  | -0.3403E-17 |
|         |      | 4       | -0.3000E 00                 | 0.3000E 00 | -0.1000E 01 | 0.3000E 00 | 0.1371E-07 | 0.3631E-07  | 0.6245E-16  |
|         |      | 5       | -0.3000E 00                 | 0.3000E 00 | -0.1000E 01 | 0.3000E 00 | 0.1500E-08 | -0.3095E-08 | 0.5939E-17  |
|         |      | 6       | -0.3000E 00                 | 0.3000E 00 | -0.1000E 01 | 0.3000E 00 | 0.1500E-08 | -0.3095E-08 | 0.5551E-16  |
|         |      | 7       | -0.3000E 00                 | 0.3000E 00 | -0.1000E 01 | 0.3000E 00 | 0.5568E-08 | 0.4446E-07  | -0.3489E-17 |
|         |      | 8       | -0.3000E 00                 | 0.3000E 00 | -0.1000E 01 | 0.3000E 00 | 0.5568E-08 | 0.3631E-07  | 0.6245E-16  |

|  |  | INTEGR. | CUMULATIVE ELASTIC STRAINS |             |            |             |             |             |             |
|--|--|---------|----------------------------|-------------|------------|-------------|-------------|-------------|-------------|
|  |  | POINT   | THERMAL STRAINS            | XX          | YY         | ZZ          | XY          | XZ          | YZ          |
|  |  | 1       | 0.2000E 01                 | -0.5364E-06 | 0.1907E-05 | -0.4768E-06 | 0.2220E-07  | 0.9483E-08  | 0.4634E-17  |
|  |  | 2       | 0.2000E 01                 | -0.4768E-06 | 0.1907E-05 | -0.5364E-06 | 0.2220E-07  | -0.1506E-06 | -0.2913E-17 |
|  |  | 3       | 0.2000E 01                 | -0.5364E-06 | 0.1907E-05 | -0.4768E-06 | 0.2641E-07  | 0.3199E-07  | -0.7394E-17 |
|  |  | 4       | 0.2000E 01                 | -0.4768E-06 | 0.1907E-05 | -0.5364E-06 | 0.2641E-07  | 0.1505E-06  | -0.6752E-17 |
|  |  | 5       | 0.2000E 01                 | -0.4768E-06 | 0.1907E-05 | -0.6557E-06 | -0.1379E-06 | 0.4683E-08  | 0.8507E-17  |
|  |  | 6       | 0.2000E 01                 | -0.1013E-05 | 0.2861E-05 | -0.8941E-06 | 0.1379E-06  | -0.1506E-06 | -0.1011E-16 |
|  |  | 7       | 0.2000E 01                 | -0.4768E-06 | 0.1907E-05 | -0.6557E-06 | 0.1450E-06  | 0.3199E-07  | 0.1720E-17  |
|  |  | 8       | 0.2000E 01                 | -0.1013E-05 | 0.2861E-05 | -0.8941E-06 | 0.1450E-06  | 0.1505E-06  | -0.6752E-17 |

## PLASTIC WORK AND STRAINS

| ELEMENT NO.   | I.D.                    | INTEGR. POINT | INCREMENTAL PLASTIC WORK | INCREMENTAL PLASTIC STRAINS |             |             |             |     |     |
|---------------|-------------------------|---------------|--------------------------|-----------------------------|-------------|-------------|-------------|-----|-----|
|               |                         |               |                          | XX                          | YY          | ZZ          | XY          | XZ  | YZ  |
| 1             | 3                       | 1             | 0.0                      | 0.0                         | 0.0         | 0.0         | 0.0         | 0.0 | 0.0 |
|               |                         | 2             | 0.0                      | 0.0                         | 0.0         | 0.0         | 0.0         | 0.0 | 0.0 |
|               |                         | 3             | 0.0                      | 0.0                         | 0.0         | 0.0         | 0.0         | 0.0 | 0.0 |
|               |                         | 4             | 0.0                      | 0.0                         | 0.0         | 0.0         | 0.0         | 0.0 | 0.0 |
|               |                         | 5             | 0.0                      | 0.0                         | 0.0         | 0.0         | 0.0         | 0.0 | 0.0 |
|               |                         | 6             | 0.0                      | 0.0                         | 0.0         | 0.0         | 0.0         | 0.0 | 0.0 |
|               |                         | 7             | 0.0                      | 0.0                         | 0.0         | 0.0         | 0.0         | 0.0 | 0.0 |
|               |                         | 8             | 0.0                      | 0.0                         | 0.0         | 0.0         | 0.0         | 0.0 | 0.0 |
| INTEGR. POINT | CUMULATIVE PLASTIC WORK | XX            | YY                       | ZZ                          | XY          | XZ          | YZ          |     |     |
| 1             | 0.9500E 01              | -0.1500E 01   | 0.3000E 01               | -0.1500E 01                 | 0.1143E-07  | -0.8790E-07 | 0.7053E-16  |     |     |
| 2             | 0.9500E 01              | -0.1500E 01   | 0.3000E 01               | -0.1500E 01                 | 0.1143E-07  | -0.1787E-06 | -0.1151E-15 |     |     |
| 3             | 0.9500E 01              | -0.1500E 01   | 0.3000E 01               | -0.1500E 01                 | 0.9505E-08  | 0.1148E-06  | 0.4178E-16  |     |     |
| 4             | 0.9500E 01              | -0.1500E 01   | 0.3000E 01               | -0.1500E 01                 | 0.9505E-08  | 0.1889E-06  | -0.1430E-15 |     |     |
| 5             | 0.9500E 01              | -0.1500E 01   | 0.3000E 01               | -0.1500E 01                 | 0.7941E-07  | -0.8790E-07 | -0.7045E-16 |     |     |
| 6             | 0.9500E 01              | -0.1500E 01   | 0.3000E 01               | -0.1500E 01                 | -0.7941E-07 | -0.1787E-06 | -0.2089E-15 |     |     |
| 7             | 0.9500E 01              | -0.1500E 01   | 0.3000E 01               | -0.1500E 01                 | 0.8302E-07  | 0.1148E-06  | 0.4129E-16  |     |     |
| 8             | 0.9500E 01              | -0.1500E 01   | 0.3000E 01               | -0.1500E 01                 | 0.3302E-07  | 0.1889E-06  | -0.1430E-15 |     |     |

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# CUMULATIVE STRESS QUANTITIES

| ELEMENT NO. | I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | *****CUMULATIVE STRESS CENTER***** |            |             |             |             |             |
|-------------|------|---------------|-------------------------|------------------------------------|------------|-------------|-------------|-------------|-------------|
|             |      |               |                         | XX                                 | YY         | ZZ          | XY          | XZ          | YZ          |
| 1           | 3    | 1             | 0.2000E-01              | -0.6667E-00                        | 0.1333E-01 | -0.6667E-00 | 0.2078E-07  | -0.5620E-08 | -0.3013E-16 |
|             |      | 2             | 0.2000E-01              | -0.6667E-00                        | 0.1333E-01 | -0.6667E-00 | 0.2078E-07  | -0.5777E-07 | -0.5944E-16 |
|             |      | 3             | 0.2000E-01              | -0.6667E-00                        | 0.1333E-01 | -0.6667E-00 | 0.2082E-07  | 0.5959E-07  | 0.1720E-16  |
|             |      | 4             | 0.2000E-01              | -0.6667E-00                        | 0.1333E-01 | -0.6667E-00 | 0.2082E-07  | 0.7067E-07  | -0.7200E-16 |
|             |      | 5             | 0.2000E-01              | -0.6667E-00                        | 0.1333E-01 | -0.6667E-00 | -0.2437E-07 | -0.5620E-08 | 0.2027E-16  |
|             |      | 6             | 0.2000E-01              | -0.6667E-00                        | 0.1333E-01 | -0.6667E-00 | -0.2437E-07 | -0.5777E-07 | -0.5187E-16 |
|             |      | 7             | 0.2000E-01              | -0.6667E-00                        | 0.1333E-01 | -0.6667E-00 | 0.3189E-07  | 0.5459E-07  | -0.1643E-16 |
|             |      | 8             | 0.2000E-01              | -0.6667E-00                        | 0.1333E-01 | -0.6667E-00 | 0.3189E-07  | 0.7067E-07  | -0.7200E-16 |

| INTEGR. POINT | EFFECTIVE STRESS | *****CUMULATIVE STRESSES***** |            |             |             |             |             |
|---------------|------------------|-------------------------------|------------|-------------|-------------|-------------|-------------|
|               |                  | XX                            | YY         | ZZ          | XY          | XZ          | YZ          |
| 1             | 0.1858E-05       | 0.1032E-06                    | 0.1983E-05 | 0.1490E-06  | 0.1708E-07  | 0.7295E-08  | 0.3564E-17  |
| 2             | 0.1868E-05       | 0.1450E-06                    | 0.1983E-05 | 0.1032E-06  | 0.1708E-07  | 0.1159E-06  | 0.2244E-17  |
| 3             | 0.1858E-05       | 0.1032E-06                    | 0.1983E-05 | 0.1490E-06  | 0.2032E-07  | 0.2461E-07  | 0.5680E-17  |
| 4             | 0.1868E-05       | 0.1450E-06                    | 0.1983E-05 | 0.1032E-06  | 0.2032E-07  | 0.1158E-06  | -0.5194E-17 |
| 5             | 0.1515E-05       | 0.8024E-07                    | 0.1914E-05 | 0.5731E-07  | 0.1061E-06  | 0.7295E-08  | 0.6544E-17  |
| 6             | 0.2943E-05       | -0.2293E-06                   | 0.2751E-05 | -0.1376E-06 | -0.1061E-06 | -0.1159E-06 | -0.7778E-17 |
| 7             | 0.1917E-05       | 0.8024E-07                    | 0.1914E-05 | -0.5731E-07 | 0.1115E-06  | 0.2461E-07  | 0.5518E-17  |
| 8             | 0.2949E-05       | -0.2293E-06                   | 0.2751E-05 | -0.1376E-06 | 0.1115E-06  | 0.1158E-06  | -0.5194E-17 |

## PLASTIC AND CREEP STRAINS

ELEMENT NO. = 1 ID = 3

| INT | E-P  | SUM  | INCREMENTAL | TOTAL       | SURFACE    | *** EFFECTIVE PLASTIC STRAINS *** |            |            | *** EFFECTIVE CREEP STRAINS *** |            |            |
|-----|------|------|-------------|-------------|------------|-----------------------------------|------------|------------|---------------------------------|------------|------------|
| PNT | CODE | CODE | TEMPERATURE | TEMPERATURE | YIELD SIZE | INCREMENTAL                       | SUM        | INCR.      | INCREMENTAL                     | SUM        | INCR.      |
| 1   | 0    | -2   | -0.1600E-01 | 0.5400E-01  | 0.2000E-01 | 0.0                               | 0.3000E-01 | 0.3000E-01 | 0.0                             | 0.1000E-01 | 0.1000E-01 |
| 2   | 0    | -2   | -0.1600E-01 | 0.5400E-01  | 0.2000E-01 | 0.0                               | 0.3000E-01 | 0.3000E-01 | 0.0                             | 0.1000E-01 | 0.1000E-01 |
| 3   | 0    | -2   | -0.1600E-01 | 0.5400E-01  | 0.2000E-01 | 0.0                               | 0.3000E-01 | 0.3000E-01 | 0.0                             | 0.1000E-01 | 0.1000E-01 |
| 4   | 0    | -2   | -0.1600E-01 | 0.5400E-01  | 0.2000E-01 | 0.0                               | 0.3000E-01 | 0.3000E-01 | 0.0                             | 0.1000E-01 | 0.1000E-01 |
| 5   | 0    | -2   | -0.1600E-01 | 0.5400E-01  | 0.2000E-01 | 0.0                               | 0.3000E-01 | 0.3000E-01 | 0.0                             | 0.1000E-01 | 0.1000E-01 |
| 6   | 0    | -2   | -0.1600E-01 | 0.5400E-01  | 0.2000E-01 | 0.0                               | 0.3000E-01 | 0.3000E-01 | 0.0                             | 0.1000E-01 | 0.1000E-01 |
| 7   | 0    | -2   | -0.1600E-01 | 0.5400E-01  | 0.2000E-01 | 0.0                               | 0.3000E-01 | 0.3000E-01 | 0.0                             | 0.1000E-01 | 0.1000E-01 |
| 8   | 0    | -2   | -0.1600E-01 | 0.5400E-01  | 0.2000E-01 | 0.0                               | 0.3000E-01 | 0.3000E-01 | 0.0                             | 0.1000E-01 | 0.1000E-01 |



CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 8

| INCREMENT | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| TEMP.     | 0.80000E 01 | 0.80000E 01 | 0.80000E 01 | 0.80000E 01 | 0.80000E 01 | 0.80000E 01 | 0.80000E 01 | 0.80000E 01 |

RESIDUAL URM = -0.57783E 00  
 RESIDUAL URM = 0.37551E 00  
 RESIDUAL URM = 0.28783E 00  
 RESIDUAL URM = -0.52030E 01  
 RESIDUAL URM = 0.35544E 02  
 RESIDUAL URM = 0.59061E 03  
 RESIDUAL URM = -0.21042E 03  
 RESIDUAL URM = 0.34777E 04  
 RESIDUAL URM = 0.23E64E 05

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END OF LOAD INCREMENT 8

INCREMENT 8  
 MECHANICAL LOAD CURVE FACTORS = 0.4000E 01, -0.6000E 00  
 CREEP TIME INCREMENT = 0.0  
 NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 8  
 8 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT  
 SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 0  
 SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 9  
 SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.1000E 04, ACTUAL ERROR = 0.2386E 05

# CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE **<br>NO. I.D. | FORCES         |               |                | DISPLACEMENTS  |               |                |
|------------------------|----------------|---------------|----------------|----------------|---------------|----------------|
|                        | U              | V             | W              | U              | V             | W              |
| 1 1                    | 0.3674118E-06  | 0.1562520E 00 | -0.9569303E-06 | -0.8571777E-07 | 0.0           | -0.6000000E 00 |
| 2 2                    | -0.3797257E-06 | 0.1562513E 00 | -0.9587029E-06 | -0.5999989E 00 | 0.0           | -0.6000000E 00 |
| 3 3                    | 0.4694947E-06  | 0.1562513E 00 | -0.1030540E-05 | -0.5999982E 00 | 0.4000000E 01 | -0.6000000E 00 |
| 4 4                    | 0.4713356E-06  | 0.1562520E 00 | -0.1029843E-05 | -0.7790741E-06 | 0.4000000E 01 | -0.6000000E 00 |
| 5 5                    | 0.3840629E-06  | 0.1562517E 00 | 0.9587020E-06  | 0.0            | 0.0           | 0.0            |
| 6 6                    | 0.3827633E-06  | 0.1562518E 00 | 0.9569303E-06  | -0.5999994E 00 | 0.0           | 0.0            |
| 7 7                    | -0.4075300E-06 | 0.1562518E 00 | 0.1029850E-05  | -0.5999994E 00 | 0.4000000E 01 | 0.0            |
| 8 8                    | 0.4617271E-06  | 0.1562517E 00 | 0.1030547E-05  | 0.3664492E-06  | 0.4000000E 01 | 0.0            |

14.1-39

## THERMAL AND ELASTIC STRAINS

| ELEMENT |      |       | INTEGR.         | INCREMENTAL |             | INCREMENTAL ELASTIC STRAINS |             |             |             |  |  |
|---------|------|-------|-----------------|-------------|-------------|-----------------------------|-------------|-------------|-------------|--|--|
| NO.     | I.D. | POINT | THERMAL STRAINS | XX          | YY          | ZZ                          | XY          | XZ          | YZ          |  |  |
| 1       | 3    | 1     | -0.1000E 01     | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | -0.1584E-07 | -0.4146E-07 | -0.2286E-16 |  |  |
|         |      | 2     | -0.1000E 01     | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | -0.1584E-07 | -0.8291E-07 | -0.1904E-15 |  |  |
|         |      | 3     | -0.1000E 01     | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | -0.3243E-07 | -0.9302E-08 | -0.1016E-16 |  |  |
|         |      | 4     | -0.1000E 01     | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | -0.3243E-07 | -0.8409E-07 | 0.3464E-16  |  |  |
|         |      | 5     | -0.1000E 01     | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | -0.1005E-06 | -0.4146E-07 | -0.1450E-16 |  |  |
|         |      | 6     | -0.1000E 01     | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | 0.1005E-06  | 0.8291E-07  | 0.1700E-15  |  |  |
|         |      | 7     | -0.1000E 01     | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | -0.1075E-06 | -0.9002E-08 | -0.1527E-16 |  |  |
|         |      | 8     | -0.1000E 01     | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | -0.1075E-06 | -0.8409E-07 | -0.2531E-16 |  |  |
|         |      |       | INTEGR.         | CUMULATIVE  |             | CUMULATIVE ELASTIC STRAINS  |             |             |             |  |  |
|         |      | POINT | THERMAL STRAINS | XX          | YY          | ZZ                          | XY          | XZ          | YZ          |  |  |
|         |      | 1     | 0.1000E 01      | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | 0.6365E-08  | -0.3198E-07 | -0.1823E-16 |  |  |
|         |      | 2     | 0.1000E 01      | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | 0.6365E-08  | -0.6773E-07 | 0.1875E-15  |  |  |
|         |      | 3     | 0.1000E 01      | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | -0.6017E-08 | 0.2244E-07  | -0.2781E-17 |  |  |
|         |      | 4     | 0.1000E 01      | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | -0.6017E-08 | 0.6645E-07  | 0.2789E-16  |  |  |
|         |      | 5     | 0.1000E 01      | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | -0.2939E-07 | -0.3198E-07 | -0.5191E-17 |  |  |
|         |      | 6     | 0.1000E 01      | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | -0.2939E-07 | -0.6773E-07 | 0.1605E-15  |  |  |
|         |      | 7     | 0.1000E 01      | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | 0.3744E-07  | 0.2299E-07  | -0.7547E-17 |  |  |
|         |      | 8     | 0.1000E 01      | 0.1500E 00  | -0.5000E 00 | 0.1500E 00                  | 0.3744E-07  | 0.6645E-07  | 0.2256E-16  |  |  |

## PLASTIC WORK AND STRAINS

| ELEMENT NO. | I.D. | INTEGR. POINT | INCREMENTAL PLASTIC WORK | INCREMENTAL PLASTIC STRAINS |             |             |             |             |             |
|-------------|------|---------------|--------------------------|-----------------------------|-------------|-------------|-------------|-------------|-------------|
|             |      |               |                          | XX                          | YY          | ZZ          | XY          | XZ          | YZ          |
| 1           | 3    | 1             | 0.1562E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.2308E-08 | -0.6578E-09 | -0.1183E-16 |
|             |      | 2             | 0.1562E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.2308E-08 | -0.1124E-07 | 0.4554E-16  |
|             |      | 3             | 0.1562E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.4705E-08 | -0.1274E-07 | -0.5014E-17 |
|             |      | 4             | 0.1562E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.4705E-08 | -0.4664E-08 | -0.2695E-16 |
|             |      | 5             | 0.1562E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.1249E-07 | -0.6578E-09 | -0.6319E-17 |
|             |      | 6             | 0.1562E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.1249E-07 | -0.1124E-07 | 0.5141E-16  |
|             |      | 7             | 0.1562E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.1270E-07 | -0.1274E-07 | -0.5550E-17 |
|             |      | 8             | 0.1562E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | 0.1270E-07  | 0.4664E-08  | 0.2620E-16  |
|             |      | INTEGR. POINT | CUMULATIVE PLASTIC WORK  | CUMULATIVE PLASTIC STRAINS  |             |             |             |             |             |
|             |      |               |                          | XX                          | YY          | ZZ          | XY          | XZ          | YZ          |
|             |      | 1             | 0.9656E 01               | -0.1250E 01                 | 0.2500E 01  | -0.1250E 01 | 0.9123E-08  | -0.8856E-07 | 0.5870E-16  |
|             |      | 2             | 0.9656E 01               | -0.1250E 01                 | 0.2500E 01  | -0.1250E 01 | 0.9123E-08  | -0.1900E-06 | -0.5400E-16 |
|             |      | 3             | 0.9656E 01               | -0.1250E 01                 | 0.2500E 01  | -0.1250E 01 | 0.4300E-08  | 0.1020E-06  | 0.3677E-16  |
|             |      | 4             | 0.9656E 01               | -0.1250E 01                 | 0.2500E 01  | -0.1250E 01 | 0.4300E-08  | 0.1936E-06  | -0.1167E-15 |
|             |      | 5             | 0.9656E 01               | -0.1250E 01                 | 0.2500E 01  | -0.1250E 01 | -0.9230E-07 | -0.8856E-07 | -0.4132E-16 |
|             |      | 6             | 0.9656E 01               | -0.1250E 01                 | 0.2500E 01  | -0.1250E 01 | -0.9230E-07 | -0.1900E-06 | -0.1574E-15 |
|             |      | 7             | 0.9656E 01               | -0.1250E 01                 | 0.2500E 01  | -0.1250E 01 | 0.9632E-07  | 0.1020E-06  | 0.3514E-16  |
|             |      | 8             | 0.9656E 01               | -0.1250E 01                 | 0.2500E 01  | -0.1250E 01 | -0.9632E-07 | -0.1936E-06 | -0.1174E-15 |

CRACKING  
 OF FOOT  
 CHAIRS

14.1.40

# CUMULATIVE STRESS QUANTITIES

| ELEMENT NO. | I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | *****CUMULATIVE STRESS CENTER***** |            |             |             |             |             |
|-------------|------|---------------|-------------------------|------------------------------------|------------|-------------|-------------|-------------|-------------|
|             |      |               |                         | XX                                 | YY         | ZZ          | XY          | XZ          | YZ          |
| 1           | 3    | 1             | 0.1500E 01              | -0.5000E 00                        | 0.1000E 01 | -0.5000E 00 | 0.1924E 07  | -0.6959E 08 | -0.2224E 16 |
|             |      | 2             | 0.1500E 01              | -0.5000E 00                        | 0.1000E 01 | -0.5000E 00 | 0.1924E 07  | -0.5827E 07 | -0.2908E 16 |
|             |      | 3             | 0.1500E 01              | -0.5000E 00                        | 0.1000E 01 | -0.5000E 00 | 0.1768E 07  | 0.5110E 07  | 0.1385E 16  |
|             |      | 4             | 0.1500E 01              | -0.5000E 00                        | 0.1000E 01 | -0.5000E 00 | 0.1768E 07  | -0.7378E 07 | -0.5486E 16 |
|             |      | 5             | 0.1500E 01              | -0.5000E 00                        | 0.1000E 01 | -0.5000E 00 | -0.3297E 07 | -0.6054E 08 | 0.1605E 16  |
|             |      | 6             | 0.1500E 01              | -0.5000E 00                        | 0.1000E 01 | -0.5000E 00 | -0.3297E 07 | -0.5827E 07 | -0.5759E 16 |
|             |      | 7             | 0.1500E 01              | -0.5000E 00                        | 0.1000E 01 | -0.5000E 00 | 0.4036E 07  | 0.5110E 07  | -0.1523E 16 |
|             |      | 8             | 0.1500E 01              | -0.5000E 00                        | 0.1000E 01 | -0.5000E 00 | 0.4036E 07  | 0.7378E 07  | -0.5534E 16 |

| INTEGR. POINT | EFFECTIVE STRESS | *****CUMULATIVE STRESSES***** |             |             |             |             |             |
|---------------|------------------|-------------------------------|-------------|-------------|-------------|-------------|-------------|
|               |                  | XX                            | YY          | ZZ          | XY          | XZ          | YZ          |
| 1             | 0.6250E 00       | -0.1433E 05                   | -0.6250E 00 | -0.3725E 05 | 0.6120E 08  | -0.3075E 07 | -0.1752E 16 |
| 2             | 0.6250E 00       | -0.1863E 05                   | -0.6250E 00 | -0.4155E 05 | 0.6120E 08  | -0.6512E 07 | -0.1803E 15 |
| 3             | 0.6250E 00       | -0.1433E 05                   | -0.6250E 00 | -0.3725E 05 | -0.5786E 08 | 0.2210E 07  | -0.2674E 17 |
| 4             | 0.6250E 00       | -0.1863E 05                   | -0.6250E 00 | -0.4155E 05 | -0.5786E 08 | 0.6389E 07  | 0.2681E 16  |
| 5             | 0.6250E 00       | -0.1290E 05                   | -0.6250E 00 | -0.3725E 05 | -0.2826E 07 | -0.3075E 07 | -0.5761E 17 |
| 6             | 0.6250E 00       | -0.2149E 05                   | -0.6250E 00 | -0.4238E 05 | -0.2826E 07 | -0.6512E 07 | 0.1543E 15  |
| 7             | 0.6250E 00       | -0.1290E 05                   | -0.6250E 00 | -0.3725E 05 | 0.3600E 07  | 0.2210E 07  | -0.7257E 17 |
| 8             | 0.6250E 00       | -0.2149E 05                   | -0.6250E 00 | -0.4238E 05 | 0.3600E 07  | 0.6389E 07  | 0.2170E 16  |

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## PLASTIC AND CREEP STRAINS

| ELEMENT NO. = 1 ID = 3 |      |      |             |             |            |            |             |             |            |             |             |
|------------------------|------|------|-------------|-------------|------------|------------|-------------|-------------|------------|-------------|-------------|
| INT                    | E-P  | SUM  | INCREMENTAL | TOTAL       | SURFACE    | YIELD      | SIZE        | INCREMENTAL | SUM INCR.  | CUMULATIVE  | INCREMENTAL |
| PNT                    | CODE | CODE | TEMPERATURE | TEMPERATURE | YIELD      | SIZE       | INCREMENTAL | SUM INCR.   | CUMULATIVE | INCREMENTAL | SUM INCR.   |
| 1                      | 1    | 2    | 0.2600E 01  | 0.8000E 01  | 0.2125E 01 | 0.5000E 00 | 0.3500E 01  | 0.2500E 01  | 0.0        | 0.1000E 01  | 0.1000E 01  |
| 2                      | 1    | 2    | 0.2600E 01  | 0.8000E 01  | 0.2125E 01 | 0.5000E 00 | 0.3500E 01  | 0.2500E 01  | 0.0        | 0.1000E 01  | 0.1000E 01  |
| 3                      | 1    | 2    | 0.2600E 01  | 0.8000E 01  | 0.2125E 01 | 0.5000E 00 | 0.3500E 01  | 0.2500E 01  | 0.0        | 0.1000E 01  | 0.1000E 01  |
| 4                      | 1    | 2    | 0.2600E 01  | 0.8000E 01  | 0.2125E 01 | 0.5000E 00 | 0.3500E 01  | 0.2500E 01  | 0.0        | 0.1000E 01  | 0.1000E 01  |
| 5                      | 1    | 2    | 0.2600E 01  | 0.8000E 01  | 0.2125E 01 | 0.5000E 00 | 0.3500E 01  | 0.2500E 01  | 0.0        | 0.1000E 01  | 0.1000E 01  |
| 7                      | 1    | 2    | 0.2600E 01  | 0.8000E 01  | 0.2125E 01 | 0.5000E 00 | 0.3500E 01  | 0.2500E 01  | 0.0        | 0.1000E 01  | 0.1000E 01  |
| 8                      | 1    | 2    | 0.2600E 01  | 0.8000E 01  | 0.2125E 01 | 0.5000E 00 | 0.3500E 01  | 0.2500E 01  | 0.0        | 0.1000E 01  | 0.1000E 01  |

CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 9

INCREMENT 9

NODE 1.0 1 2 3 4 5 6 7 8  
TEMP. 0.90000E 01 0.90000E 01 0.90000E 01 0.90000E 01 0.90000E 01 0.90000E 01 0.90000E 01 0.90000E 01

RESIDUAL URM = 0.34131E-00  
RESIDUAL URM = 0.35190E-00  
RESIDUAL URM = 0.22534E-00  
RESIDUAL URM = 0.77806E-02  
RESIDUAL URM = 0.10929E-02  
RESIDUAL URM = 0.76270E-03  
RESIDUAL URM = 0.97871E-05

END OF LOAD INCREMENT 9

INCREMENT 9

MECHANICAL LOAD CURVE FACTORS = 0.1500E 01, -0.2000E 00

CREEP TIME INCREMENT = 0.1000E 02

NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 8

0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 0

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 7

SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.1000E-04, ACTUAL ERROR = 0.9787E-05

CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** | NO. I.D. | U              | V              | W              | U              | V             | W              |
|------------|----------|----------------|----------------|----------------|----------------|---------------|----------------|
| 1          | 1        | 0.1964421E-05  | 0.3124941E-00  | -0.4442135E-05 | 0.9001636E-06  | 0.0           | -0.2000000E 00 |
| 2          | 2        | -0.1961625E-05 | 0.3124934E-00  | -0.4442720E-05 | -0.1999977E 00 | 0.0           | -0.2000000E 00 |
| 3          | 3        | -0.1542345E-05 | -0.3124933E-00 | -0.4298388E-05 | -0.1999955E 00 | 0.1500000E 01 | -0.2000000E 00 |
| 4          | 4        | 0.1537517E-05  | -0.3124940E-00 | -0.4296536E-05 | -0.1400732E-05 | 0.1500000E 01 | -0.2000000E 00 |
| 5          | 5        | 0.1879454E-05  | 0.3124936E-00  | 0.4442718E-05  | 0.0            | 0.0           | 0.0            |
| 6          | 6        | -0.1876372E-05 | -0.3124937E-00 | 0.4442138E-05  | -0.1999971E-05 | 0.0           | 0.0            |
| 7          | 7        | -0.1675200E-05 | -0.3124937E-00 | 0.4296938E-05  | -0.1999985E 00 | 0.1500000E 01 | 0.0            |
| 8          | 8        | 0.1675151E-05  | -0.3124936E-00 | 0.4298386E-05  | 0.1571990E-05  | 0.1500000E 01 | 0.0            |

## THERMAL AND ELASTIC STRAINS

| ELEMENT NO.   | I.D.                       | INTEGR. POINT              | INCREMENTAL THERMAL STRAINS | INCREMENTAL ELASTIC STRAINS |             |             |             |             |             |
|---------------|----------------------------|----------------------------|-----------------------------|-----------------------------|-------------|-------------|-------------|-------------|-------------|
|               |                            |                            |                             | XX                          | YY          | ZZ          | XY          | XZ          | YZ          |
| 1             | 3                          | 1                          | -0.5000E 00                 | 0.1500E 00                  | -0.5000E 00 | 0.1500E 00  | 0.1199E-06  | 0.7986E-07  | -0.3409E-17 |
|               |                            | 2                          | -0.5000E 00                 | 0.1500E 00                  | -0.5000E 00 | 0.1500E 00  | 0.1199E-06  | 0.2413E-06  | -0.6175E-16 |
|               |                            | 3                          | -0.5000E 00                 | 0.1500E 00                  | -0.5000E 00 | 0.1500E 00  | -0.1293E-06 | -0.7196E-07 | -0.1286E-16 |
|               |                            | 4                          | -0.5000E 00                 | 0.1500E 00                  | -0.5000E 00 | 0.1500E 00  | -0.1293E-06 | 0.2352E-06  | -0.0651E-17 |
|               |                            | 5                          | -0.5000E 00                 | 0.1500E 00                  | -0.5000E 00 | 0.1500E 00  | -0.2013E-06 | -0.7566E-07 | -0.1008E-16 |
|               |                            | 6                          | -0.5000E 00                 | 0.1500E 00                  | -0.5000E 00 | 0.1500E 00  | -0.2013E-06 | -0.2413E-06 | -0.5801E-16 |
|               |                            | 7                          | -0.5000E 00                 | 0.1500E 00                  | -0.5000E 00 | 0.1500E 00  | 0.1779E-06  | -0.7196E-07 | -0.1320E-17 |
|               |                            | 8                          | -0.5000E 00                 | 0.1500E 00                  | -0.5000E 00 | 0.1500E 00  | 0.1779E-06  | -0.2352E-06 | 0.1142E-17  |
| INTEGR. POINT | CUMULATIVE THERMAL STRAINS | CUMULATIVE ELASTIC STRAINS |                             |                             |             |             |             | XZ          | YZ          |
|               |                            | XX                         | YY                          | ZZ                          | XY          | XY          | XY          |             |             |
| 1             | 0.5000E 00                 | 0.3000E 00                 | -0.1000E 01                 | 0.3000E 00                  | 0.1263E-06  | 0.4789E-07  | -0.2144E-16 |             |             |
| 2             | 0.5000E 00                 | 0.3000E 00                 | -0.1000E 01                 | 0.3000E 00                  | 0.1263E-06  | -0.3090E-06 | 0.1257E-15  |             |             |
| 3             | 0.5000E 00                 | 0.3000E 00                 | -0.1000E 01                 | 0.3000E 00                  | -0.1353E-06 | -0.4897E-07 | -0.1564E-16 |             |             |
| 4             | 0.5000E 00                 | 0.3000E 00                 | -0.1000E 01                 | 0.3000E 00                  | -0.1353E-06 | 0.3017E-05  | 0.1923E-16  |             |             |
| 5             | 0.5000E 00                 | 0.3000E 00                 | -0.1000E 01                 | 0.3000E 00                  | -0.2307E-06 | 0.4789E-07  | -0.1607E-16 |             |             |
| 6             | 0.5000E 00                 | 0.3000E 00                 | -0.1000E 01                 | 0.3000E 00                  | -0.2307E-06 | -0.3050E-06 | -0.1015E-15 |             |             |
| 7             | 0.5000E 00                 | 0.3000E 00                 | -0.1000E 01                 | 0.3000E 00                  | 0.2153E-06  | -0.4897E-07 | -0.8867E-17 |             |             |
| 8             | 0.5000E 00                 | 0.3000E 00                 | -0.1000E 01                 | 0.3000E 00                  | 0.2153E-06  | 0.3017E-06  | 0.2137E-16  |             |             |

## PLASTIC WORK AND STRAINS

| ELEMENT NO.   | I.D.                    | INTEGR. POINT              | INCREMENTAL PLASTIC WORK | INCREMENTAL PLASTIC STRAINS |             |             |             |             |             |
|---------------|-------------------------|----------------------------|--------------------------|-----------------------------|-------------|-------------|-------------|-------------|-------------|
|               |                         |                            |                          | XX                          | YY          | ZZ          | XY          | XZ          | YZ          |
| 1             | 3                       | 1                          | 0.4688E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | 0.1547E-07  | 0.8334E-08  | -0.1274E-16 |
|               |                         | 2                          | 0.4688E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | 0.1547E-07  | -0.3991E-07 | 0.5528E-16  |
|               |                         | 3                          | 0.4688E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.2826E-07 | -0.2378E-07 | -0.5988E-17 |
|               |                         | 4                          | 0.4688E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.2826E-07 | 0.3574E-07  | -0.2385E-16 |
|               |                         | 5                          | 0.4688E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.3278E-07 | 0.8334E-08  | -0.8204E-17 |
|               |                         | 6                          | 0.4688E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.3278E-07 | -0.3991E-07 | 0.5655E-16  |
|               |                         | 7                          | 0.4688E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.2926E-07 | -0.2378E-07 | -0.6499E-17 |
|               |                         | 8                          | 0.4688E 00               | 0.2500E 00                  | -0.5000E 00 | 0.2500E 00  | -0.2926E-07 | 0.3574E-07  | 0.2353E-16  |
| INTEGR. POINT | CUMULATIVE PLASTIC WORK | CUMULATIVE PLASTIC STRAINS |                          |                             |             |             |             | XZ          | YZ          |
|               |                         | XX                         | YY                       | ZZ                          | XY          | XY          | XY          |             |             |
| 1             | 0.1013E 02              | -0.1000E 01                | 0.2000E 01               | -0.1000E 01                 | 0.2459E-07  | -0.3023E-07 | 0.4596E-16  |             |             |
| 2             | 0.1013E 02              | -0.1000E 01                | 0.2000E 01               | -0.1000E 01                 | 0.2459E-07  | -0.2299E-06 | -0.1432E-16 |             |             |
| 3             | 0.1013E 02              | -0.1000E 01                | 0.2000E 01               | -0.1000E 01                 | -0.2346E-07 | 0.7827E-07  | 0.2978E-16  |             |             |
| 4             | 0.1013E 02              | -0.1000E 01                | 0.2000E 01               | -0.1000E 01                 | -0.2346E-07 | 0.2273E-06  | -0.9283E-16 |             |             |
| 5             | 0.1013E 02              | -0.1000E 01                | 0.2000E 01               | -0.1000E 01                 | -0.1251E-06 | -0.6023E-07 | 0.3312E-16  |             |             |
| 6             | 0.1013E 02              | -0.1000E 01                | 0.2000E 01               | -0.1000E 01                 | -0.1251E-06 | -0.2299E-06 | -0.1009E-15 |             |             |
| 7             | 0.1013E 02              | -0.1000E 01                | 0.2000E 01               | -0.1000E 01                 | 0.1256E-06  | 0.7827E-07  | 0.2324E-16  |             |             |
| 8             | 0.1013E 02              | -0.1000E 01                | 0.2000E 01               | -0.1000E 01                 | 0.1256E-06  | 0.2273E-06  | -0.9390E-16 |             |             |

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# CUMULATIVE STRESS QUANTITIES

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | EFFECTIVE<br>STRESS CENTER | *****CUMULATIVE STRESS CENTER***** |             |             |             |             |             |
|---------------------|------------------|----------------------------|------------------------------------|-------------|-------------|-------------|-------------|-------------|
|                     |                  |                            | XX                                 | YY          | ZZ          | XY          | XZ          | YZ          |
| 1 3                 | 1                | 0.1000E 01                 | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | 0.2955E 07  | -0.5024E 09 | 0.1375E 16  |
|                     | 2                | 0.1000E 01                 | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | 0.2955E 07  | -0.8488E 07 | 0.7774E 17  |
|                     | 3                | 0.1000E 01                 | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | -0.1160E 08 | 0.3524E 07  | 0.9194E 17  |
|                     | 4                | 0.1000E 01                 | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | -0.1160E 08 | -0.4627E 07 | -0.3894E 16 |
|                     | 5                | 0.1000E 01                 | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | -0.5482E 07 | -0.5025E 09 | 0.1058E 16  |
|                     | 6                | 0.1000E 01                 | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | -0.5482E 07 | -0.8488E 07 | -0.1390E 16 |
|                     | 7                | 0.1000E 01                 | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | -0.5587E 07 | -0.3524E 07 | -0.4897E 17 |
|                     | 8                | 0.1000E 01                 | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | 0.5487E 07  | 0.4627E 07  | -0.3965E 16 |
| INTEGR. EFFECTIVE   |                  |                            | *****CUMULATIVE STRESSES*****      |             |             |             |             |             |
| POINT STRESS        |                  |                            | XX                                 | YY          | ZZ          | XY          | XZ          | YZ          |
| 1 3                 | 1                | 0.1250E 01                 | -0.7737E 05                        | -0.1250E 01 | -0.1702E 04 | 0.1214E 06  | 0.4604E 07  | -0.2080E 16 |
|                     | 2                | 0.1250E 01                 | -0.6304E 05                        | -0.1250E 01 | -0.1662E 04 | 0.1214E 06  | -0.2972E 06 | -0.1205E 15 |
|                     | 3                | 0.1250E 01                 | -0.7737E 05                        | -0.1250E 01 | -0.1762E 04 | -0.1301E 06 | -0.4709E 07 | -0.1504E 16 |
|                     | 4                | 0.1250E 01                 | -0.6304E 05                        | -0.1250E 01 | -0.1662E 04 | -0.1301E 06 | 0.2901E 06  | 0.1304E 16  |
|                     | 5                | 0.1250E 01                 | -0.8454E 05                        | -0.1250E 01 | -0.1834E 04 | 0.2218E 06  | -0.4604E 07 | -0.1540E 16 |
|                     | 6                | 0.1250E 01                 | -0.5731E 05                        | -0.1250E 01 | -0.1734E 04 | -0.2218E 06 | -0.2972E 06 | 0.9799E 16  |
|                     | 7                | 0.1250E 01                 | -0.8454E 05                        | -0.1250E 01 | -0.1834E 04 | 0.2070E 06  | -0.4709E 07 | -0.1520E 17 |
|                     | 8                | 0.1250E 01                 | -0.5731E 05                        | -0.1250E 01 | -0.1734E 04 | 0.2070E 06  | 0.2901E 06  | 0.2055E 16  |

## CREEP WORK AND STRAINS

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | INCREMENTAL<br>CREEP WORK | *****INCREMENTAL CREEP STRAINS***** |             |             |             |             |             |
|---------------------|------------------|---------------------------|-------------------------------------|-------------|-------------|-------------|-------------|-------------|
|                     |                  |                           | XX                                  | YY          | ZZ          | XY          | XZ          | YZ          |
| 1 3                 | 1                | 0.9375E 00                | 0.5000E 00                          | -0.1000E 01 | 0.5000E 00  | 0.3053E 07  | 0.1667E 07  | -0.2548E 16 |
|                     | 2                | 0.9375E 00                | 0.5000E 00                          | -0.1000E 01 | 0.5000E 00  | 0.3093E 07  | -0.7982E 07 | 0.1106E 15  |
|                     | 3                | 0.9375E 00                | 0.5000E 00                          | -0.1000E 01 | 0.5000E 00  | -0.5652E 07 | -0.4756E 07 | -0.1398E 16 |
|                     | 4                | 0.9375E 00                | 0.5000E 00                          | -0.1000E 01 | 0.5000E 00  | -0.5652E 07 | 0.6748E 07  | 0.4765E 16  |
|                     | 5                | 0.9375E 00                | 0.5000E 00                          | -0.1000E 01 | 0.5000E 00  | -0.6556E 07 | 0.1607E 07  | -0.1641E 16 |
|                     | 6                | 0.9375E 00                | -0.5000E 00                         | -0.1000E 01 | 0.5000E 00  | -0.6556E 07 | -0.7982E 07 | 0.1131E 15  |
|                     | 7                | 0.9375E 00                | 0.5000E 00                          | -0.1000E 01 | 0.5000E 00  | 0.5852E 07  | -0.4756E 07 | -0.1300E 16 |
|                     | 8                | 0.9375E 00                | 0.5000E 00                          | -0.1000E 01 | 0.5000E 00  | 0.5852E 07  | 0.6748E 07  | 0.4715E 16  |
| INTEGR. CUMULATIVE  |                  |                           | *****CUMULATIVE CREEP STRAINS*****  |             |             |             |             |             |
| POINT CREEP WORK    |                  |                           | XX                                  | YY          | ZZ          | XY          | XZ          | YZ          |
| 1 3                 | 1                | 0.3687E 01                | -0.2205E 05                         | -0.3570E 06 | -0.2563E 05 | -0.8296E 07 | -0.7914E 07 | -0.3536E 17 |
|                     | 2                | 0.3687E 01                | -0.2444E 05                         | -0.2980E 06 | 0.2742E 05  | 0.3296E 07  | -0.7648E 07 | 0.5901E 16  |
|                     | 3                | 0.3687E 01                | -0.2205E 05                         | -0.3576E 06 | 0.2563E 05  | 0.2018E 08  | 0.3371E 07  | -0.2424E 17 |
|                     | 4                | 0.3687E 01                | -0.2444E 05                         | -0.2980E 06 | 0.2742E 05  | -0.2018E 08 | -0.1170E 06 | -0.3163E 17 |
|                     | 5                | 0.3687E 01                | -0.1907E 05                         | -0.8941E 06 | 0.2801E 05  | -0.7206E 07 | 0.7514E 07  | -0.3163E 17 |
|                     | 6                | 0.3687E 01                | -0.2146E 05                         | -0.1252E 05 | 0.3397E 05  | -0.7206E 07 | -0.7548E 07 | 0.4752E 16  |
|                     | 7                | 0.3687E 01                | -0.1907E 05                         | -0.8941E 06 | 0.2801E 05  | 0.8531E 07  | -0.3371E 07 | -0.1723E 17 |
|                     | 8                | 0.3687E 01                | -0.2146E 05                         | -0.1252E 05 | 0.3397E 05  | 0.8531E 07  | 0.1170E 06  | -0.3793E 17 |

# PLASTIC AND CREEP STRAINS

| ELEMENT NO. 1 10 3 |      |      |             |             |            |             |             |             | **** EFFECTIVE PLASTIC STRAINS **** |             |             | **** EFFECTIVE CREEP STRAINS **** |             |             |
|--------------------|------|------|-------------|-------------|------------|-------------|-------------|-------------|-------------------------------------|-------------|-------------|-----------------------------------|-------------|-------------|
| INT                | E-P  | SUM  | INCREMENTAL | TOTAL       | SURFACE    | INCREMENTAL | SUM         | INCR.       | INCREMENTAL                         | SUM         | INCR.       | INCREMENTAL                       | SUM         | INCR.       |
| PNT                | CODE | CODE | TEMPERATURE | TEMPERATURE | YIELD SIZE | TEMPERATURE | TEMPERATURE | TEMPERATURE | TEMPERATURE                         | TEMPERATURE | TEMPERATURE | TEMPERATURE                       | TEMPERATURE | TEMPERATURE |
| 1                  | 0    | 2    | 0.1000E 01  | 0.9000E 01  | 0.2250E 01 | 0.5000E 00  | 0.4000E 01  | 0.2000E 01  | 0.1000E 01                          | 0.2000E 01  | 0.2779E -05 | 0.1000E 01                        | 0.2000E 01  | 0.2779E -05 |
| 2                  | 0    | 2    | 0.1000E 01  | 0.9000E 01  | 0.2250E 01 | 0.5000E 00  | 0.4000E 01  | 0.2000E 01  | 0.1000E 01                          | 0.2000E 01  | 0.2776E -05 | 0.1000E 01                        | 0.2000E 01  | 0.2776E -05 |
| 3                  | 0    | 2    | 0.1000E 01  | 0.9000E 01  | 0.2250E 01 | 0.5000E 00  | 0.4000E 01  | 0.2000E 01  | 0.1000E 01                          | 0.2000E 01  | 0.3012E -05 | 0.1000E 01                        | 0.2000E 01  | 0.3012E -05 |
| 4                  | 0    | 2    | 0.1000E 01  | 0.9000E 01  | 0.2250E 01 | 0.5000E 00  | 0.4000E 01  | 0.2000E 01  | 0.1000E 01                          | 0.2000E 01  | 0.2865E -05 | 0.1000E 01                        | 0.2000E 01  | 0.2865E -05 |
| 5                  | 0    | 2    | 0.1000E 01  | 0.9000E 01  | 0.2250E 01 | 0.5000E 00  | 0.4000E 01  | 0.2000E 01  | 0.1000E 01                          | 0.2000E 01  | 0.3439E -05 | 0.1000E 01                        | 0.2000E 01  | 0.3439E -05 |
| 6                  | 0    | 2    | 0.1000E 01  | 0.9000E 01  | 0.2250E 01 | 0.5000E 00  | 0.4000E 01  | 0.2000E 01  | 0.1000E 01                          | 0.2000E 01  | 0.2864E -05 | 0.1000E 01                        | 0.2000E 01  | 0.2864E -05 |
| 7                  | 0    | 2    | 0.1000E 01  | 0.9000E 01  | 0.2250E 01 | 0.5000E 00  | 0.4000E 01  | 0.2000E 01  | 0.1000E 01                          | 0.2000E 01  | 0.3441E -05 | 0.1000E 01                        | 0.2000E 01  | 0.3441E -05 |
| 8                  | 0    | 2    | 0.1000E 01  | 0.9000E 01  | 0.2250E 01 | 0.5000E 00  | 0.4000E 01  | 0.2000E 01  | 0.1000E 01                          | 0.2000E 01  |             | 0.1000E 01                        | 0.2000E 01  |             |

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# CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 10

INCREMENT 10

| NODE I.D. | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| TEMP.     | 0.10000E-02 | 0.10000E-02 | 0.10000E-02 | 0.10000E-02 | 0.10000E-02 | 0.10000E-02 | 0.10000E-02 | 0.10000E-02 |

RESIDUAL NORM = 0.51704E-00  
 RESIDUAL VIRM = 0.34321E-00  
 RESIDUAL NORM = 0.51704E-00  
 RESIDUAL NORM = 0.79491E-05

END OF LOAD INCREMENT 10

INCREMENT 10  
 MECHANICAL LOAD CURVE FACTORS = 0.3650E 01, 0.0  
 CREEP TIME INCREMENT = 0.0  
 NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 0  
 0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 8 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT  
 SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 0  
 SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 4  
 SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.1000E-04, ACTUAL ERROR = 0.7949E-05

## CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** |      | ** ***** FORCES ***** |                |                | ***** DISPLACEMENTS ***** |               |     |
|------------|------|-----------------------|----------------|----------------|---------------------------|---------------|-----|
| NO.        | I.D. | U                     | V              | W              | U                         | V             | W   |
| 1          | 1    | -0.5002525E-05        | -0.2500135E 00 | -0.2373478E-05 | 0.7787124E-06             | 0.0           | 0.0 |
| 2          | 2    | 0.5355882E-05         | -0.2500144E 00 | -0.2102488E-05 | 0.1162291E-04             | 0.0           | 0.0 |
| 3          | 3    | -0.5136731E-05        | -0.2500144E-00 | -0.2081742E-05 | -0.1046725E-04            | 0.3650000E 01 | 0.0 |
| 4          | 4    | -0.5204512E-05        | 0.2500138E 00  | -0.2096423E-05 | -0.3874529E-06            | 0.3650000E 01 | 0.0 |
| 5          | 5    | -0.5148855E-05        | -0.2500139E 00 | 0.2102486E-05  | 0.0                       | 0.0           | 0.0 |
| 6          | 3    | -0.4704005E-05        | -0.2500136E 00 | 0.2373479E-05  | 0.9357929E-05             | 0.0           | 0.0 |
| 7          | 7    | 0.5263706E-05         | 0.2500137E 00  | 0.2096424E-05  | 0.9494371E-05             | 0.3650000E 01 | 0.0 |
| 8          | 8    | -0.5104511E-05        | 0.2500137E 00  | 0.2081741E-05  | 0.1915079E-05             | 0.3650000E 01 | 0.0 |



## THERMAL AND ELASTIC STRAINS

| ELEMENT<br>NO.   | I.D.                          | INTEGR.<br>POINT           | INCREMENTAL<br>THERMAL STRAINS | INCREMENTAL ELASTIC STRAINS |             |             |             |             |             |
|------------------|-------------------------------|----------------------------|--------------------------------|-----------------------------|-------------|-------------|-------------|-------------|-------------|
|                  |                               |                            |                                | XX                          | YY          | ZZ          | XY          | XZ          | YZ          |
| 1                | 3                             | 1                          | 0.6500E 00                     | -0.4500E 00                 | 0.1500E 01  | -0.4500E 00 | 0.2963E-06  | 0.2139E-06  | 0.3123E-16  |
|                  |                               | 2                          | 0.6500E 00                     | -0.4500E 00                 | 0.1500E 01  | -0.4500E 00 | 0.2963E-06  | 0.2139E-06  | 0.2251E-15  |
|                  |                               | 3                          | 0.6500E 00                     | -0.4500E 00                 | 0.1500E 01  | -0.4500E 00 | 0.4087E-06  | 0.7357E-06  | 0.1258E-16  |
|                  |                               | 4                          | 0.6500E 00                     | -0.4500E 00                 | 0.1500E 01  | -0.4500E 00 | 0.4087E-06  | -0.3016E-06 | -0.8500E-16 |
|                  |                               | 5                          | 0.6500E 00                     | -0.4500E 00                 | 0.1500E 01  | -0.4500E 00 | 0.1857E-06  | 0.2139E-06  | 0.2775E-16  |
|                  |                               | 6                          | 0.6500E 00                     | -0.4500E 00                 | 0.1500E 01  | -0.4500E 00 | 0.1857E-06  | 0.1032E-06  | -0.1041E-15 |
|                  |                               | 7                          | 0.6500E 00                     | -0.4500E 00                 | 0.1500E 01  | -0.4500E 00 | -0.6286E-06 | 0.7357E-06  | 0.1821E-16  |
|                  |                               | 8                          | 0.6500E 00                     | -0.4500E 00                 | 0.1500E 01  | -0.4500E 00 | -0.6286E-06 | 0.3016E-06  | 0.7834E-16  |
| INTEGR.<br>POINT | CUMULATIVE<br>THERMAL STRAINS | CUMULATIVE ELASTIC STRAINS |                                |                             |             |             |             | XZ          | YZ          |
|                  |                               | XX                         | YY                             | ZZ                          | XY          |             |             |             |             |
| 1                | 0.1150E 01                    | -0.1500E 00                | 0.5000E 00                     | -0.1500E 00                 | 0.4226E-06  |             | 0.2618E-06  | 0.9590E-17  |             |
| 2                | 0.1150E 01                    | -0.1500E 00                | 0.5000E 00                     | -0.1500E 00                 | 0.4226E-06  |             | -0.2058E-06 | -0.9936E-16 |             |
| 3                | 0.1150E 01                    | -0.1500E 00                | 0.5000E 00                     | -0.1500E 00                 | 0.2734E-06  |             | 0.6867E-06  | 0.3067E-17  |             |
| 4                | 0.1150E 01                    | -0.1500E 00                | 0.5000E 00                     | -0.1500E 00                 | 0.2734E-06  |             | 0.3905E-10  | -0.5577E-16 |             |
| 5                | 0.1150E 01                    | -0.1500E 00                | 0.5000E 00                     | -0.1500E 00                 | -0.4501E-07 |             | 0.2618E-06  | 0.1168E-15  |             |
| 6                | 0.1150E 01                    | -0.1500E 00                | 0.5000E 00                     | -0.1500E 00                 | -0.4501E-07 |             | 0.2058E-06  | -0.2174E-17 |             |
| 7                | 0.1150E 01                    | -0.1500E 00                | 0.5000E 00                     | -0.1500E 00                 | -0.4133E-06 |             | 0.6867E-06  | 0.1348E-17  |             |
| 8                | 0.1150E 01                    | -0.1500E 00                | 0.5000E 00                     | -0.1500E 00                 | -0.4133E-06 |             | 0.3905E-10  | -0.5756E-15 |             |

## PLASTIC WORK AND STRAINS

| ELEMENT<br>NO.   | I.D.                       | INTEGR.<br>POINT           | INCREMENTAL<br>PLASTIC WORK | INCREMENTAL PLASTIC STRAINS |             |     |             |             |     |
|------------------|----------------------------|----------------------------|-----------------------------|-----------------------------|-------------|-----|-------------|-------------|-----|
|                  |                            |                            |                             | XX                          | YY          | ZZ  | XY          | XZ          | YZ  |
| 1                | 3                          | 1                          | 0.0                         | 0.0                         | 0.0         | 0.0 | 0.0         | 0.0         | 0.0 |
|                  |                            | 2                          | 0.0                         | 0.0                         | 0.0         | 0.0 | 0.0         | 0.0         | 0.0 |
|                  |                            | 3                          | 0.0                         | 0.0                         | 0.0         | 0.0 | 0.0         | 0.0         | 0.0 |
|                  |                            | 4                          | 0.0                         | 0.0                         | 0.0         | 0.0 | 0.0         | 0.0         | 0.0 |
|                  |                            | 5                          | 0.0                         | 0.0                         | 0.0         | 0.0 | 0.0         | 0.0         | 0.0 |
|                  |                            | 6                          | 0.0                         | 0.0                         | 0.0         | 0.0 | 0.0         | 0.0         | 0.0 |
|                  |                            | 7                          | 0.0                         | 0.0                         | 0.0         | 0.0 | 0.0         | 0.0         | 0.0 |
|                  |                            | 8                          | 0.0                         | 0.0                         | 0.0         | 0.0 | 0.0         | 0.0         | 0.0 |
| INTEGR.<br>POINT | CUMULATIVE<br>PLASTIC WORK | CUMULATIVE PLASTIC STRAINS |                             |                             |             |     |             | XZ          | YZ  |
|                  |                            | XX                         | YY                          | ZZ                          | XY          |     |             |             |     |
| 1                | 0.1013E 02                 | -0.1000E 01                | 0.2000E 01                  | -0.1000E 01                 | 0.2459E-07  |     | -0.8023E-07 | 0.4596E-16  |     |
| 2                | 0.1013E 02                 | -0.1000E 01                | 0.2000E 01                  | -0.1000E 01                 | 0.2459E-07  |     | -0.2299E-06 | -0.1432E-16 |     |
| 3                | 0.1013E 02                 | -0.1000E 01                | 0.2000E 01                  | -0.1000E 01                 | -0.2346E-07 |     | 0.7827E-07  | 0.2978E-16  |     |
| 4                | 0.1013E 02                 | -0.1000E 01                | 0.2000E 01                  | -0.1000E 01                 | -0.2346E-07 |     | 0.2273E-06  | -0.9283E-16 |     |
| 5                | 0.1013E 02                 | -0.1000E 01                | 0.2000E 01                  | -0.1000E 01                 | -0.1251E-06 |     | -0.8023E-07 | 0.3312E-16  |     |
| 6                | 0.1013E 02                 | -0.1000E 01                | 0.2000E 01                  | -0.1000E 01                 | -0.1251E-06 |     | -0.2299E-06 | -0.1009E-15 |     |
| 7                | 0.1013E 02                 | -0.1000E 01                | 0.2000E 01                  | -0.1000E 01                 | 0.1250E-06  |     | 0.7827E-07  | 0.2924E-16  |     |
| 8                | 0.1013E 02                 | -0.1000E 01                | 0.2000E 01                  | -0.1000E 01                 | -0.1250E-06 |     | 0.2273E-06  | -0.9350E-16 |     |

# CUMULATIVE STRESS QUANTITIES

| ELEMENT |      | INTEGR.    | EFFECTIVE                     | *****CUMULATIVE STRESS CENTER***** |             |             |             |             |             |
|---------|------|------------|-------------------------------|------------------------------------|-------------|-------------|-------------|-------------|-------------|
| NO.     | I.D. | POINT      | STRESS CENTER                 | XX                                 | YY          | ZZ          | XY          | XZ          | YZ          |
| 1       | 3    | 1          | 0.1000E 01                    | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | 0.2555E-07  | -0.5024E-09 | 0.1375E-16  |
|         |      | 2          | 0.1000E 01                    | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | 0.2955E-07  | -0.8488E-07 | 0.7774E-17  |
|         |      | 3          | 0.1000E 01                    | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | -0.1160E-08 | 0.3524E-07  | 0.9194E-17  |
|         |      | 4          | 0.1000E 01                    | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | -0.1160E-08 | -0.0627E-07 | -0.3894E-16 |
|         |      | 5          | 0.1000E 01                    | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | -0.5482E-07 | -0.5025E-09 | 0.1058E-16  |
|         |      | 6          | 0.1000E 01                    | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | -0.5482E-07 | -0.8488E-07 | -0.1950E-16 |
|         |      | 7          | 0.1000E 01                    | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | -0.5987E-07 | -0.3524E-07 | -0.8047E-17 |
|         |      | 8          | 0.1000E 01                    | -0.3333E 00                        | 0.6667E 00  | -0.3333E 00 | 0.5987E-07  | 0.9627E-07  | -0.3965E-16 |
| -----   |      |            |                               |                                    |             |             |             |             |             |
| INTEGR. |      | EFFECTIVE  | *****CUMULATIVE STRESSES***** |                                    |             |             |             |             |             |
| POINT   |      | STRESS     | XX                            | YY                                 | ZZ          | XY          | XZ          | YZ          |             |
| 1       |      | 0.1903E 01 | 0.1903E-04                    | 0.1000E 01                         | -0.9399E-05 | 0.6501E-06  | 0.4028E-06  | 0.1475E-16  |             |
| 2       |      | 0.1000E 01 | 0.2155E-04                    | 0.1000E 01                         | -0.6419E-05 | 0.6501E-06  | -0.3166E-06 | 0.1529E-15  |             |
| 3       |      | 0.1000E 01 | 0.1403E-04                    | 0.1000E 01                         | -0.9399E-05 | 0.4206E-06  | 0.1056E-05  | -0.4710E-17 |             |
| 4       |      | 0.1000E 01 | 0.2155E-04                    | 0.1000E 01                         | -0.6419E-05 | 0.4206E-06  | 0.6008E-10  | -0.1012E-15 |             |
| 5       |      | 0.1000E 01 | 0.2063E-04                    | 0.1000E 01                         | -0.8941E-05 | -0.6924E-07 | -0.3028E-06 | 0.1797E-16  |             |
| 6       |      | 0.1000E 01 | 0.2063E-04                    | 0.1000E 01                         | -0.9858E-05 | -0.6924E-07 | -0.3166E-06 | -0.3344E-17 |             |
| 7       |      | 0.1000E 01 | 0.2063E-04                    | 0.1000E 01                         | -0.8941E-05 | -0.6359E-06 | 0.1056E-05  | 0.1438E-16  |             |
| 8       |      | 0.1000E 01 | 0.2063E-04                    | 0.1000E 01                         | -0.9858E-05 | -0.6359E-06 | 0.6008E-10  | -0.8805E-16 |             |

## PLASTIC AND CREEP STRAINS

| ELEMENT NO. = 1 ID = 3 |      |      |             |                 |            |             |            |            |             |                                   |            |  |                                 |  |  |
|------------------------|------|------|-------------|-----------------|------------|-------------|------------|------------|-------------|-----------------------------------|------------|--|---------------------------------|--|--|
| INT                    |      | E-P  |             | SUM INCREMENTAL |            | TOTAL       |            | SURFACE    |             | *** EFFECTIVE PLASTIC STRAINS *** |            |  | *** EFFECTIVE CREEP STRAINS *** |  |  |
| PNT                    | CODE | CODE | TEMPERATURE | TEMPERATURE     | YIELD SIZE | INCREMENTAL | SUM INCR.  | CUMULATIVE | INCREMENTAL | SUM INCR.                         | CUMULATIVE |  |                                 |  |  |
| 1                      | -1   | -2   | 0.1000E 01  | 0.1000E 02      | 0.2250E 01 | 0.0         | 0.4000E 01 | 0.2000E 01 | 0.0         | 0.2000E 01                        | 0.2779E-05 |  |                                 |  |  |
| 2                      | -1   | -2   | 0.1000E 01  | 0.1000E 02      | 0.2250E 01 | 0.0         | 0.4000E 01 | 0.2000E 01 | 0.0         | 0.2000E 01                        | 0.3012E-05 |  |                                 |  |  |
| 3                      | -1   | -2   | 0.1000E 01  | 0.1000E 02      | 0.2250E 01 | 0.0         | 0.4000E 01 | 0.2000E 01 | 0.0         | 0.2000E 01                        | 0.2776E-05 |  |                                 |  |  |
| 4                      | -1   | -2   | 0.1000E 01  | 0.1000E 02      | 0.2250E 01 | 0.0         | 0.4000E 01 | 0.2000E 01 | 0.0         | 0.2000E 01                        | 0.3012E-05 |  |                                 |  |  |
| 5                      | -1   | -2   | 0.1000E 01  | 0.1000E 02      | 0.2250E 01 | 0.0         | 0.4000E 01 | 0.2000E 01 | 0.0         | 0.2000E 01                        | 0.2865E-05 |  |                                 |  |  |
| 6                      | -1   | -2   | 0.1000E 01  | 0.1000E 02      | 0.2250E 01 | 0.0         | 0.4000E 01 | 0.2000E 01 | 0.0         | 0.2000E 01                        | 0.3439E-05 |  |                                 |  |  |
| 7                      | -1   | -2   | 0.1000E 01  | 0.1000E 02      | 0.2250E 01 | 0.0         | 0.4000E 01 | 0.2000E 01 | 0.0         | 0.2000E 01                        | 0.2804E-05 |  |                                 |  |  |
| 8                      | -1   | -2   | 0.1000E 01  | 0.1000E 02      | 0.2250E 01 | 0.0         | 0.4000E 01 | 0.2000E 01 | 0.0         | 0.2000E 01                        | 0.3441E-05 |  |                                 |  |  |

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# CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 11

INCREMENT 11

NODE 1-8  
TEMP. 0.45000E 01 0.45000E 01 0.45000E 01 0.45000E 01 0.45000E 01 0.45000E 01 0.45000E 01 0.45000E 01

RESIDUAL NJRM = 0.30604E 00  
RESIDUAL NJRM = 0.41439E 00  
RESIDUAL NJRM = 0.51504E 00  
RESIDUAL NJRM = 0.11151E 00  
RESIDUAL NJRM = 0.22868E 00  
RESIDUAL NJRM = 0.45789E 00  
RESIDUAL NJRM = 0.56770E 00  
RESIDUAL NJRM = 0.41712E 00  
RESIDUAL NJRM = 0.33018E 00  
RESIDUAL NJRM = 0.32329E 00  
RESIDUAL NJRM = 0.32588E 00  
RESIDUAL NJRM = 0.19274E 00  
RESIDUAL NJRM = 0.17634E 01  
RESIDUAL NJRM = 0.29735E 02  
RESIDUAL NJRM = 0.45309E 03  
RESIDUAL NJRM = 0.71251E 04  
RESIDUAL NJRM = 0.11278E 04  
RESIDUAL NJRM = 0.13168E 05

END OF LOAD INCREMENT 11

INCREMENT 11

MECHANICAL LOAD CURVE FACTORS = 0.5500E 01, 0.3000E 00

CREEP TIME INCREMENT = 0.1000E 02

NO. ELASTIC INTEGRATION POINTS = 8, NO. PLASTIC INTEGRATION POINTS = 0

NO. INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 1

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 8

SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.1000E 04, ACTUAL ERROR = 0.1817E 05

## CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** |     | ***** FORCES ***** |                |                | ***** DISPLACEMENTS ***** |               |                |
|------------|-----|--------------------|----------------|----------------|---------------------------|---------------|----------------|
| NO.        | 1-8 | U                  | V              | W              | U                         | V             | W              |
| 1          | 1   | 0.1209611E-05      | -0.8125274E 00 | 0.1100972E-04  | 0.2574659E-05             | 0.0           | -0.3000000E 00 |
| 2          | 2   | 0.1499921E-05      | -0.8125231E 00 | 0.1100884E-04  | 0.2999790E 00             | 0.0           | 0.000000E 00   |
| 3          | 3   | -0.1227371E-05     | 0.8125232E 00  | 0.1106237E-04  | -0.2999306E 00            | 0.5500000E 01 | -0.3000000E 00 |
| 4          | 4   | 0.1216057E-05      | 0.8125225E 00  | 0.1106372E-04  | 0.2338675E-07             | 0.5500000E 01 | -0.3000000E 00 |
| 5          | 5   | 0.1137362E-05      | -0.8125222E 00 | -0.1106884E-04 | 0.0                       | 0.0           | 0.0            |
| 6          | 6   | -0.1126412E-05     | -0.8125221E 00 | -0.1106972E-04 | -0.2999831E 00            | 0.0           | 0.0            |
| 7          | 7   | -0.1102298E-05     | 0.8125221E 00  | -0.1106372E-04 | -0.2999819E 00            | 0.5500000E 01 | 0.0            |
| 8          | 8   | 0.1093662E-05      | 0.8125221E 00  | -0.1106238E-04 | 0.4153544E-05             | 0.5500000E 01 | 0.0            |

## THERMAL AND ELASTIC STRAINS

| ELEMENT NO. | I.D. | INTEGR. POINT | INTEGRAL THERMAL STRAINS | XX          | YY         | ZZ          | XY          | XZ          | YZ          |
|-------------|------|---------------|--------------------------|-------------|------------|-------------|-------------|-------------|-------------|
| 1           | 3    | 1             | 0.3500E 00               | -0.1500E 00 | 0.5000E 00 | -0.1500E 00 | -0.3610E-06 | -0.4486E-07 | 0.1734E-16  |
|             |      | 2             | 0.3500E 00               | -0.1500E 00 | 0.5000E 00 | -0.1500E 00 | -0.3610E-06 | -0.1110E-06 | -0.1183E-15 |
|             |      | 3             | 0.3500E 00               | -0.1500E 00 | 0.5000E 00 | -0.1500E 00 | -0.3508E-06 | -0.9048E-06 | 0.1935E-16  |
|             |      | 4             | 0.3500E 00               | -0.1500E 00 | 0.5000E 00 | -0.1500E 00 | -0.3508E-06 | 0.9233E-07  | 0.1288E-15  |
|             |      | 5             | 0.3500E 00               | -0.1500E 00 | 0.5000E 00 | -0.1500E 00 | -0.2035E-06 | 0.4486E-07  | -0.1306E-16 |
|             |      | 6             | 0.3500E 00               | -0.1500E 00 | 0.5000E 00 | -0.1511E 00 | -0.2052E-06 | 0.1110E-06  | -0.2249E-15 |
|             |      | 7             | 0.3500E 00               | -0.1500E 00 | 0.5000E 00 | -0.1500E 00 | 0.6464E-06  | -0.9048E-06 | -0.1203E-16 |
|             |      | 8             | 0.3500E 00               | -0.1500E 00 | 0.5000E 00 | -0.1500E 00 | 0.6464E-06  | -0.9233E-07 | -0.1077E-15 |

| INTEGR. POINT | CUMULATIVE THERMAL STRAINS | XX          | YY         | ZZ          | XY         | XZ          | YZ          |
|---------------|----------------------------|-------------|------------|-------------|------------|-------------|-------------|
| 1             | 0.1500E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | 0.6151E-07 | 0.2169E-06  | 0.2693E-16  |
| 2             | 0.1500E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | 0.6151E-07 | -0.4481E-07 | 0.1890E-16  |
| 3             | 0.1500E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | 0.7739E-07 | -0.2181E-06 | 0.1524E-16  |
| 4             | 0.1500E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | 0.7740E-07 | 0.9237E-07  | 0.9301E-16  |
| 5             | 0.1500E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | 0.2502E-06 | 0.2169E-06  | -0.1396E-17 |
| 6             | 0.1500E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | 0.2502E-06 | 0.4481E-07  | -0.2201E-15 |
| 7             | 0.1500E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | 0.2530E-06 | -0.2181E-06 | -0.4082E-17 |
| 8             | 0.1500E 01                 | -0.3000E 00 | 0.1000E 01 | -0.3000E 00 | 0.2330E-06 | 0.9237E-07  | 0.5011E-16  |

## PLASTIC WORK AND STRAINS

| ELEMENT NO. | I.D. | INTEGR. POINT | INTEGRAL PLASTIC WORK | XX  | YY  | ZZ  | XY  | XZ  | YZ  |
|-------------|------|---------------|-----------------------|-----|-----|-----|-----|-----|-----|
| 1           | 3    | 1             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             |      | 2             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             |      | 3             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             |      | 4             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             |      | 5             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             |      | 6             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             |      | 7             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|             |      | 8             | 0.0                   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| INTEGR. POINT | CUMULATIVE PLASTIC WORK | XX          | YY         | ZZ          | XY         | XZ          | YZ          |
|---------------|-------------------------|-------------|------------|-------------|------------|-------------|-------------|
| 1             | 0.1013E 02              | -0.1000E 01 | 0.2000E 01 | -0.1000E 01 | 0.2459E-07 | -0.8023E-07 | 0.4596E-16  |
| 2             | 0.1013E 02              | -0.1000E 01 | 0.2000E 01 | -0.1000E 01 | 0.2459E-07 | 0.2749E-06  | -0.1432E-16 |
| 3             | 0.1013E 02              | -0.1000E 01 | 0.2000E 01 | -0.1000E 01 | 0.2346E-07 | 0.7827E-07  | 0.2578E-16  |
| 4             | 0.1013E 02              | -0.1000E 01 | 0.2000E 01 | -0.1000E 01 | 0.2346E-07 | 0.2275E-06  | -0.9283E-16 |
| 5             | 0.1013E 02              | -0.1000E 01 | 0.2000E 01 | -0.1000E 01 | 0.1251E-06 | -0.8723E-07 | 0.3312E-16  |
| 6             | 0.1013E 02              | -0.1000E 01 | 0.2000E 01 | -0.1000E 01 | 0.1251E-06 | 0.2255E-06  | -0.1005E-15 |
| 7             | 0.1013E 02              | -0.1000E 01 | 0.2000E 01 | -0.1000E 01 | 0.1251E-06 | 0.7827E-07  | 0.2724E-16  |
| 8             | 0.1013E 02              | -0.1000E 01 | 0.2000E 01 | -0.1000E 01 | 0.1251E-06 | 0.2273E-06  | 0.9390E-16  |

# CUMULATIVE STRESS QUANTITIES

| ELEMENT |      | INTEGR. POINT | EFFECTIVE STRESS CENTER | *****CUMULATIVE STRESS CENTER***** |            |             |             |             |            |
|---------|------|---------------|-------------------------|------------------------------------|------------|-------------|-------------|-------------|------------|
| NO.     | I.D. |               |                         | XX                                 | YY         | ZZ          | XY          | XZ          | YZ         |
| 1       | 3    | 1             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.2955E-07  | -0.5024E-09 | 0.1375E-16 |
|         |      | 2             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.2955E-07  | -0.5024E-09 | 0.1375E-16 |
|         |      | 3             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | -0.1160E-08 | 0.3524E-07  | 0.9194E-17 |
|         |      | 4             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | -0.1160E-08 | 0.3524E-07  | 0.9194E-17 |
|         |      | 5             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | -0.5482E-07 | -0.5024E-09 | 0.1375E-16 |
|         |      | 6             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | -0.5482E-07 | -0.5024E-09 | 0.1375E-16 |
|         |      | 7             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.5987E-07  | 0.3524E-07  | 0.8897E-17 |
|         |      | 8             | 0.1000E 01              | -0.3333E 00                        | 0.6667E 00 | -0.3333E 00 | 0.5987E-07  | 0.3524E-07  | 0.8897E-17 |

| INTEGR. |            | EFFECTIVE   | *****CUMULATIVE STRESSES***** |            |             |             |             |  |  |
|---------|------------|-------------|-------------------------------|------------|-------------|-------------|-------------|--|--|
| POINT   | STRESS     | XX          | YY                            | ZZ         | XY          | XZ          | YZ          |  |  |
| 1       | 0.3250E 01 | -0.5215E-05 | 0.3250E 01                    | 0.4061E-04 | 0.1538E-06  | 0.5423E-06  | 0.6731E-16  |  |  |
| 2       | 0.3250E 01 | -0.3725E-05 | 0.3250E 01                    | 0.4135E-04 | 0.1538E-06  | -0.2370E-06 | 0.4726E-16  |  |  |
| 3       | 0.3250E 01 | -0.5215E-05 | 0.3250E 01                    | 0.4061E-04 | 0.1935E-06  | 0.5423E-06  | 0.6731E-16  |  |  |
| 4       | 0.3250E 01 | -0.3725E-05 | 0.3250E 01                    | 0.4135E-04 | -0.1935E-06 | 0.2370E-06  | 0.1575E-15  |  |  |
| 5       | 0.3250E 01 | -0.3725E-05 | 0.3250E 01                    | 0.4731E-04 | -0.6255E-06 | 0.5423E-06  | -0.3490E-17 |  |  |
| 6       | 0.3250E 01 | -0.5215E-05 | 0.3250E 01                    | 0.4731E-04 | 0.6255E-06  | -0.2370E-06 | 0.5654E-15  |  |  |
| 7       | 0.3250E 01 | -0.3725E-05 | 0.3250E 01                    | 0.4731E-04 | 0.5826E-06  | -0.5451E-06 | -0.3704E-17 |  |  |
| 8       | 0.3250E 01 | -0.5215E-05 | 0.3250E 01                    | 0.4731E-04 | 0.5826E-06  | 0.2370E-06  | 0.1253E-15  |  |  |

## CREEP WORK AND STRAINS

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| ELEMENT |      | INTEGR. POINT | INCREMENTAL CREEP WORK | *****INCREMENTAL CREEP STRAINS***** |            |             |             |             |             |  |
|---------|------|---------------|------------------------|-------------------------------------|------------|-------------|-------------|-------------|-------------|--|
| NO.     | I.D. |               |                        | XX                                  | YY         | ZZ          | XY          | XZ          | YZ          |  |
| 1       | 3    | 1             | 0.2125E 01             | -0.5000E 00                         | 0.1000E 01 | -0.5000E 00 | 0.9751E-06  | -0.4041E-06 | 0.2213E-16  |  |
|         |      | 2             | 0.2125E 01             | -0.5000E 00                         | 0.1000E 01 | -0.5000E 00 | 0.9751E-06  | -0.4749E-05 | -0.2243E-15 |  |
|         |      | 3             | 0.2125E 01             | -0.5000E 00                         | 0.1000E 01 | -0.5000E 00 | 0.6308E-06  | 0.1585E-05  | -0.7077E-17 |  |
|         |      | 4             | 0.2125E 01             | -0.5000E 00                         | 0.1000E 01 | -0.5000E 00 | 0.6308E-06  | 0.4041E-10  | -0.1518E-15 |  |
|         |      | 5             | 0.2125E 01             | -0.5000E 00                         | 0.1000E 01 | -0.5000E 00 | -0.1039E-06 | 0.6041E-06  | 0.2695E-16  |  |
|         |      | 6             | 0.2125E 01             | -0.5000E 00                         | 0.1000E 01 | -0.5000E 00 | -0.1039E-06 | -0.4749E-05 | -0.5016E-17 |  |
|         |      | 7             | 0.2125E 01             | -0.5000E 00                         | 0.1000E 01 | -0.5000E 00 | -0.5538E-06 | 0.1585E-05  | 0.2157E-16  |  |
|         |      | 8             | 0.2125E 01             | -0.5000E 00                         | 0.1000E 01 | -0.5000E 00 | -0.9538E-06 | 0.4041E-10  | -0.1528E-15 |  |

| INTEGR. |            | CUMULATIVE  |            | *****CUMULATIVE CREEP STRAINS***** |             |             |             |  |  |  |
|---------|------------|-------------|------------|------------------------------------|-------------|-------------|-------------|--|--|--|
| POINT   | CREEP WORK | XX          | YY         | ZZ                                 | XY          | XZ          | YZ          |  |  |  |
| 1       | 0.5813E 01 | -0.5000E 00 | 0.1000E 01 | -0.5000E 00                        | 0.1058E-05  | 0.6832E-06  | 0.1862E-16  |  |  |  |
| 2       | 0.5813E 01 | -0.5000E 00 | 0.1000E 01 | -0.5000E 00                        | 0.1058E-05  | -0.5514E-06 | -0.1003E-15 |  |  |  |
| 3       | 0.5813E 01 | -0.5000E 00 | 0.1000E 01 | -0.5000E 00                        | 0.6328E-06  | 0.1618E-05  | -0.9501E-17 |  |  |  |
| 4       | 0.5813E 01 | -0.5000E 00 | 0.1000E 01 | -0.5000E 00                        | 0.6328E-06  | 0.1171E-06  | -0.1549E-15 |  |  |  |
| 5       | 0.5813E 01 | -0.5000E 00 | 0.1000E 01 | -0.5000E 00                        | -0.1765E-06 | 0.6832E-06  | 0.2379E-16  |  |  |  |
| 6       | 0.5813E 01 | -0.5000E 00 | 0.1000E 01 | -0.5000E 00                        | -0.1765E-06 | -0.5514E-06 | -0.4250E-16 |  |  |  |
| 7       | 0.5813E 01 | -0.5000E 00 | 0.1000E 01 | -0.5000E 00                        | -0.8684E-06 | 0.1618E-05  | 0.1385E-16  |  |  |  |
| 8       | 0.5813E 01 | -0.5000E 00 | 0.1000E 01 | -0.5000E 00                        | -0.8684E-06 | 0.1171E-06  | -0.1366E-15 |  |  |  |

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OF POOR QUALITY

# PLASTIC AND CREEP STRAINS

| ELEMENT NO. 1 |      |      | ID 3        |             |            |            |             |            | **** EFFECTIVE PLASTIC STRAINS **** |            |             | ***** EFFECTIVE CREEP STRAINS ***** |            |            |
|---------------|------|------|-------------|-------------|------------|------------|-------------|------------|-------------------------------------|------------|-------------|-------------------------------------|------------|------------|
| INT           | E-P  | SUM  | INCREMENTAL | TOTAL       | SURFACE    | YIELD SIZE | INCREMENTAL | SUM        | INCR.                               | CUMULATIVE | INCREMENTAL | SUM                                 | INCR.      | CUMULATIVE |
| PNT           | CODE | CODE | TEMPERATURE | TEMPERATURE | YIELD SIZE | YIELD SIZE | INCREMENTAL | SUM        | INCR.                               | CUMULATIVE | INCREMENTAL | SUM                                 | INCR.      | CUMULATIVE |
| 1             | 0    | -2   | -0.5500E 01 | 0.4500E 01  | 0.2250E 01 | 0.0        | 0.4000E 01  | 0.2000E 01 | 0.1000E 01                          | 0.3000E 01 | 0.1000E 01  | 0.3000E 01                          | 0.1000E 01 | 0.1000E 01 |
| 2             | 0    | -2   | -0.5500E 01 | 0.4500E 01  | 0.2250E 01 | 0.0        | 0.4000E 01  | 0.2000E 01 | 0.1000E 01                          | 0.3000E 01 | 0.1000E 01  | 0.3000E 01                          | 0.1000E 01 | 0.1000E 01 |
| 3             | 0    | -2   | -0.5500E 01 | 0.4500E 01  | 0.2250E 01 | 0.0        | 0.4000E 01  | 0.2000E 01 | 0.1000E 01                          | 0.3000E 01 | 0.1000E 01  | 0.3000E 01                          | 0.1000E 01 | 0.1000E 01 |
| 4             | 0    | -2   | -0.5500E 01 | 0.4500E 01  | 0.2250E 01 | 0.0        | 0.4000E 01  | 0.2000E 01 | 0.1000E 01                          | 0.3000E 01 | 0.1000E 01  | 0.3000E 01                          | 0.1000E 01 | 0.1000E 01 |
| 5             | 0    | -2   | -0.5500E 01 | 0.4500E 01  | 0.2250E 01 | 0.0        | 0.4000E 01  | 0.2000E 01 | 0.1000E 01                          | 0.3000E 01 | 0.1000E 01  | 0.3000E 01                          | 0.1000E 01 | 0.1000E 01 |
| 6             | 0    | -2   | -0.5500E 01 | 0.4500E 01  | 0.2250E 01 | 0.0        | 0.4000E 01  | 0.2000E 01 | 0.1000E 01                          | 0.3000E 01 | 0.1000E 01  | 0.3000E 01                          | 0.1000E 01 | 0.1000E 01 |
| 7             | 0    | -2   | -0.5500E 01 | 0.4500E 01  | 0.2250E 01 | 0.0        | 0.4000E 01  | 0.2000E 01 | 0.1000E 01                          | 0.3000E 01 | 0.1000E 01  | 0.3000E 01                          | 0.1000E 01 | 0.1000E 01 |
| 8             | 0    | -2   | -0.5500E 01 | 0.4500E 01  | 0.2250E 01 | 0.0        | 0.4000E 01  | 0.2000E 01 | 0.1000E 01                          | 0.3000E 01 | 0.1000E 01  | 0.3000E 01                          | 0.1000E 01 | 0.1000E 01 |

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## 12

~~-12~~

NOTE 1.9.

```
RESIDUAL NJRM = 0.56433E-00
RESIDUAL NJRM = 0.37032E 00
RESIDUAL NJRM = 0.37065E 00
RESIDUAL NJRM = 0.25410E-00
RESIDUAL NJRM = 0.87729E-01
RESIDUAL NJRM = 0.14111E-02
RESIDUAL NJRM = -0.21369E-02
RESIDUAL NJRM = 0.27629E-02
RESIDUAL NJRM = 0.35405E-02
RESIDUAL NJRM = 0.53045E-03
RESIDUAL NJRM = 0.11136E-03
RESIDUAL NJRM = 0.70427E-06
```

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END OF LOAD INCREMENT 12

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MECHANICAL LOAD CURVE FACTORS • 0.6550E-01, 0.0

CREEP TIME INCREMENT = 0.0

NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 8

~~- 8 - INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 - INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT~~

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 1

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 2

SPECIFIED MAX. UNBALANCED-FORCE-ERROR = -0.1000E-04, - ACTUAL-ERROR = -0.7043E-06

### CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| NODE NO. |      | FORCES         |                |                | DISPLACEMENTS  |               |     |
|----------|------|----------------|----------------|----------------|----------------|---------------|-----|
| NO.      | I.D. | U              | V              | W              | U              | V             | W   |
| 1        | 1    | 0.2142770E-08  | -0.9667405E 00 | 0.1495326E-04  | 0.1641336E-05  | 0.0           | 0.0 |
| 2        | 2    | -0.1524495E-07 | -0.9687413E 00 | 0.1495146E-04  | 0.6139278E-05  | 0.0           | 0.0 |
| 3        | 3    | -0.1476065E-06 | -0.9687414E 00 | -0.1409604E-04 | 0.3443511E-05  | 0.6549999E 01 | 0.0 |
| 4        | 4    | 0.1592043E-06  | 0.9687406E 00  | 0.1469580E-04  | -0.2984252E-06 | 0.6549999E 01 | 0.0 |
| 5        | 5    | 0.2302028E-06  | -0.9667418E 00 | -0.1495146E-04 | 0.0            | 0.0           | 0.0 |
| 6        | 6    | -0.2376722E-06 | -0.9667408E 00 | 0.1495326E-04  | -0.1192093E-05 | 0.0           | 0.0 |
| 7        | 7    | -0.1540110E-06 | 0.9687406E 00  | -0.1469580E-04 | 0.3443511E-05  | 0.6549999E 01 | 0.0 |
| 8        | 8    | 0.1635452E-06  | 0.9687408E 00  | -0.1469604E-04 | 0.2570155E-05  | 0.6549999E 01 | 0.0 |

## THERMAL AND ELASTIC STRAINS

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | INCREMENTAL<br>THERMAL STRAINS | INCREMENTAL ELASTIC STRAINS |             |            |             |             |             |
|---------------------|------------------|--------------------------------|-----------------------------|-------------|------------|-------------|-------------|-------------|
|                     |                  |                                | XX                          | YY          | ZZ         | XY          | XZ          | YZ          |
| 1 3                 | 1                | 0.5500E 00                     | 0.1270E-04                  | -0.4041E-04 | 0.1329E-04 | -0.2561E-06 | -0.2212E-06 | -0.6053E-17 |
|                     | 2                | 0.5500E 00                     | 0.1132E-04                  | -0.3552E-04 | 0.1109E-04 | -0.2560E-06 | -0.2021E-06 | -0.3205E-17 |
|                     | 3                | 0.5500E 00                     | 0.1270E-04                  | -0.4041E-04 | 0.1329E-04 | 0.2833E-06  | 0.2189E-06  | -0.1573E-17 |
|                     | 4                | 0.5500E 00                     | 0.1132E-04                  | -0.3552E-04 | 0.1109E-04 | 0.2833E-06  | -0.1984E-06 | -0.4282E-16 |
|                     | 5                | 0.5500E 00                     | 0.1276E-04                  | -0.3638E-04 | 0.1234E-04 | 0.1673E-06  | 0.2212E-06  | 0.7932E-17  |
|                     | 6                | 0.5500E 00                     | 0.1287E-04                  | -0.3892E-04 | 0.1204E-04 | 0.1673E-06  | 0.2321E-06  | 0.1305E-15  |
|                     | 7                | 0.5500E 00                     | 0.1276E-04                  | -0.3892E-04 | 0.1234E-04 | -0.1339E-06 | 0.2189E-06  | 0.9601E-17  |
|                     | 8                | 0.5500E 00                     | 0.1287E-04                  | -0.3892E-04 | 0.1204E-04 | -0.1339E-06 | -0.1944E-06 | -0.3402E-16 |

| INTEGR.<br>POINT | CUMULATIVE<br>THERMAL STRAINS | CUMULATIVE ELASTIC STRAINS |            |             |             |             |             |
|------------------|-------------------------------|----------------------------|------------|-------------|-------------|-------------|-------------|
|                  |                               | XX                         | YY         | ZZ          | XY          | XZ          | YZ          |
| 1                | 0.2050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | -0.1945E-06 | -0.4244E-06 | 0.2087E-16  |
| 2                | 0.2050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | -0.1945E-06 | 0.1073E-06  | 0.9698E-17  |
| 3                | 0.2050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | -0.2060E-06 | -0.8280E-06 | 0.1371E-16  |
| 4                | 0.2050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | 0.2060E-06  | -0.1060E-06 | 0.2019E-16  |
| 5                | 0.2050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | -0.8294E-07 | -0.4244E-06 | 0.5586E-17  |
| 6                | 0.2050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | -0.8294E-07 | 0.1073E-06  | -0.5504E-16 |
| 7                | 0.2050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | 0.9911E-07  | 0.8275E-06  | 0.5519E-17  |
| 8                | 0.2050E 01                    | -0.3000E 00                | 0.1000E 01 | -0.3000E 00 | 0.9911E-07  | -0.1060E-06 | 0.1609E-16  |

## PLASTIC WORK AND STRAINS

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | INCREMENTAL<br>PLASTIC WORK | INCREMENTAL PLASTIC STRAINS |            |             |             |             |             |
|---------------------|------------------|-----------------------------|-----------------------------|------------|-------------|-------------|-------------|-------------|
|                     |                  |                             | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
| 1 3                 | 1                | 0.1781E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.6330E-07 | 0.8260E-07  | 0.1516E-16  |
|                     | 2                | 0.1781E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.6330E-07 | 0.2809E-07  | 0.3205E-17  |
|                     | 3                | 0.1781E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | 0.5416E-07  | -0.9474E-07 | 0.6945E-17  |
|                     | 4                | 0.1781E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.5416E-07 | 0.3210E-07  | 0.4496E-16  |
|                     | 5                | 0.1781E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.1178E-06 | 0.8260E-07  | -0.1043E-17 |
|                     | 6                | 0.1781E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.1178E-06 | 0.2809E-07  | -0.1236E-15 |
|                     | 7                | 0.1781E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | 0.1168E-06  | -0.9474E-07 | -0.9475E-18 |
|                     | 8                | 0.1781E 01                  | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | 0.1168E-06  | -0.4210E-07 | 0.3836E-16  |

| INTEGR.<br>POINT | CUMULATIVE<br>PLASTIC WORK | CUMULATIVE PLASTIC STRAINS |            |             |             |             |             |
|------------------|----------------------------|----------------------------|------------|-------------|-------------|-------------|-------------|
|                  |                            | XX                         | YY         | ZZ          | XY          | XZ          | YZ          |
| 1                | 0.1191E 02                 | -0.1250E 01                | 0.2500E 01 | -0.1250E 01 | -0.3871E-07 | 0.2375E-08  | 0.6112E-16  |
| 2                | 0.1191E 02                 | -0.1250E 01                | 0.2500E 01 | -0.1250E 01 | -0.3871E-07 | -0.2918E-06 | 0.5114E-17  |
| 3                | 0.1191E 02                 | -0.1250E 01                | 0.2500E 01 | -0.1250E 01 | 0.3870E-07  | -0.1647E-07 | 0.3872E-16  |
| 4                | 0.1191E 02                 | -0.1250E 01                | 0.2500E 01 | -0.1250E 01 | 0.3870E-07  | 0.1952E-06  | -0.4784E-16 |
| 5                | 0.1191E 02                 | -0.1250E 01                | 0.2500E 01 | -0.1250E 01 | -0.2429E-06 | -0.2375E-08 | 0.3217E-16  |
| 6                | 0.1191E 02                 | -0.1250E 01                | 0.2500E 01 | -0.1250E 01 | -0.2429E-06 | -0.2118E-06 | -0.2244E-15 |
| 7                | 0.1191E 02                 | -0.1250E 01                | 0.2500E 01 | -0.1250E 01 | 0.2424E-06  | -0.1647E-07 | 0.2831E-16  |
| 8                | 0.1191E 02                 | -0.1250E 01                | 0.2500E 01 | -0.1250E 01 | -0.2424E-06 | 0.1952E-06  | -0.5554E-16 |



# CUMULATIVE STRESS QUANTITIES

| ELEMENT NO.                     | I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | ***** CUMULATIVE STRESS CENTER ***** |            |             |             |             |            |
|---------------------------------|------|---------------|-------------------------|--------------------------------------|------------|-------------|-------------|-------------|------------|
|                                 |      |               |                         | XX                                   | YY         | ZZ          | XY          | XZ          | YZ         |
| 1                               | 1    | 1             | 0.1500E-01              | -0.5000E-00                          | 0.1000E-01 | -0.5000E-00 | 0.1265E-07  | 0.5457E-07  | 0.2386E-16 |
|                                 |      | 2             | 0.1500E-01              | -0.5000E-00                          | 0.1000E-01 | -0.5000E-00 | 0.1265E-07  | 0.6615E-07  | 0.1391E-16 |
|                                 |      | 3             | 0.1500E-01              | -0.5000E-00                          | 0.1000E-01 | -0.5000E-00 | 0.3495E-07  | 0.2792E-07  | 0.1516E-16 |
|                                 |      | 4             | 0.1500E-01              | -0.5000E-00                          | 0.1000E-01 | -0.5000E-00 | 0.3495E-07  | 0.7487E-07  | 0.8949E-17 |
|                                 |      | 5             | 0.1500E-01              | -0.5000E-00                          | 0.1000E-01 | -0.5000E-00 | -0.1334E-06 | 0.5457E-07  | 0.9888E-17 |
|                                 |      | 6             | 0.1500E-01              | -0.5000E-00                          | 0.1000E-01 | -0.5000E-00 | -0.1334E-06 | 0.6615E-07  | 0.1023E-15 |
|                                 |      | 7             | 0.1500E-01              | -0.5000E-00                          | 0.1000E-01 | -0.5000E-00 | 0.1377E-06  | 0.2792E-07  | 0.8279E-17 |
|                                 |      | 8             | 0.1500E-01              | -0.5000E-00                          | 0.1000E-01 | -0.5000E-00 | 0.1377E-06  | 0.7487E-07  | 0.1408E-16 |
| ***** CUMULATIVE STRESSES ***** |      |               |                         | XX                                   | YY         | ZZ          | XY          | XZ          | YZ         |
|                                 |      | 1             | 0.3875E-01              | -0.8883E-06                          | 0.3875E-01 | 0.5952E-04  | -0.5798E-06 | -0.1265E-07 | 0.2225E-16 |
|                                 |      | 2             | 0.3875E-01              | -0.1333E-05                          | 0.3875E-01 | 0.5508E-04  | 0.5798E-06  | 0.3199E-06  | 0.2891E-16 |
|                                 |      | 3             | 0.3875E-01              | -0.8883E-06                          | 0.3875E-01 | 0.5952E-04  | 0.6139E-06  | 0.2468E-08  | 0.6038E-16 |
|                                 |      | 4             | 0.3875E-01              | -0.1333E-05                          | 0.3875E-01 | 0.5508E-04  | 0.6139E-06  | -0.3160E-06 | 0.6018E-16 |
|                                 |      | 5             | 0.3875E-01              | -0.8883E-06                          | 0.3875E-01 | 0.6085E-04  | 0.2472E-06  | 0.1265E-07  | 0.1903E-16 |
|                                 |      | 6             | 0.3875E-01              | -0.8883E-06                          | 0.3875E-01 | 0.6174E-04  | -0.2472E-06 | 0.3199E-06  | 0.2051E-15 |
|                                 |      | 7             | 0.3875E-01              | 0.8883E-06                           | 0.3875E-01 | 0.6085E-04  | 0.2954E-06  | 0.2466E-08  | 0.2063E-16 |
|                                 |      | 8             | 0.3875E-01              | 0.8883E-06                           | 0.3875E-01 | 0.6174E-04  | 0.2954E-06  | 0.3160E-06  | 0.4775E-16 |

14.1-55

## PLASTIC AND CREEP STRAINS

| ELEMENT NO. = 1 ID = 3 |      |      |             |             |            |             |             |            |             |             |            |            |
|------------------------|------|------|-------------|-------------|------------|-------------|-------------|------------|-------------|-------------|------------|------------|
| INT                    | E-P  | SUM  | INCREMENTAL | TOTAL       | SURFACE    | YIELD SIZE  | INCREMENTAL | SUM INCR.  | CUMULATIVE  | INCREMENTAL | SUM INCR.  | CUMULATIVE |
| PNT                    | CODE | CODE | TEMPERATURE | TEMPERATURE | YIELD SIZE | INCREMENTAL | SUM INCR.   | CUMULATIVE | INCREMENTAL | SUM INCR.   | CUMULATIVE |            |
| 1                      | 1    | 2    | 0.7500E-01  | 0.1200E-02  | 0.2375E-01 | 0.5000E-00  | 0.4500E-01  | 0.2500E-01 | 0.0         | 0.3000E-01  | 0.1000E-01 |            |
| 2                      | 1    | 2    | 0.7500E-01  | 0.1200E-02  | 0.2375E-01 | 0.5000E-00  | 0.4500E-01  | 0.2500E-01 | 0.0         | 0.3000E-01  | 0.1000E-01 |            |
| 3                      | 1    | 2    | 0.7500E-01  | 0.1200E-02  | 0.2375E-01 | 0.5000E-00  | 0.4500E-01  | 0.2500E-01 | 0.0         | 0.3000E-01  | 0.1000E-01 |            |
| 4                      | 1    | 2    | 0.7500E-01  | 0.1200E-02  | 0.2375E-01 | 0.5000E-00  | 0.4500E-01  | 0.2500E-01 | 0.0         | 0.3000E-01  | 0.1000E-01 |            |
| 5                      | 1    | 2    | 0.7500E-01  | 0.1200E-02  | 0.2375E-01 | 0.5000E-00  | 0.4500E-01  | 0.2500E-01 | 0.0         | 0.3000E-01  | 0.1000E-01 |            |
| 6                      | 1    | 2    | 0.7500E-01  | 0.1200E-02  | 0.2375E-01 | 0.5000E-00  | 0.4500E-01  | 0.2500E-01 | 0.0         | 0.3000E-01  | 0.1000E-01 |            |
| 7                      | 1    | 2    | 0.7500E-01  | 0.1200E-02  | 0.2375E-01 | 0.5000E-00  | 0.4500E-01  | 0.2500E-01 | 0.0         | 0.3000E-01  | 0.1000E-01 |            |
| 8                      | 1    | 2    | 0.7500E-01  | 0.1200E-02  | 0.2375E-01 | 0.5000E-00  | 0.4500E-01  | 0.2500E-01 | 0.0         | 0.3000E-01  | 0.1000E-01 |            |

INCREMENT 13  
 NODE 1.D. 1 2 3 4 5 6 7 8  
 TEMP. -0.13000E-02 0.13000E-02 0.13000E-02 0.13000E-02 0.13000E-02 0.13000E-02 0.13000E-02 0.13000E-02

RESIDUAL NORM = 0.53475E-00  
 RESIDUAL NORM = 0.36659E-00  
 RESIDUAL NORM = 0.24213E-00  
 RESIDUAL NORM = 0.95973E-02  
 RESIDUAL NORM = 0.32429E-02  
 RESIDUAL NORM = 0.27001E-03  
 RESIDUAL NORM = 0.26758E-04  
 RESIDUAL NORM = 0.49350E-05

END OF LOAD INCREMENT 13

INCREMENT 13  
 MECHANICAL LOAD CURVE FACTORS = 0.7500E 01, 0.2000E 00  
 CREEP TIME INCREMENT = 0.0  
 NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 8  
 0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT  
 SPECIFIED MAX. NO. STIFFNESS-UPDATES = 3, NO. UPDATES PERFORMED = 0  
 SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 8  
 SPECIFIED MAX. UNBALANCED-FORCE ERROR = 0.1000E-04, ACTUAL ERROR = 0.4935E-05

14.1-86

CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** |      | ***** FORCES ***** |                |               |  | ***** DISPLACEMENTS ***** |               |               |  |
|------------|------|--------------------|----------------|---------------|--|---------------------------|---------------|---------------|--|
| NO.        | I.D. | U                  | V              | h             |  | U                         | V             | h             |  |
| 1          | 1    | -0.1142594E-05     | -0.1125026E 01 | 0.3116721E-05 |  | 0.2073369E-05             | 0.0           | 0.2000000E 00 |  |
| 2          | 2    | 0.1140224E-05      | -0.1125027E 01 | 0.3107535E-05 |  | 0.2000180E 00             | 0.0           | 0.2000000E 00 |  |
| 3          | 3    | -0.3569586E-06     | -0.1125027E 01 | 0.3331095E-05 |  | 0.2000154E 00             | 0.7500000E 01 | 0.2000000E 00 |  |
| 4          | 4    | -0.8725429E-06     | 0.1125026E 01  | 0.3339867E-05 |  | 0.5643701E-06             | 0.7500000E 01 | 0.2000000E 00 |  |
| 5          | 5    | -0.5349151E-06     | -0.1125026E 01 | 0.3117535E-05 |  | 0.0                       | 0.0           | 0.0           |  |
| 6          | 6    | 0.5513160E-06      | -0.1125026E 01 | 0.3116722E-05 |  | 0.2000135E 00             | 0.0           | 0.0           |  |
| 7          | 7    | 0.9042738E-06      | 0.1125026E 01  | 0.3339866E-05 |  | 0.2000152E 00             | 0.7500000E 01 | 0.0           |  |
| 8          | 8    | -0.8027211E-06     | 0.1125026E 01  | 0.3331096E-05 |  | 0.3649180E-05             | 0.7500000E 01 | 0.0           |  |

# THERMAL AND ELASTIC STRAINS

| ELEMENT |      | INTEGR. POINT | INCREMENTAL THERMAL STRAINS | INCREMENTAL ELASTIC STRAINS |            |             |             |             |             |
|---------|------|---------------|-----------------------------|-----------------------------|------------|-------------|-------------|-------------|-------------|
| NO.     | I.D. |               |                             | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
| 1       | 1    | 1             | 0.4500E 00                  | -0.6437E-05                 | 0.4059E-04 | -0.2414E-04 | 0.3010E-06  | 0.6941E-07  | -0.1169E-17 |
|         |      | 2             | 0.4500E 00                  | -0.4530E-05                 | 0.3439E-04 | -0.2092E-04 | 0.3010E-06  | 0.9780E-07  | -0.5420E-17 |
|         |      | 3             | 0.4500E 00                  | -0.6437E-05                 | 0.4059E-04 | -0.2414E-04 | -0.3209E-06 | -0.7678E-07 | -0.3489E-17 |
|         |      | 4             | 0.4500E 00                  | -0.4530E-05                 | 0.3439E-04 | -0.2092E-04 | -0.3209E-06 | 0.8658E-07  | -0.1548E-16 |
|         |      | 5             | 0.4500E 00                  | -0.5126E-05                 | 0.3725E-04 | -0.2247E-04 | 0.1337E-06  | -0.6941E-07 | 0.2608E-17  |
|         |      | 6             | 0.4500E 00                  | -0.5245E-05                 | 0.3654E-04 | -0.2229E-04 | 0.1337E-06  | -0.9786E-07 | 0.4438E-16  |
|         |      | 7             | 0.4500E 00                  | -0.5126E-05                 | 0.3725E-04 | -0.2247E-04 | -0.1576E-06 | -0.7678E-07 | 0.2965E-17  |
|         |      | 8             | 0.4500E 00                  | -0.5245E-05                 | 0.3654E-04 | -0.2229E-04 | -0.1576E-06 | -0.8658E-07 | -0.9740E-17 |
|         |      | INTEGR. POINT | CUMULATIVE THERMAL STRAINS  | CUMULATIVE ELASTIC STRAINS  |            |             |             |             |             |
|         |      |               |                             | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
|         |      | 1             | 0.2500E 01                  | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | 0.1064E-06  | 0.6517E-07  | 0.1970E-16  |
|         |      | 2             | 0.2500E 01                  | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | 0.1064E-06  | 0.4473E-08  | 0.4277E-17  |
|         |      | 3             | 0.2500E 01                  | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.1150E-06 | -0.7595E-07 | -0.1022E-16 |
|         |      | 4             | 0.2500E 01                  | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.1150E-06 | -0.1944E-07 | 0.4709E-17  |
|         |      | 5             | 0.2500E 01                  | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | 0.5075E-07  | 0.6517E-07  | 0.5194E-17  |
|         |      | 6             | 0.2500E 01                  | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.5075E-07 | -0.4473E-08 | -0.5126E-16 |
|         |      | 7             | 0.2500E 01                  | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.5844E-07 | -0.7595E-07 | 0.9905E-17  |
|         |      | 8             | 0.2500E 01                  | -0.3000E 00                 | 0.1000E 01 | -0.3000E 00 | -0.5844E-07 | -0.1944E-07 | 0.6346E-17  |

## PLASTIC WORK AND STRAINS

| ELEMENT |      | INTEGR. POINT | INCREMENTAL PLASTIC WORK | INCREMENTAL PLASTIC STRAINS |            |             |             |             |             |
|---------|------|---------------|--------------------------|-----------------------------|------------|-------------|-------------|-------------|-------------|
| NO.     | I.D. |               |                          | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
| 1       | 1    | 1             | 0.2094E 01               | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.3408E-07 | 0.1161E-07  | -0.1158E-16 |
|         |      | 2             | 0.2094E 01               | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.3409E-07 | 0.7242E-07  | 0.2385E-17  |
|         |      | 3             | 0.2094E 01               | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | 0.2862E-07  | -0.2533E-07 | 0.6525E-17  |
|         |      | 4             | 0.2094E 01               | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.2862E-07 | -0.7866E-07 | -0.1374E-16 |
|         |      | 5             | 0.2094E 01               | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | 0.2673E-07  | 0.1161E-07  | 0.4331E-17  |
|         |      | 6             | 0.2094E 01               | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | 0.2673E-07  | 0.7242E-07  | -0.3744E-16 |
|         |      | 7             | 0.2094E 01               | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.2471E-07 | -0.2533E-07 | -0.5255E-17 |
|         |      | 8             | 0.2094E 01               | -0.2500E 00                 | 0.5000E 00 | -0.2500E 00 | -0.2471E-07 | -0.7866E-07 | 0.1403E-16  |
|         |      | INTEGR. POINT | CUMULATIVE PLASTIC WORK  | CUMULATIVE PLASTIC STRAINS  |            |             |             |             |             |
|         |      |               |                          | XX                          | YY         | ZZ          | XY          | XZ          | YZ          |
|         |      | 1             | 0.1400E 02               | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | -0.7279E-07 | 0.1398E-07  | 0.7270E-16  |
|         |      | 2             | 0.1400E 02               | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | -0.7280E-07 | -0.1294E-06 | -0.2724E-17 |
|         |      | 3             | 0.1400E 02               | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | 0.5932E-07  | -0.4181E-07 | 0.4525E-16  |
|         |      | 4             | 0.1400E 02               | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | 0.5931E-07  | 0.1166E-06  | -0.3410E-16 |
|         |      | 5             | 0.1400E 02               | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | -0.2162E-06 | -0.1398E-07 | -0.3640E-16 |
|         |      | 6             | 0.1400E 02               | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | -0.2162E-06 | -0.1294E-06 | -0.2619E-15 |
|         |      | 7             | 0.1400E 02               | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | 0.2177E-06  | -0.4181E-07 | 0.3357E-16  |
|         |      | 8             | 0.1400E 02               | -0.1500E 01                 | 0.3000E 01 | -0.1500E 01 | 0.2177E-06  | -0.1166E-06 | -0.4146E-16 |

ORIGINAL PAGE IS  
OF POOR QUALITY

# CUMULATIVE STRESS QUANTITIES

\*\*\*\*\* CUMULATIVE STRESS CENTER \*\*\*\*\*

| ELEMENT NO. I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | XX          | YY         | ZZ          | XY          | XZ          | YZ          |
|------------------|---------------|-------------------------|-------------|------------|-------------|-------------|-------------|-------------|
| 1                | 3             |                         |             |            |             |             |             |             |
|                  | 1             | 0.2000E-01              | -0.6667E-00 | 0.1333E-01 | -0.6667E-00 | -0.3537E-07 | 0.6230E-07  | 0.3158E-16  |
|                  | 2             | 0.2000E-01              | -0.6667E-00 | 0.1333E-01 | -0.6667E-00 | -0.3537E-07 | -0.1787E-07 | 0.1550E-16  |
|                  | 3             | 0.2000E-01              | -0.6667E-00 | 0.1333E-01 | -0.6667E-00 | 0.5403E-07  | -0.4481E-07 | 0.1951E-16  |
|                  | 4             | 0.2000E-01              | -0.6667E-00 | 0.1333E-01 | -0.6667E-00 | -0.5402E-07 | 0.2243E-07  | -0.2133E-18 |
|                  | 5             | 0.2000E-01              | -0.6667E-00 | 0.1333E-01 | -0.6667E-00 | -0.1155E-06 | 0.6230E-07  | 0.1278E-16  |
|                  | 6             | 0.2000E-01              | -0.6667E-00 | 0.1333E-01 | -0.6667E-00 | -0.1155E-06 | -0.1767E-07 | -0.1272E-15 |
|                  | 7             | 0.2000E-01              | -0.6667E-00 | 0.1333E-01 | -0.6667E-00 | 0.1213E-06  | 0.4481E-07  | 0.1178E-16  |
|                  | 8             | 0.2000E-01              | -0.6667E-00 | 0.1333E-01 | -0.6667E-00 | 0.1213E-06  | 0.2243E-07  | -0.4696E-17 |

\*\*\*\*\* CUMULATIVE STRESSES \*\*\*\*\*

| INTEGR. POINT | EFFECTIVE STRESS | XX         | YY         | ZZ         | XY          | XZ          | YZ          |
|---------------|------------------|------------|------------|------------|-------------|-------------|-------------|
| 1             | 0.4500E-01       | 0.0        | 0.4500E-01 | 0.8769E-05 | 0.3685E-06  | 0.2256E-06  | 0.5820E-16  |
| 2             | 0.4500E-01       | 0.4126E-05 | 0.4500E-01 | 0.1238E-04 | -0.3684E-06 | 0.3274E-07  | -0.1481E-16 |
| 3             | 0.4500E-01       | 0.0        | 0.4500E-01 | 0.8769E-05 | -0.3979E-06 | -0.2624E-06 | 0.3539E-16  |
| 4             | 0.4500E-01       | 0.4126E-05 | 0.4500E-01 | 0.1238E-04 | -0.3979E-06 | -0.6728E-07 | 0.1630E-16  |
| 5             | 0.4500E-01       | 0.6190E-05 | 0.4500E-01 | 0.1547E-04 | 0.1757E-06  | -0.2256E-06 | -0.3183E-16 |
| 6             | 0.4500E-01       | 0.3095E-05 | 0.4500E-01 | 0.1496E-04 | 0.1757E-06  | 0.3279E-07  | -0.1774E-15 |
| 7             | 0.4500E-01       | 0.6190E-05 | 0.4500E-01 | 0.1547E-04 | -0.2023E-06 | -0.2624E-06 | 0.3429E-16  |
| 8             | 0.4500E-01       | 0.3095E-05 | 0.4500E-01 | 0.1496E-04 | -0.2023E-06 | -0.6728E-07 | -0.2197E-16 |

## PLASTIC AND CREEP STRAINS

ELEMENT NO. = 1 ID = 3 \*\*\*\*\* EFFECTIVE PLASTIC STRAINS \*\*\*\*\* \*\*\*\*\* EFFECTIVE CREEP STRAINS \*\*\*\*\*

| INT | E-P  | SUM  | INCREMENTAL | TOTAL       | SURFACE    | YIELD SIZE | INCREMENTAL | SUM INCR.  | CUMULATIVE | INCREMENTAL | SUM INCR.  | CUMULATIVE |
|-----|------|------|-------------|-------------|------------|------------|-------------|------------|------------|-------------|------------|------------|
| PNT | CODE | CODE | TEMPERATURE | TEMPERATURE |            |            |             |            |            |             |            |            |
| 1   | 0    | 2    | 0.1000E-01  | 0.1300E-02  | 0.2500E-01 | 0.5000E-00 | 0.5000E-01  | 0.3000E-01 | 0.3000E-01 | 0.0         | 0.3000E-01 | 0.1000E-01 |
| 2   | 0    | 2    | 0.1000E-01  | 0.1300E-02  | 0.2500E-01 | 0.5000E-00 | 0.5000E-01  | 0.3000E-01 | 0.3000E-01 | 0.0         | 0.3000E-01 | 0.1000E-01 |
| 3   | 0    | 2    | 0.1000E-01  | 0.1300E-02  | 0.2500E-01 | 0.5000E-00 | 0.5000E-01  | 0.3000E-01 | 0.3000E-01 | 0.0         | 0.3000E-01 | 0.1000E-01 |
| 4   | 0    | 2    | 0.1000E-01  | 0.1300E-02  | 0.2500E-01 | 0.5000E-00 | 0.5000E-01  | 0.3000E-01 | 0.3000E-01 | 0.0         | 0.3000E-01 | 0.1000E-01 |
| 5   | 0    | 2    | 0.1000E-01  | 0.1300E-02  | 0.2500E-01 | 0.5000E-00 | 0.5000E-01  | 0.3000E-01 | 0.3000E-01 | 0.0         | 0.3000E-01 | 0.1000E-01 |
| 6   | 0    | 2    | 0.1000E-01  | 0.1300E-02  | 0.2500E-01 | 0.5000E-00 | 0.5000E-01  | 0.3000E-01 | 0.3000E-01 | 0.0         | 0.3000E-01 | 0.1000E-01 |
| 7   | 0    | 2    | 0.1000E-01  | 0.1300E-02  | 0.2500E-01 | 0.5000E-00 | 0.5000E-01  | 0.3000E-01 | 0.3000E-01 | 0.0         | 0.3000E-01 | 0.1000E-01 |
| 8   | 0    | 2    | 0.1000E-01  | 0.1300E-02  | 0.2500E-01 | 0.5000E-00 | 0.5000E-01  | 0.3000E-01 | 0.3000E-01 | 0.0         | 0.3000E-01 | 0.1000E-01 |

# CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 14

INCREMENT 14

| NODE I.D.     | 1             | 2           | 3           | 4           | 5           | 6           | 7           | 8           |
|---------------|---------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| TEMP.         | 0.14000E 02   | 0.14000E 02 | 0.14000E 02 | 0.14000E 02 | 0.14000E 02 | 0.14000E 02 | 0.14000E 02 | 0.14000E 02 |
| RESIDUAL URM  | = 0.32064E 00 |             |             |             |             |             |             |             |
| RESIDUAL NIRM | = 0.35148E 00 |             |             |             |             |             |             |             |
| RESIDUAL VIRM | = 0.39220E 00 |             |             |             |             |             |             |             |
| RESIDUAL UIRM | = 0.27593E 00 |             |             |             |             |             |             |             |
| RESIDUAL UIRM | = 0.11152E 00 |             |             |             |             |             |             |             |
| RESIDUAL UIRM | = 0.35421E 02 |             |             |             |             |             |             |             |
| RESIDUAL UIRM | = 0.29984E 02 |             |             |             |             |             |             |             |
| RESIDUAL UIRM | = 0.41262E 03 |             |             |             |             |             |             |             |
| RESIDUAL UIRM | = 0.43503E 05 |             |             |             |             |             |             |             |

END OF LOAD INCREMENT 14

INCREMENT 14  
MECHANICAL LOAD CURVE FACTORS = 0.8800E 01, 0.0

14.1-59  
CREEP TIME INCREMENT = 0.0  
NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 8  
0 - INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 - INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT  
SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 0  
SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 9  
SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.1000E 04, ACTUAL ERROR = 0.4350E 05

## CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NOJIE ** | ** NO. I.D. ** | *** U ***      | *** V ***      | *** W ***      | ***** U *****  | ***** V ***** | ***** W ***** |
|-------------|----------------|----------------|----------------|----------------|----------------|---------------|---------------|
| 1           | 1              | -0.7336850E-06 | -0.1312485E 01 | 0.1410311E-04  | 0.9445503E-06  | 0.0           | 0.0           |
| 2           | 2              | 0.7280888E-06  | -0.1312486E 01 | 0.1409979E-04  | 0.4351139E-05  | 0.0           | 0.0           |
| 3           | 3              | 0.1008990E-05  | -0.1312486E 01 | 0.1358109E-04  | 0.2324581E-05  | 0.8799999E 01 | 0.0           |
| 4           | 4              | -0.1005115E-05 | 0.1312485E 01  | 0.1357950E-04  | -0.1128140E-05 | 0.8799999E 01 | 0.0           |
| 5           | 5              | -0.6970147E-06 | -0.1312484E 01 | -0.1409979E-04 | 0.0            | 0.0           | 0.0           |
| 6           | 6              | 0.7029308E-06  | -0.1312484E 01 | -0.1410311E-04 | -0.1192053E-05 | 0.0           | 0.0           |
| 7           | 7              | 0.1170668E-05  | 0.1312484E 01  | -0.1357950E-04 | 0.2980232E-05  | 0.8799999E 01 | 0.0           |
| 8           | 8              | -0.1174866E-05 | 0.1312484E 01  | -0.1358109E-04 | 0.1090041E-05  | 0.8799999E 01 | 0.0           |

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## THERMAL AND ELASTIC STRAINS

| ELEMENT<br>NO.   | I.D.                          | INTEGR.<br>POINT           | INCREMENTAL<br>THERMAL STRAINS | INCREMENTAL ELASTIC STRAINS |             |             |             |             |             |
|------------------|-------------------------------|----------------------------|--------------------------------|-----------------------------|-------------|-------------|-------------|-------------|-------------|
|                  |                               |                            |                                | XX                          | YY          | ZZ          | XY          | XZ          | YZ          |
| 1                | 3                             | 1                          | 0.3000E 00                     | 0.9596E-05                  | -0.4292E-04 | 0.2098E-04  | -0.4284E-06 | -0.1574E-06 | -0.2382E-17 |
|                  |                               | 2                          | 0.3000E 00                     | 0.7272E-05                  | -0.3433E-04 | 0.1717E-04  | -0.4284E-06 | -0.4854E-07 | -0.2659E-16 |
|                  |                               | 3                          | 0.3000E 00                     | 0.9596E-05                  | -0.4292E-04 | 0.2098E-04  | 0.4325E-06  | 0.1632E-06  | -0.2321E-17 |
|                  |                               | 4                          | 0.3000E 00                     | 0.7272E-05                  | -0.3433E-04 | 0.1717E-04  | 0.4325E-06  | -0.3523E-07 | -0.3524E-16 |
|                  |                               | 5                          | 0.3000E 00                     | 0.7391E-05                  | -0.3719E-04 | 0.1812E-04  | -0.2224E-06 | -0.1574E-06 | -0.5716E-17 |
|                  |                               | 6                          | 0.3000E 00                     | 0.7629E-05                  | -0.3529E-04 | 0.1717E-04  | -0.2224E-06 | 0.4854E-07  | -0.1552E-16 |
|                  |                               | 7                          | 0.3000E 00                     | 0.7391E-05                  | -0.3719E-04 | 0.1812E-04  | 0.2341E-06  | 0.1632E-06  | -0.7122E-17 |
|                  |                               | 8                          | 0.3000E 00                     | 0.7629E-05                  | -0.3529E-04 | 0.1717E-04  | -0.2341E-06 | -0.3523E-07 | -0.4123E-16 |
| INTEGR.<br>POINT | CUMULATIVE<br>THERMAL STRAINS | CUMULATIVE ELASTIC STRAINS |                                |                             |             |             |             | XZ          | YZ          |
|                  |                               | XX                         | YY                             | ZZ                          | XY          | XZ          | YZ          |             |             |
| 1                | 0.2800E 01                    | -0.3000E 00                | 0.1000E 01                     | -0.3000E 00                 | -0.3219E-06 | -0.9226E-07 | 0.1732E-16  |             |             |
| 2                | 0.2800E 01                    | -0.3000E 00                | 0.1000E 01                     | -0.3000E 00                 | -0.3219E-06 | 0.5801E-07  | -0.2231E-16 |             |             |
| 3                | 0.2800E 01                    | -0.3000E 00                | 0.1000E 01                     | -0.3000E 00                 | -0.3176E-06 | -0.8720E-07 | -0.7903E-17 |             |             |
| 4                | 0.2800E 01                    | -0.3000E 00                | 0.1000E 01                     | -0.3000E 00                 | -0.3176E-06 | -0.5466E-07 | -0.5055E-16 |             |             |
| 5                | 0.2800E 01                    | -0.3000E 00                | 0.1000E 01                     | -0.3000E 00                 | -0.1717E-06 | -0.9226E-07 | 0.3478E-17  |             |             |
| 6                | 0.2800E 01                    | -0.3000E 00                | 0.1000E 01                     | -0.3000E 00                 | -0.1717E-06 | 0.5801E-07  | -0.6579E-16 |             |             |
| 7                | 0.2800E 01                    | -0.3000E 00                | 0.1000E 01                     | -0.3000E 00                 | 0.1757E-06  | 0.8720E-07  | 0.2783E-17  |             |             |
| 8                | 0.2800E 01                    | -0.3000E 00                | 0.1000E 01                     | -0.3000E 00                 | 0.1757E-06  | -0.5466E-07 | -0.3488E-16 |             |             |

## PLASTIC WORK AND STRAINS

| ELEMENT<br>NO.   | I.D.                       | INTEGR.<br>POINT           | INCREMENTAL<br>PLASTIC WORK | INCREMENTAL PLASTIC STRAINS |             |             |             |             |             |
|------------------|----------------------------|----------------------------|-----------------------------|-----------------------------|-------------|-------------|-------------|-------------|-------------|
|                  |                            |                            |                             | XX                          | YY          | ZZ          | XY          | XZ          | YZ          |
| 1                | 3                          | 1                          | 0.4875E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.1930E-06 | -0.5107E-07 | -0.1473E-16 |
|                  |                            | 2                          | 0.4875E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.1930E-06 | 0.7978E-07  | -0.2589E-16 |
|                  |                            | 3                          | 0.4875E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | 0.1697E-06  | 0.2615E-07  | 0.7525E-17  |
|                  |                            | 4                          | 0.4875E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | 0.1697E-06  | -0.8824E-07 | -0.2548E-16 |
|                  |                            | 5                          | 0.4875E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.6215E-07 | -0.5107E-07 | -0.5716E-17 |
|                  |                            | 6                          | 0.4875E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | -0.6215E-07 | 0.7978E-07  | -0.4793E-16 |
|                  |                            | 7                          | 0.4875E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | 0.5531E-07  | -0.2615E-07 | -0.6254E-17 |
|                  |                            | 8                          | 0.4875E 01                  | -0.5000E 00                 | 0.1000E 01  | -0.5000E 00 | 0.5531E-07  | -0.8824E-07 | -0.2556E-16 |
| INTEGR.<br>POINT | CUMULATIVE<br>PLASTIC WORK | CUMULATIVE PLASTIC STRAINS |                             |                             |             |             |             | XZ          | YZ          |
|                  |                            | XX                         | YY                          | ZZ                          | XY          | XZ          | YZ          |             |             |
| 1                | 0.1888E 02                 | -0.2000E 01                | 0.4000E 01                  | -0.2000E 01                 | -0.2658E-06 | -0.3709E-07 | 0.9242E-16  |             |             |
| 2                | 0.1888E 02                 | -0.2000E 01                | 0.4000E 01                  | -0.2000E 01                 | -0.2658E-06 | -0.4960E-07 | -0.2862E-16 |             |             |
| 3                | 0.1888E 02                 | -0.2000E 01                | 0.4000E 01                  | -0.2000E 01                 | 0.2291E-06  | -0.1566E-07 | 0.5277E-16  |             |             |
| 4                | 0.1888E 02                 | -0.2000E 01                | 0.4000E 01                  | -0.2000E 01                 | 0.2291E-06  | 0.2827E-07  | -0.5458E-16 |             |             |
| 5                | 0.1888E 02                 | -0.2000E 01                | 0.4000E 01                  | -0.2000E 01                 | 0.2730E-06  | -0.3709E-07 | -0.4212E-16 |             |             |
| 6                | 0.1888E 02                 | -0.2000E 01                | 0.4000E 01                  | -0.2000E 01                 | -0.2783E-06 | -0.4960E-07 | -0.3098E-15 |             |             |
| 7                | 0.1888E 02                 | -0.2000E 01                | 0.4000E 01                  | -0.2000E 01                 | 0.2730E-06  | -0.1566E-07 | 0.3482E-16  |             |             |
| 8                | 0.1888E 02                 | -0.2000E 01                | 0.4000E 01                  | -0.2000E 01                 | 0.2730E-06  | 0.2827E-07  | -0.6702E-16 |             |             |

14.1-60

# CUMULATIVE STRESS QUANTITIES

| ELEMENT NO.                   | I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | *****CUMULATIVE STRESS CENTER***** |            |             |              |              |              |
|-------------------------------|------|---------------|-------------------------|------------------------------------|------------|-------------|--------------|--------------|--------------|
|                               |      |               |                         | XX                                 | YY         | ZZ          | XY           | XZ           | YZ           |
| 1                             | 3    | 1             | 0.2500E 01              | -0.8333E 00                        | 0.1667E 01 | -0.8333E 00 | -0.9970E -07 | 0.4528E -07  | 0.3815E -16  |
|                               |      | 2             | 0.2500E 01              | -0.8333E 00                        | 0.1667E 01 | -0.8333E 00 | -0.9970E -07 | 0.8728E -08  | 0.5872E -17  |
|                               |      | 3             | 0.2500E 01              | -0.8333E 00                        | 0.1667E 01 | -0.8333E 00 | 0.1106E -06  | -0.3609E -07 | 0.2202E -16  |
|                               |      | 4             | 0.2500E 01              | -0.8333E 00                        | 0.1667E 01 | -0.8333E 00 | -0.1106E -06 | 0.6557E -08  | 0.8275E -17  |
|                               |      | 5             | 0.2500E 01              | -0.8333E 00                        | 0.1667E 01 | -0.8333E 00 | -0.1363E -06 | 0.4528E -07  | 0.1468E -16  |
|                               |      | 6             | 0.2500E 01              | -0.8333E 00                        | 0.1667E 01 | -0.8333E 00 | -0.1363E -06 | 0.8727E -08  | -0.1432E -15 |
|                               |      | 7             | 0.2500E 01              | -0.8333E 00                        | 0.1667E 01 | -0.8333E 00 | 0.1397E -06  | -0.3609E -07 | 0.1387E -16  |
|                               |      | 8             | 0.2500E 01              | -0.8333E 00                        | 0.1667E 01 | -0.8333E 00 | 0.1357E -06  | -0.6997E -08 | -0.1522E -16 |
| *****CUMULATIVE STRESSES***** |      |               |                         | XX                                 | YY         | ZZ          | XY           | XZ           | YZ           |
|                               |      | 1             | 0.5250E 01              | 0.3611E -05                        | 0.5250E 01 | 0.5777E -04 | -0.1300E -05 | -0.3726E -06 | 0.5995E -16  |
|                               |      | 2             | 0.5250E 01              | 0.4212E -05                        | 0.5250E 01 | 0.5175E -04 | -0.1300E -05 | 0.2343E -06  | 0.3011E -16  |
|                               |      | 3             | 0.5250E 01              | 0.3611E -05                        | 0.5250E 01 | 0.5777E -04 | 0.1282E -05  | 0.3522E -06  | 0.3191E -16  |
|                               |      | 4             | 0.5250E 01              | 0.4212E -05                        | 0.5250E 01 | 0.5175E -04 | 0.1282E -05  | -0.2208E -06 | -0.1233E -15 |
|                               |      | 5             | 0.5250E 01              | 0.3005E -05                        | 0.5250E 01 | 0.5657E -04 | -0.6933E -06 | 0.3426E -06  | 0.1405E -16  |
|                               |      | 6             | 0.5250E 01              | 0.3611E -05                        | 0.5250E 01 | 0.5536E -04 | -0.6933E -06 | 0.2343E -06  | -0.2657E -15 |
|                               |      | 7             | 0.5250E 01              | 0.3005E -05                        | 0.5250E 01 | 0.5657E -04 | 0.7096E -06  | 0.3522E -06  | 0.1124E -16  |
|                               |      | 8             | 0.5250E 01              | 0.3611E -05                        | 0.5250E 01 | 0.5536E -04 | 0.7096E -06  | 0.2208E -06  | 0.1409E -16  |

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## PLASTIC AND CREEP STRAINS

| ELEMENT NO. = 1 ID = 3 |      |      |             |             |            |                                       |                                     |            |             |            |            |  |
|------------------------|------|------|-------------|-------------|------------|---------------------------------------|-------------------------------------|------------|-------------|------------|------------|--|
| INT                    | E-P  | SUM  | INCREMENTAL | TOTAL       | SURFACE    | ***** EFFECTIVE PLASTIC STRAINS ***** | ***** EFFECTIVE CREEP STRAINS ***** |            |             |            |            |  |
| PNT                    | CODE | CODE | TEMPERATURE | TEMPERATURE | YIELD SIZE | INCREMENTAL                           | SUM INCR.                           | CUMULATIVE | INCREMENTAL | SUM INCR.  | CUMULATIVE |  |
| 1                      | 0    | 2    | 0.1000E 01  | 0.1400E 02  | 0.2750E 01 | 0.1000E 01                            | 0.6000E 01                          | 0.4000E 01 | 0.0         | 0.3000E 01 | 0.1000E 01 |  |
| 2                      | 0    | 2    | 0.1000E 01  | 0.1400E 02  | 0.2750E 01 | 0.1000E 01                            | 0.6000E 01                          | 0.4000E 01 | 0.0         | 0.3000E 01 | 0.1000E 01 |  |
| 3                      | 0    | 2    | 0.1000E 01  | 0.1400E 02  | 0.2750E 01 | 0.1000E 01                            | 0.6000E 01                          | 0.4000E 01 | 0.0         | 0.3000E 01 | 0.1000E 01 |  |
| 4                      | 0    | 2    | 0.1000E 01  | 0.1400E 02  | 0.2750E 01 | 0.1000E 01                            | 0.6000E 01                          | 0.4000E 01 | 0.0         | 0.3000E 01 | 0.1000E 01 |  |
| 5                      | 0    | 2    | 0.1000E 01  | 0.1400E 02  | 0.2750E 01 | 0.1000E 01                            | 0.6000E 01                          | 0.4000E 01 | 0.0         | 0.3000E 01 | 0.1000E 01 |  |
| 6                      | 0    | 2    | 0.1000E 01  | 0.1400E 02  | 0.2750E 01 | 0.1000E 01                            | 0.6000E 01                          | 0.4000E 01 | 0.0         | 0.3000E 01 | 0.1000E 01 |  |
| 7                      | 0    | 2    | 0.1000E 01  | 0.1400E 02  | 0.2750E 01 | 0.1000E 01                            | 0.6000E 01                          | 0.4000E 01 | 0.0         | 0.3000E 01 | 0.1000E 01 |  |
| 8                      | 0    | 2    | 0.1000E 01  | 0.1400E 02  | 0.2750E 01 | 0.1000E 01                            | 0.6000E 01                          | 0.4000E 01 | 0.0         | 0.3000E 01 | 0.1000E 01 |  |

# CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 15

INCREMENT 15

NODE I.D. 1 2 3 4 5 6 7 8  
TEMP. 0.15000E 02 0.15000E 02 0.15000E 02 0.15000E 02 0.15000E 02 0.15000E 02 0.15000E 02

RESIDUAL NCRM = 0.54585E 00  
RESIDUAL NCRM = 0.15785E 00  
RESIDUAL NJRM = 0.58146E 00  
RESIDUAL NJRM = 0.44445E 00  
RESIDUAL NCRM = 0.32565E 00  
RESIDUAL NCRM = 0.39909E 00  
RESIDUAL NCRM = 0.46458E 00  
RESIDUAL NCRM = 0.45702E 00  
RESIDUAL NCRM = 0.36182E 00  
RESIDUAL NJRM = 0.19075E 00  
RESIDUAL NJRM = 0.49476E 01  
RESIDUAL NJRM = 0.12981E 01  
RESIDUAL NCRM = 0.22420E 02  
RESIDUAL NJRM = 0.74818E 03  
RESIDUAL NJRM = 0.85368E 04  
RESIDUAL NCRM = 0.55954E 04  
RESIDUAL NJRM = 0.40735E 05

END OF LOAD INCREMENT 15

INCREMENT 15

MECHANICAL LOAD CURVE FACTORS = 0.1629E 02, 0.0

CREEP TIME INCREMENT = 0.1000E 02

NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 8  
0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 1

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 7

SPECIFIED MAX. UNBALANCED-FORCE ERROR = 0.1000E 04, ACTUAL ERROR = 0.4074E 05

## CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** | ** I.D. ** | FORCES         |                |                | DISPLACEMENTS |               |     |
|------------|------------|----------------|----------------|----------------|---------------|---------------|-----|
| NO.        | I.D.       | U              | V              | W              | U             | V             | W   |
| 1          | 1          | -0.8119260E-06 | -0.1500008E 01 | 0.3159538E-05  | 0.2084478E-05 | 0.0           | 0.0 |
| 2          | 2          | 0.8342445E-06  | -0.1500009E 01 | 0.3158444E-05  | 0.1121855E-04 | 0.0           | 0.0 |
| 3          | 3          | -0.1030263E-05 | -0.1500009E 01 | 0.3249560E-05  | 0.8567446E-05 | 0.1628749E 02 | 0.0 |
| 4          | 4          | -0.1051801E-05 | 0.1500008E 01  | 0.3247693E-05  | 0.3959394E-06 | 0.1628749E 02 | 0.0 |
| 5          | 5          | -0.7660256E-06 | -0.1500009E 01 | -0.3198448E-05 | 0.0           | 0.0           | 0.0 |
| 6          | 6          | -0.7334748E-06 | -0.1500009E 01 | -0.3159538E-05 | 0.6942575E-05 | 0.0           | 0.0 |
| 7          | 7          | 0.7807619E-06  | 0.1500009E 01  | -0.3247690E-05 | 0.9634905E-05 | 0.1628749E 02 | 0.0 |
| 8          | 8          | -0.8089860E-06 | 0.1500009E 01  | -0.3249558E-05 | 0.2143021E-05 | 0.1628749E 02 | 0.0 |



## THERMAL AND ELASTIC STRAINS

| ELEMENT<br>NO. | I.D. | INTEGR.<br>POINT | INCREMENTAL<br>THERMAL STRAINS | ***** INCREMENTAL ELASTIC STRAINS ***** |            |             |             |             |             |
|----------------|------|------------------|--------------------------------|---|------------|-------------|-------------|-------------|-------------|
|                |      |                  |                                | XX                                      | YY         | ZZ          | XY          | XZ          | YZ          |
| 1              | 3    | 1                | 0.2362E 01                     | -0.3000E 00                             | 0.1000E 01 | -0.3000E 00 | 0.4786E-06  | 0.2284E-06  | 0.2271E-16  |
|                |      | 2                | 0.2362E 01                     | -0.3000E 00                             | 0.1000E 01 | -0.3000E 00 | 0.4786E-06  | 0.3205E-07  | 0.3638E-16  |
|                |      | 3                | 0.2362E 01                     | -0.3000E 00                             | 0.1000E 01 | -0.3000E 00 | -0.5218E-06 | -0.2272E-06 | -0.2415E-17 |
|                |      | 4                | 0.2362E 01                     | -0.3000E 00                             | 0.1000E 01 | -0.3000E 00 | -0.5218E-06 | 0.3381E-07  | 0.4090E-16  |
|                |      | 5                | 0.2362E 01                     | -0.3000E 00                             | 0.1000E 01 | -0.3000E 00 | -0.2181E-06 | -0.2784E-06 | -0.2685E-16 |
|                |      | 6                | 0.2362E 01                     | -0.3000E 00                             | 0.1000E 01 | -0.3000E 00 | 0.2181E-06  | -0.3205E-07 | 0.2513E-16  |
|                |      | 7                | 0.2362E 01                     | -0.3000E 00                             | 0.1000E 01 | -0.3000E 00 | -0.2608E-06 | -0.2272E-06 | 0.1042E-17  |
|                |      | 8                | 0.2362E 01                     | -0.3000E 00                             | 0.1000E 01 | -0.3000E 00 | 0.2608E-06  | 0.3381E-07  | 0.4484E-16  |
|                |      | INTEGR.<br>POINT | CUMULATIVE<br>THERMAL STRAINS  | ***** CUMULATIVE ELASTIC STRAINS *****  |            |             |             |             |             |
|                |      |                  |                                | XX                                      | YY         | ZZ          | XY          | XZ          | YZ          |
|                |      | 1                | 0.5162E 01                     | -0.6000E 00                             | 0.2000E 01 | -0.6000E 00 | 0.1566E-06  | 0.1302E-06  | 0.4003E-16  |
|                |      | 2                | 0.5162E 01                     | -0.6000E 00                             | 0.2000E 01 | -0.6000E 00 | 0.1566E-06  | 0.2596E-07  | 0.1437E-16  |
|                |      | 3                | 0.5162E 01                     | -0.6000E 00                             | 0.2000E 01 | -0.6000E 00 | -0.2042E-06 | -0.1399E-06 | -0.5484E-17 |
|                |      | 4                | 0.5162E 01                     | -0.6000E 00                             | 0.2000E 01 | -0.6000E 00 | -0.2042E-06 | -0.2085E-07 | -0.1037E-16 |
|                |      | 5                | 0.5162E 01                     | -0.6000E 00                             | 0.2000E 01 | -0.6000E 00 | 0.4641E-07  | 0.1362E-06  | -0.2338E-16 |
|                |      | 6                | 0.5162E 01                     | -0.6000E 00                             | 0.2000E 01 | -0.6000E 00 | -0.4641E-07 | -0.2596E-07 | -0.4060E-16 |
|                |      | 7                | 0.5162E 01                     | -0.6000E 00                             | 0.2000E 01 | -0.6000E 00 | -0.8510E-07 | -0.1399E-06 | -0.3825E-17 |
|                |      | 8                | 0.5162E 01                     | -0.6000E 00                             | 0.2000E 01 | -0.6000E 00 | -0.8510E-07 | -0.2085E-07 | 0.9556E-17  |

## PLASTIC WORK AND STRAINS

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| ELEMENT NO. I.D. |   | INTEGR. POINT | INCREMENTAL PLASTIC WORK | ***** INCREMENTAL PLASTIC STRAINS ***** |            |             |             |             |             |
|------------------|---|---------------|--------------------------|---|------------|-------------|-------------|-------------|-------------|
|                  |   |               |                          | XX                                      | YY         | ZZ          | XY          | XZ          | YZ          |
| 1                | 3 | 1             | 0.5625E 01               | -0.5000E 00                             | 0.1000E 01 | -0.5000E 00 | -0.6275E-07 | 0.1241E-07  | -0.2141E-16 |
|                  |   | 2             | 0.5625E 01               | -0.5000E 00                             | 0.1000E 01 | -0.5000E 00 | -0.6275E-07 | 0.5174E-07  | -0.9820E-17 |
|                  |   | 3             | 0.5625E 01               | -0.5000E 00                             | 0.1000E 01 | -0.5000E 00 | 0.2338E-07  | -0.2261E-07 | -0.1096E-17 |
|                  |   | 4             | 0.5625E 01               | -0.5000E 00                             | 0.1000E 01 | -0.5000E 00 | 0.2338E-07  | -0.4690E-07 | -0.9914E-17 |
|                  |   | 5             | 0.5625E 01               | -0.5000E 00                             | 0.1000E 01 | -0.5000E 00 | -0.2342E-07 | 0.1241E-07  | -0.2040E-16 |
|                  |   | 6             | 0.5625E 01               | -0.5000E 00                             | 0.1000E 01 | -0.5000E 00 | -0.2342E-07 | 0.5174E-07  | -0.6052E-17 |
|                  |   | 7             | 0.5625E 01               | -0.5000E 00                             | 0.1000E 01 | -0.5000E 00 | -0.9054E-09 | -0.2261E-07 | -0.1935E-17 |
|                  |   | 8             | 0.5625E 01               | -0.5000E 00                             | 0.1000E 01 | -0.5000E 00 | -0.9054E-09 | -0.4690E-07 | -0.1087E-16 |
|                  |   | INTEGR. POINT | CUMULATIVE PLASTIC WORK  | ***** CUMULATIVE PLASTIC STRAINS *****  |            |             |             |             |             |
|                  |   |               |                          | XX                                      | YY         | ZZ          | XY          | XZ          | YZ          |
|                  |   | 1             | 0.2450E 02               | -0.2500E 01                             | 0.5000E 01 | -0.2500E 01 | -0.3285E-06 | -0.2467E-07 | 0.1138E-15  |
|                  |   | 2             | 0.2450E 02               | -0.2500E 01                             | 0.5000E 01 | -0.2500E 01 | -0.3285E-06 | -0.2140E-08 | -0.3744E-16 |
|                  |   | 3             | 0.2450E 02               | -0.2500E 01                             | 0.5000E 01 | -0.2500E 01 | 0.2524E-06  | -0.3827E-07 | 0.5168E-16  |
|                  |   | 4             | 0.2450E 02               | -0.2500E 01                             | 0.5000E 01 | -0.2500E 01 | 0.2524E-06  | -0.1863E-07 | -0.5549E-16 |
|                  |   | 5             | 0.2450E 02               | -0.2500E 01                             | 0.5000E 01 | -0.2500E 01 | -0.3017E-06 | -0.2467E-07 | -0.2172E-16 |
|                  |   | 6             | 0.2450E 02               | -0.2500E 01                             | 0.5000E 01 | -0.2500E 01 | -0.3017E-06 | 0.2138E-08  | -0.3159E-15 |
|                  |   | 7             | 0.2450E 02               | -0.2500E 01                             | 0.5000E 01 | -0.2500E 01 | 0.2721E-06  | -0.3827E-07 | 0.3788E-16  |
|                  |   | 8             | 0.2450E 02               | -0.2500E 01                             | 0.5000E 01 | -0.2500E 01 | -0.2721E-06 | -0.1863E-07 | -0.7789E-16 |

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# CUMULATIVE STRESS QUANTITIES

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | EFFECTIVE<br>STRESS CENTER | *****CUMULATIVE STRESS CENTER***** |            |             |             |             |             |
|---------------------|------------------|----------------------------|------------------------------------|------------|-------------|-------------|-------------|-------------|
|                     |                  |                            | XX                                 | YY         | ZZ          | XY          | XZ          | YZ          |
| 1 3                 | 1                | 0.3000E 01                 | -0.1000E 01                        | 0.2000E 01 | -0.1000E 01 | -0.1206E-06 | 0.4942E-07  | 0.4529E-16  |
|                     | 2                | 0.3000E 01                 | -0.1000E 01                        | 0.2000E 01 | -0.1000E 01 | -0.1206E-06 | 0.2598E-07  | 0.3932E-17  |
|                     | 3                | 0.3000E 01                 | -0.1000E 01                        | 0.2000E 01 | -0.1000E 01 | 0.1184E-06  | -0.4363E-07 | 0.2165E-16  |
|                     | 4                | 0.3000E 01                 | -0.1000E 01                        | 0.2000E 01 | -0.1000E 01 | -0.1184E-06 | -0.2263E-07 | -0.1159E-16 |
|                     | 5                | 0.3000E 01                 | -0.1000E 01                        | 0.2000E 01 | -0.1000E 01 | -0.1441E-06 | 0.4942E-07  | 0.7879E-17  |
|                     | 6                | 0.3000E 01                 | -0.1000E 01                        | 0.2000E 01 | -0.1000E 01 | -0.1441E-06 | 0.2597E-07  | -0.1452E-15 |
|                     | 7                | 0.3000E 01                 | -0.1000E 01                        | 0.2000E 01 | -0.1000E 01 | -0.1394E-06 | -0.4363E-07 | -0.1322E-16 |
|                     | 8                | 0.3000E 01                 | -0.1000E 01                        | 0.2000E 01 | -0.1000E 01 | 0.1394E-06  | -0.2263E-07 | -0.1084E-16 |

| INTEGR. POINT | EFFECTIVE STRESS | *****CUMULATIVE STRESSES***** |            |            |             |             |             |
|---------------|------------------|-------------------------------|------------|------------|-------------|-------------|-------------|
|               |                  | XX                            | YY         | ZZ         | XY          | XZ          | YZ          |
| 1             | 0.6000E 01       | 0.2407E-05                    | 0.6000E 01 | 0.1135E-04 | 0.3615E-06  | 0.3143E-06  | 0.9237E-16  |
| 2             | 0.6000E 01       | 0.3095E-05                    | 0.6000E 01 | 0.1204E-04 | 0.3615E-06  | 0.5991E-07  | 0.3247E-16  |
| 3             | 0.6000E 01       | 0.2407E-05                    | 0.6000E 01 | 0.1135E-04 | -0.4712E-06 | -0.3230E-06 | 0.1265E-16  |
| 4             | 0.6000E 01       | 0.3095E-05                    | 0.6000E 01 | 0.1204E-04 | -0.4712E-06 | -0.4812E-07 | 0.2392E-16  |
| 5             | 0.6000E 01       | 0.3095E-05                    | 0.6000E 01 | 0.1410E-04 | -0.1071E-06 | -0.3143E-06 | -0.5344E-16 |
| 6             | 0.6000E 01       | 0.5158E-05                    | 0.6000E 01 | 0.1410E-04 | 0.1071E-06  | 0.5991E-07  | -0.5342E-16 |
| 7             | 0.6000E 01       | 0.3095E-05                    | 0.6000E 01 | 0.1410E-04 | -0.1964E-06 | -0.3230E-06 | 0.8327E-17  |
| 8             | 0.6000E 01       | 0.5158E-05                    | 0.6000E 01 | 0.1410E-04 | 0.1964E-06  | -0.4812E-07 | -0.2298E-16 |

## CREEP WORK AND STRAINS

| ELEMENT |      | INTEGR. | INCREMENTAL | *****INCREMENTAL CREEP STRAINS***** |            |             |             |             |             |
|---------|------|---------|-------------|-------------------------------------|------------|-------------|-------------|-------------|-------------|
| NO.     | I.D. | POINT   | CREEP WORK  | XX                                  | YY         | ZZ          | XY          | XZ          | YZ          |
| 1       | 3    | 1       | 0.1758E 02  | -0.1562E 01                         | 0.3125E 01 | -0.1562E 01 | -0.1961E-06 | 0.3879E-07  | 0.6691E-16  |
|         |      | 2       | 0.1758E 02  | -0.1562E 01                         | 0.3125E 01 | -0.1562E 01 | -0.1961E-06 | 0.1617E-06  | -0.2756E-16 |
|         |      | 3       | 0.1758E 02  | -0.1562E 01                         | 0.3125E 01 | -0.1562E 01 | 0.7306E-07  | -0.7066E-07 | -0.3424E-17 |
|         |      | 4       | 0.1758E 02  | -0.1562E 01                         | 0.3125E 01 | -0.1562E 01 | 0.7306E-07  | -0.1466E-06 | -0.3098E-16 |
|         |      | 5       | 0.1758E 02  | -0.1562E 01                         | 0.3125E 01 | -0.1562E 01 | -0.7319E-07 | 0.3879E-07  | -0.6377E-16 |
|         |      | 6       | 0.1758E 02  | -0.1562E 01                         | 0.3125E 01 | -0.1562E 01 | -0.7319E-07 | -0.1617E-06 | -0.1400E-16 |
|         |      | 7       | 0.1758E 02  | -0.1562E 01                         | 0.3125E 01 | -0.1562E 01 | -0.2829E-08 | -0.7066E-07 | -0.5046E-17 |
|         |      | 8       | 0.1758E 02  | -0.1562E 01                         | 0.3125E 01 | -0.1562E 01 | -0.2829E-08 | -0.1466E-06 | -0.3357E-16 |

| INTEGR.<br>POINT | CUMULATIVE<br>CREEP WORK | *****CUMULATIVE CREEP STRAINS***** |            |             |             |             |             |
|------------------|--------------------------|------------------------------------|------------|-------------|-------------|-------------|-------------|
|                  |                          | XX                                 | YY         | ZZ          | XY          | XZ          | YZ          |
| 1                | 0.2339E 02               | -0.2062E 01                        | 0.4125E 01 | -0.2063E 01 | -0.8720E-06 | 0.7220E-06  | 0.8553E-16  |
| 2                | 0.2339E 02               | -0.2062E 01                        | 0.4125E 01 | -0.2063E 01 | 0.8620E-06  | -0.3897E-06 | -0.1878E-15 |
| 3                | 0.2339E 02               | -0.2062E 01                        | 0.4125E 01 | -0.2063E 01 | 0.7059E-06  | 0.1548E-05  | -0.1242E-16 |
| 4                | 0.2339E 02               | -0.2062E 01                        | 0.4125E 01 | -0.2063E 01 | -0.7059E-06 | -0.2946E-07 | -0.1854E-15 |
| 5                | 0.2339E 02               | -0.2062E 01                        | 0.4125E 01 | -0.2063E 01 | -0.2497E-06 | 0.7220E-06  | -0.3397E-16 |
| 6                | 0.2339E 02               | -0.2062E 01                        | 0.4125E 01 | -0.2063E 01 | -0.2497E-06 | -0.3497E-06 | 0.2347E-16  |
| 7                | 0.2339E 02               | -0.2062E 01                        | 0.4125E 01 | -0.2063E 01 | -0.6713E-06 | 0.1548E-05  | -0.1380E-16 |
| 8                | 0.2339E 02               | -0.2062E 01                        | 0.4125E 01 | -0.2063E 01 | -0.8713E-06 | -0.2946E-07 | -0.1706E-15 |

# PLASTIC AND CREEP STRAINS

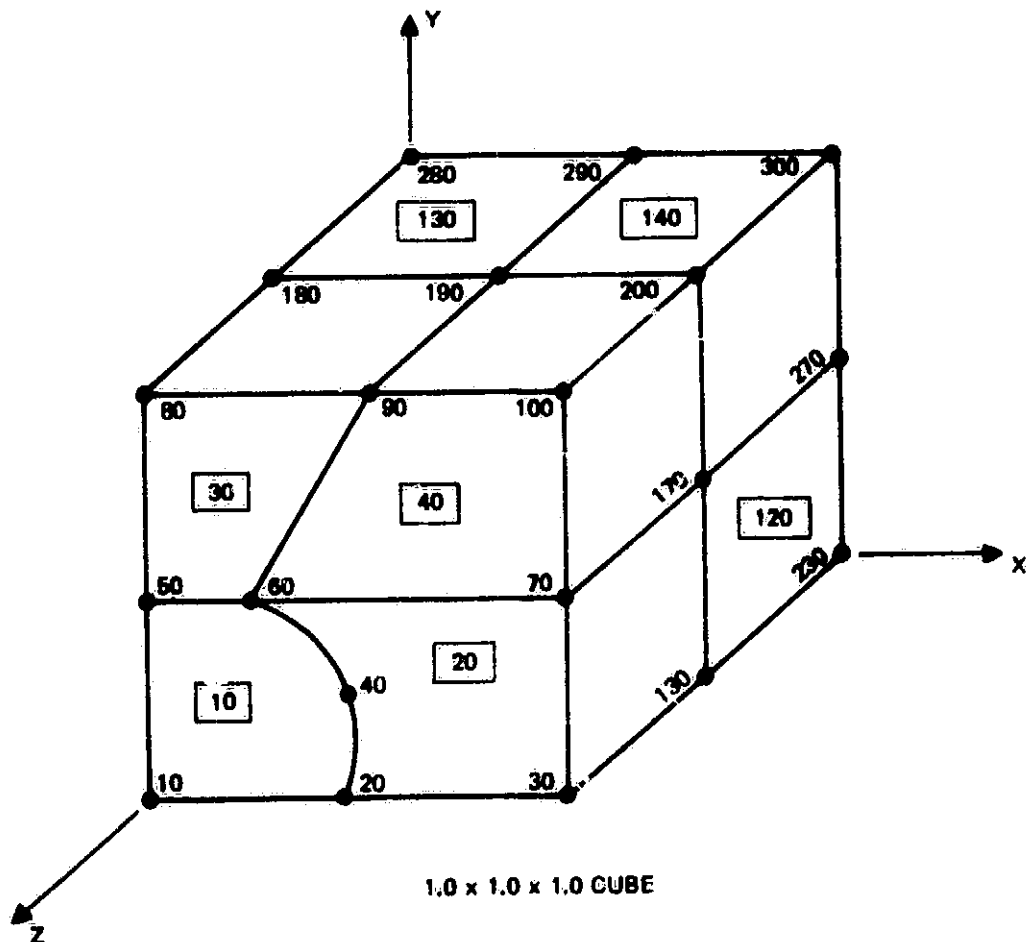
| ELEMENT NO. = 1 ID = 3 |      |      |             |             |            |                                   |            |            |                                     |            |            |  |  |
|------------------------|------|------|-------------|-------------|------------|-----------------------------------|------------|------------|-------------------------------------|------------|------------|--|--|
| INT                    | E-P  | SUM  | INCREMENTAL | TOTAL       | SURFACE    | *** EFFECTIVE PLASTIC STRAINS *** |            |            | ***** EFFECTIVE CREEP STRAINS ***** |            |            |  |  |
| PNT                    | CODE | CODE | TEMPERATURE | TEMPERATURE | YIELD SIZE | INCREMENTAL                       | SUM INCR.  | CUMULATIVE | INCREMENTAL                         | SUM INCR.  | CUMULATIVE |  |  |
| 1                      | 0    | 2    | 0.1000E 01  | 0.1500E 02  | 0.3000E 01 | 0.1000E 01                        | 0.7000E 01 | 0.5000E 01 | 0.3125E 01                          | 0.6125E 01 | 0.4125E 01 |  |  |
| 2                      | 0    | 2    | 0.1000E 01  | 0.1500E 02  | 0.3000E 01 | 0.1000E 01                        | 0.7000E 01 | 0.5000E 01 | 0.3125E 01                          | 0.6125E 01 | 0.4125E 01 |  |  |
| 3                      | 0    | 2    | 0.1000E 01  | 0.1500E 02  | 0.3000E 01 | 0.1000E 01                        | 0.7000E 01 | 0.5000E 01 | 0.3125E 01                          | 0.6125E 01 | 0.4125E 01 |  |  |
| 4                      | 0    | 2    | 0.1000E 01  | 0.1500E 02  | 0.3000E 01 | 0.1000E 01                        | 0.7000E 01 | 0.5000E 01 | 0.3125E 01                          | 0.6125E 01 | 0.4125E 01 |  |  |
| 5                      | 0    | 2    | 0.1000E 01  | 0.1500E 02  | 0.3000E 01 | 0.1000E 01                        | 0.7000E 01 | 0.5000E 01 | 0.3125E 01                          | 0.6125E 01 | 0.4125E 01 |  |  |
| 6                      | 0    | 2    | 0.1000E 01  | 0.1500E 02  | 0.3000E 01 | 0.1000E 01                        | 0.7000E 01 | 0.5000E 01 | 0.3125E 01                          | 0.6125E 01 | 0.4125E 01 |  |  |
| 7                      | 0    | 2    | 0.1000E 01  | 0.1500E 02  | 0.3000E 01 | 0.1000E 01                        | 0.7000E 01 | 0.5000E 01 | 0.3125E 01                          | 0.6125E 01 | 0.4125E 01 |  |  |
| 8                      | 0    | 2    | 0.1000E 01  | 0.1500E 02  | 0.3000E 01 | 0.1000E 01                        | 0.7000E 01 | 0.5000E 01 | 0.3125E 01                          | 0.6125E 01 | 0.4125E 01 |  |  |

14.1-65

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## 14.2 MULTI-ELEMENT CURVED BOUNDARY PROBLEM

The 3-D plane-strain problem of Section 14.1 is again analyzed using a  $1.0 \times 1.0 \times 1.0$  cube, but idealized by eight elements including curved interior boundaries. The input data listing and partial results are included at the end of this section. Because of the state of constant stress and strain throughout the cube, all integration points have equal values for stress and strain. In order to conserve space, results are listed for only two load increments and one of the elements.



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Figure 14.2-1. Multi-Element Problem Mesh

START 5 5 6  
 BOPACE 3-C CHECK (PLANE STRAIN WITH Z-LOADS, ELASTIC-PLASTIC-CREEP)

11/04/74

|      | 5      | 3    | 1    |      |        |      |       |
|------|--------|------|------|------|--------|------|-------|
| 0.5  | .00001 |      |      |      |        |      |       |
|      | 1      | 1.0  |      |      |        |      |       |
| 1.0  | 0.0    | 2.0  | 0.3  | 3.0  | 0.8    | 4.0  | 1.05  |
| 4.5  | 1.5    | 5.0  | 1.8  | 6.0  | 2.3    | 7.0  | 2.3   |
| 8.0  | 1.0    | 9.0  | 0.5  | 10.0 | 1.15   | 12.0 | 2.05  |
| 13.0 | 2.5    | 14.0 | 2.8  | 15.0 | 5.1625 |      |       |
| 1.0  | 1.0    | 2.0  | 2.0  | 3.0  | 2.5    | 4.0  | 3.0   |
| 5.0  | 3.5    | 5.4  | 1.0  | 6.0  | 4.0    | 7.0  | 2.0   |
| 8.0  | 1.25   | 9.0  | 1.25 | 10.0 | 2.0    | 12.0 | 3.875 |
| 13.0 | 4.5    | 14.0 | 5.25 | 15.0 | 3.0    |      |       |
| 0.0  | 0.3    |      |      |      |        |      |       |
|      | 1      | 1    |      |      |        |      |       |
| 0.0  | 1 0.   | 3.0  | 2.0  | 9.0  | 3.5    |      |       |
| 0.0  | 2.0    |      |      |      |        |      |       |
| 0.0  | 0.0    | 1.0  | 1.0  | 3.0  | 2.0    | 9.0  | 3.5   |
| 0.0  | 1      | 2    |      |      |        |      |       |
| 0.0  | 0.0    | 10.0 | 1.0  | 30.0 | 2.0    |      |       |
| 0.0  | 1 0.0  |      |      |      |        |      |       |
| 0.0  | 1.0    | 3.0  | 1.0  | 11.0 | 9.0    |      |       |
| 10   |        | 0.   | 0.   | 1.   |        |      |       |
| 20   |        | .5   | 0.   | 1.   |        |      |       |
| 30   |        | 1.   | 0.   | 1.   |        |      |       |
| 40   |        | .5   | .25  | 1.   |        |      |       |
| 50   |        | 0.   | .5   | 1.   |        |      |       |
| 60   |        | .25  | .5   | 1.   |        |      |       |
| 70   |        | 1.   | .5   | 1.   |        |      |       |
| 80   |        | 0.   | 1.   | 1.   |        |      |       |

14.2-2

|     |     |     |    |
|-----|-----|-----|----|
| 90  | .5  | 1.  | 1. |
| 100 | 1.  | 1.  | 1. |
| 110 | 0.  | 0.  | .5 |
| 120 | .5  | 0.  | .5 |
| 130 | 1.  | 0.  | .5 |
| 140 | .5  | .25 | .5 |
| 150 | 0.  | .5  | .5 |
| 160 | .25 | .5  | .5 |
| 170 | 1.  | .5  | .5 |
| 180 | 0.  | 1.  | .5 |
| 190 | .5  | 1.  | .5 |
| 200 | 1.  | 1.  | .5 |
| 210 | 0.  | 0.  | 0. |
| 220 | .5  | 0.  | 0. |
| 230 | 1.  | 0.  | 0. |
| 240 | .5  | .25 | 0. |
| 250 | 0.  | .5  | 0. |
| 260 | .25 | .5  | 0. |
| 270 | 1.  | .5  | 0. |
| 280 | 0.  | 1.  | 0. |
| 290 | .5  | 1.  | 0. |
| 300 | 1.  | 1.  | 0. |

|    |     |     |     |     |     |     |     |     |     |   |   |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|---|
| 10 | 1   | 10  | 20  | 60  | 50  | 110 | 120 | 160 | 150 | 1 |   |
| 0  | 40  | 0   | 0   | 0   | 140 | 0   | 0   | 0   | 0   | 0 | C |
| 20 | 1   | 20  | 30  | 70  | 60  | 120 | 130 | 170 | 160 | 1 |   |
| 0  | 0   | 0   | 40  | 0   | 0   | 0   | 140 | 0   | 0   | 0 | 0 |
| 30 | 1   | 50  | 60  | 90  | 80  | 150 | 160 | 190 | 180 | 0 |   |
| 40 | 1   | 60  | 70  | 100 | 90  | 160 | 170 | 200 | 190 | 0 |   |
| 50 | 1   | 110 | 120 | 160 | 150 | 210 | 220 | 260 | 250 | 1 |   |
| 0  | 140 | 0   | 0   | 0   | 240 | 0   | 0   | 0   | 0   | 0 | C |
| 60 | 1   | 120 | 130 | 170 | 160 | 220 | 230 | 270 | 260 | 1 |   |
| 0  | 0   | 0   | 140 | 0   | 0   | 0   | 240 | 0   | 0   | 0 | C |
| 70 | 1   | 150 | 160 | 190 | 180 | 250 | 260 | 290 | 280 | 0 |   |
| 80 | 1   | 160 | 170 | 200 | 190 | 260 | 270 | 300 | 290 | 0 |   |

|     |   |      |  |     |   |      |  |     |   |      |
|-----|---|------|--|-----|---|------|--|-----|---|------|
| 10  | 2 | -10  |  | 20  | 2 | -20  |  | 30  | 2 | -30  |
| 110 | 2 | -110 |  | 120 | 2 | -120 |  | 130 | 2 | -130 |
| 210 | 2 | -210 |  | 220 | 2 | -220 |  | 230 | 2 | -230 |
| 290 | 2 | -80  |  | 90  | 2 | -90  |  | 100 | 2 | -100 |

|     |   |      |     |   |      |     |   |      |     |   |      |  |
|-----|---|------|-----|---|------|-----|---|------|-----|---|------|--|
| 180 | 2 | -180 | 190 | 2 | -190 | 200 | 2 | -200 |     |   |      |  |
| 280 | 2 | -280 | 290 | 2 | -290 | 300 | 2 | -300 |     |   |      |  |
| 10  | 3 | -10  | 20  | 3 | -20  | 30  | 3 | -30  | 40  | 3 | -40  |  |
| 50  | 3 | -50  | 60  | 3 | -60  | 70  | 3 | -70  |     |   |      |  |
| 80  | 3 | -80  | 90  | 3 | -90  | 100 | 3 | -100 |     |   |      |  |
| 210 | 3 | -210 | 220 | 3 | -220 | 230 | 3 | -230 | 240 | 3 | -240 |  |
| 250 | 3 | -250 | 260 | 3 | -260 | 270 | 3 | -270 |     |   |      |  |
| 280 | 3 | -280 | 290 | 3 | -290 | 300 | 3 | -300 |     |   |      |  |
| 10  | 1 | -10  | 210 | 1 | -210 |     |   |      |     |   |      |  |

|     |   |    |     |   |    |     |   |    |    |   |    |  |
|-----|---|----|-----|---|----|-----|---|----|----|---|----|--|
|     | 2 |    |     |   |    |     |   |    |    |   |    |  |
| 80  | 2 | 1. | 90  | 2 | 1. | 100 | 2 | 1. |    |   |    |  |
| 180 | 2 | 1. | 190 | 2 | 1. | 200 | 2 | 1. |    |   |    |  |
| 280 | 2 | 1. | 290 | 2 | 1. | 300 | 2 | 1. |    |   |    |  |
|     |   |    |     |   |    |     |   |    |    |   |    |  |
| 10  | 3 | 1. | 20  | 3 | 1. | 30  | 3 | 1. | 40 | 3 | 1. |  |
| 50  | 3 | 1. | 60  | 3 | 1. | 70  | 3 | 1. |    |   |    |  |
| 80  | 3 | 1. | 90  | 3 | 1. | 100 | 3 | 1. |    |   |    |  |

|           |     |    |     |     |     |    |     |    |  |  |  |  |
|-----------|-----|----|-----|-----|-----|----|-----|----|--|--|--|--|
|           | 2   |    |     |     |     |    |     |    |  |  |  |  |
|           | 1.3 |    |     | 0.0 |     |    |     |    |  |  |  |  |
|           | 2.8 |    |     | 5.0 |     |    |     |    |  |  |  |  |
| INCREMENT |     | 1  |     |     |     |    |     |    |  |  |  |  |
| 10        |     | 2. | 20  | 2.  | 30  | 2. | 40  | 2. |  |  |  |  |
| 50        |     | 2. | 60  | 2.  | 70  | 2. | 80  | 2. |  |  |  |  |
| 90        |     | 2. | 100 | 2.  | 110 | 2. | 120 | 2. |  |  |  |  |
| 130       |     | 2. | 140 | 2.  | 150 | 2. | 160 | 2. |  |  |  |  |
| 170       |     | 2. | 180 | 2.  | 190 | 2. | 200 | 2. |  |  |  |  |
| 210       |     | 2. | 220 | 2.  | 230 | 2. | 240 | 2. |  |  |  |  |
| 250       |     | 2. | 260 | 2.  | 270 | 2. | 280 | 2. |  |  |  |  |
| 290       |     | 2. | 300 | 2.  |     |    |     |    |  |  |  |  |
| INCREMENT |     | 2  |     |     |     |    |     |    |  |  |  |  |
| 10        |     | 3. | 20  | 3.  | 30  | 3. | 40  | 3. |  |  |  |  |
| 50        |     | 3. | 60  | 3.  | 70  | 3. | 80  | 3. |  |  |  |  |
| 90        |     | 3. | 100 | 3.  | 110 | 3. | 120 | 3. |  |  |  |  |
| 130       |     | 3. | 140 | 3.  | 150 | 3. | 160 | 3. |  |  |  |  |
| 170       |     | 3. | 180 | 3.  | 190 | 3. | 200 | 3. |  |  |  |  |
| 210       |     | 3. | 220 | 3.  | 230 | 3. | 240 | 3. |  |  |  |  |
| 250       |     | 3. | 260 | 3.  | 270 | 3. | 280 | 3. |  |  |  |  |
| 290       |     | 3. | 300 | 3.  |     |    |     |    |  |  |  |  |

## STARTING PROBLEM

BUPAGE 3-D CHECK (PLANE STRAIN WITH Z-LOADS, ELASTIC-PLASTIC-CREEP)

11/04/74

SOLUTION METHOD CODE = 5  
 MAXIMUM NO. STIFFNESS UPDATES PER INCREMENT = 3  
 MAXIMUM TOTAL ITERATIONS PER INCREMENT = 10  
 MAXIMUM ELASTIC ITERATIONS PER INCREMENT = 2  
 MAXIMUM MAGNITUDE FOR ELASTIC-PLASTIC SUM CODE = 2  
 MAXIMUM REDUCTIONS = 1  
 CONVERGENCE REDUCTION FACTOR = 0.50000E-00  
 MAXIMUM SPECIFIED ERROR NORM = 0.10000E-04  
 FRACTION FROM END OF INCREMENT TO EVALUATE SLOPE = 0.10000E 00

NO. OF MATERIALS = 1  
 FABRICATION TEMPERATURE = 0.10000E 01

## MATERIAL NO. 1 TEMPERATURE DEPENDENT PROPERTIES

| TEMPERATURE | THERMAL STRAIN |
|-------------|----------------|
| 0.1000E 01  | 0.0            |
| 0.2000E 01  | 0.3000E-00     |
| 0.3000E 01  | 0.8000E 00     |
| 0.4000E 01  | 0.1050E 01     |
| 0.4500E-01  | 0.1500E-01     |
| 0.5000E 01  | 0.1800E 01     |
| 0.6000E 01  | 0.2300E 01     |
| 0.7000E 01  | 0.2300E-01     |
| 0.8000E 01  | 0.1000E 01     |
| 0.9000E 01  | 0.5000E 00     |
| 0.1000E-02  | 0.1150E-01     |
| 0.1200E-02  | 0.2050E 01     |
| 0.1300E 02  | 0.2500E 01     |
| 0.1400E 02  | 0.2800E-01     |
| 0.1500E 02  | 0.5162E 01     |

| TEMPERATURE | ELASTIC MOD. |
|-------------|--------------|
| 0.1000E 01  | 0.1000E 01   |
| 0.2000E 01  | 0.2000E 01   |
| 0.3000E 01  | 0.2500E-01   |
| 0.4000E 01  | 0.3000E 01   |
| 0.5000E 01  | 0.3500E 01   |
| 0.5500E-01  | 0.1000E-01   |
| 0.6000E 01  | 0.4000E 01   |
| 0.7000E 01  | 0.2000E 01   |
| 0.8000E-01  | 0.1250E-01   |
| 0.9000E 01  | 0.1250E 01   |
| 0.1000E 02  | 0.2000E 01   |
| 0.1200E-02  | 0.3375E-01   |
| 0.1300E 02  | 0.4500E 01   |
| 0.1400E 02  | 0.5250E 01   |
| 0.1500E 02  | 0.3000E-01   |

| TEMPERATURE | POISSONS RATIO |
|-------------|----------------|
| 0.0         | 0.3000E-00     |

ORIGINAL PAGE IS  
OF POOR QUALITY

14.2-5



MATERIAL NO. 1, PLASTICITY TYPE 1, KINEMATIC CODE 0

MATERIAL NO. 1, TEMPERATURE = 0.0

PARAMETER ISOTROPIC STRESS

0.0 0.20000E 01  
0.30000E 01 0.20000E 01  
0.90000E 01 0.35000E 01

PARAMETER KINEMATIC SHAPE

0.0 0.0  
0.10000E 01 0.10000E 01  
0.30000E 01 0.20000E 01  
0.90000E 01 0.35000E 01

TEMPERATURE = 0.0

PARAMETER ISOTROPIC STRESS

0.0 0.20000E 01  
0.30000E 01 0.20000E 01  
0.90000E 01 0.35000E 01

PARAMETER KINEMATIC SHAPE

0.0 0.0  
0.10000E 01 0.10000E 01  
0.30000E 01 0.20000E 01  
0.90000E 01 0.35000E 01

MATERIAL NO. 1, CREEP TYPE 2

TIME CREEP STRAIN

0.0 0.0  
0.10000E 02 0.10000E 01  
0.30000E 02 0.20000E 01

MATERIAL NO. 1, TEMPERATURE = 0.0

STRESS CREEP FACTOR

0.0 0.10000E 01  
0.30000E 01 0.10000E 01  
0.11000E 02 0.90000E 01

MATERIAL NO. 1, TEMPERATURE = 0.0

STRESS CREEP FACTOR

0.0 0.10000E 01  
0.30000E 01 0.10000E 01  
0.11000E 02 0.90000E 01

14.2-6

# CARTESIAN COORDINATE SYSTEMS DEFINED

| NUMBER | OPTION | REFERENCE<br>COORD SYS | ORIGIN..... | POINT ON<br>.....X-AXIS..... | POINT IN<br>.....XY-PLANE..... |
|--------|--------|------------------------|-------------|------------------------------|--------------------------------|
|--------|--------|------------------------|-------------|------------------------------|--------------------------------|

| ** NODE ** | NO. | I.O. | LOCATE | X1 | X2 | X3 | DISPLACE |
|------------|-----|------|--------|----|----|----|----------|
|------------|-----|------|--------|----|----|----|----------|

|    |     |   |             |             |             |             |   |
|----|-----|---|-------------|-------------|-------------|-------------|---|
| 1  | 13  | 0 | 0.0         | 0.0         | 0.0         | 0.100000 01 | 0 |
| 2  | 20  | 0 | 0.500000 00 | 0.0         | 0.100000 01 | 0           | 0 |
| 3  | 30  | 0 | 0.100000 01 | 0.0         | 0.100000 01 | 0           | 0 |
| 4  | 40  | 0 | 0.500000 00 | 0.250000 00 | 0.100000 01 | 0           | 0 |
| 5  | 50  | 0 | 0.0         | 0.500000 00 | 0.100000 01 | 0           | 0 |
| 6  | 60  | 0 | 0.250000 00 | 0.500000 00 | 0.100000 01 | 0           | 0 |
| 7  | 70  | 0 | 0.100000 01 | 0.500000 00 | 0.100000 01 | 0           | 0 |
| 8  | 80  | 0 | 0.0         | 0.100000 01 | 0.100000 01 | 0           | 0 |
| 9  | 90  | 0 | 0.500000 00 | 0.100000 01 | 0.100000 01 | 0           | 0 |
| 10 | 100 | 0 | 0.100000 01 | 0.100000 01 | 0.100000 01 | 0           | 0 |
| 11 | 110 | 0 | 0.0         | 0.0         | 0.500000 00 | 0           | 0 |
| 12 | 120 | 0 | 0.500000 00 | 0.0         | 0.500000 00 | 0           | 0 |
| 13 | 130 | 0 | 0.100000 01 | 0.0         | 0.500000 00 | 0           | 0 |
| 14 | 140 | 0 | 0.500000 00 | 0.250000 00 | 0.500000 00 | 0           | 0 |
| 15 | 150 | 0 | 0.0         | 0.500000 00 | 0.500000 00 | 0           | 0 |
| 16 | 160 | 0 | 0.250000 00 | 0.500000 00 | 0.500000 00 | 0           | 0 |
| 17 | 170 | 0 | 0.100000 01 | 0.500000 00 | 0.500000 00 | 0           | 0 |
| 18 | 180 | 0 | 0.0         | 0.100000 01 | 0.500000 00 | 0           | 0 |
| 19 | 190 | 0 | 0.500000 00 | 0.100000 01 | 0.500000 00 | 0           | 0 |
| 20 | 200 | 0 | 0.100000 01 | 0.100000 01 | 0.500000 00 | 0           | 0 |
| 21 | 210 | 0 | 0.0         | 0.0         | 0.0         | 0           | 0 |
| 22 | 220 | 0 | 0.500000 00 | 0.0         | 0.0         | 0           | 0 |
| 23 | 230 | 0 | 0.100000 01 | 0.0         | 0.0         | 0           | 0 |
| 24 | 240 | 0 | 0.500000 00 | 0.250000 00 | 0.0         | 0           | 0 |
| 25 | 250 | 0 | 0.0         | 0.500000 00 | 0.0         | 0           | 0 |
| 26 | 260 | 0 | 0.250000 00 | 0.500000 00 | 0.0         | 0           | 0 |
| 27 | 270 | 0 | 0.100000 01 | 0.500000 00 | 0.0         | 0           | 0 |
| 28 | 280 | 0 | 0.0         | 0.100000 01 | 0.0         | 0           | 0 |
| 29 | 290 | 0 | 0.500000 00 | 0.100000 01 | 0.0         | 0           | 0 |
| 30 | 300 | 0 | 0.100000 01 | 0.100000 01 | 0.0         | 0           | 0 |

ORIGINAL PAGE 1  
OF FOUR QUALITY

14.2-7

| ELEMENT<br>NO. | I.O. | MATL | N1  | N2  | N3  | N4  | N5  | N6  | N7  | N8  | VOLUME<br>(ST. LINE) | INTERMEDIATE EDGE NODES     |
|----------------|------|------|-----|-----|-----|-----|-----|-----|-----|-----|----------------------|-----------------------------|
| 1              | 10   | 1    | 10  | 20  | 60  | 70  | 120 | 130 | 160 | 170 | 0.9375E-01           | 0 40 0 0 0 140 0 0 0 0      |
| 2              | 20   | 1    | 20  | 30  | 70  | 80  | 120 | 130 | 160 | 170 | 0.1562E-00           | 0 0 0 40 0 0 0 140 0 0 0 0  |
| 3              | 30   | 1    | 50  | 60  | 90  | 80  | 150 | 160 | 190 | 180 | 0.9375E-01           |                             |
| 4              | 40   | 1    | 60  | 70  | 100 | 90  | 140 | 170 | 200 | 190 | 0.1562E-00           |                             |
| 5              | 50   | 1    | 110 | 120 | 160 | 150 | 210 | 220 | 260 | 250 | 0.9375E-01           | 0 140 0 0 0 240 0 0 0 0 0   |
| 6              | 60   | 1    | 120 | 130 | 170 | 160 | 220 | 230 | 270 | 260 | 0.1562E-00           | 0 0 0 140 0 0 0 240 0 0 0 0 |
| 7              | 70   | 1    | 150 | 160 | 190 | 180 | 250 | 260 | 290 | 280 | 0.9375E-01           |                             |
| 8              | 80   | 1    | 160 | 170 | 200 | 190 | 260 | 270 | 300 | 290 | 0.1562E-00           |                             |

SUM OF ELEMENT VOLUMES = 0.1000E 01

SPECIFIED FORCE-DISPLACEMENT-CONSTRAINT DOF

NODE-100- COMPONENT CODE

|     |   |      |
|-----|---|------|
| 10  | 2 | -10  |
| 20  | 2 | -20  |
| 30  | 2 | -30  |
| 110 | 2 | -110 |
| 120 | 2 | -120 |
| 130 | 2 | -130 |
| 210 | 2 | -210 |
| 220 | 2 | -220 |
| 230 | 2 | -230 |
| 80  | 2 | -80  |
| 90  | 2 | -90  |
| 160 | 2 | -160 |
| 180 | 2 | -180 |
| 190 | 2 | -190 |
| 200 | 2 | -200 |
| 230 | 2 | -230 |
| 250 | 2 | -250 |
| 300 | 2 | -300 |
| 10  | 3 | -10  |
| 20  | 3 | -20  |
| 30  | 3 | -30  |
| 40  | 3 | -40  |
| 50  | 3 | -50  |
| 60  | 3 | -60  |
| 70  | 3 | -70  |
| 80  | 3 | -80  |
| 90  | 3 | -90  |
| 100 | 3 | -100 |
| 210 | 3 | -210 |
| 220 | 3 | -220 |
| 230 | 3 | -230 |
| 240 | 3 | -240 |
| 250 | 3 | -250 |
| 260 | 3 | -260 |
| 270 | 3 | -270 |
| 280 | 3 | -280 |
| 290 | 3 | -290 |
| 300 | 3 | -300 |
| 10  | 1 | -10  |
| 210 | 1 | -210 |

ORIGINAL PAGE IS  
OF POOR QUALITY

NO. OF LOAD REFERENCE CURVES - 2

LOAD REFERENCE CURVE NO. 1

| NODE | COMPONENT | LOAD        |
|------|-----------|-------------|
| 80   | 2         | 0.10000E 01 |
| 90   | 2         | 0.10000E 01 |
| 100  | 2         | 0.10000E 01 |
| 180  | 2         | 0.10000E 01 |
| 190  | 2         | 0.10000E 01 |
| 200  | 2         | 0.10000E 01 |
| 280  | 2         | 0.10000E 01 |
| 290  | 2         | 0.10000E 01 |
| 300  | 2         | 0.10000E 01 |

LOAD REFERENCE CURVE NO. 2

| NODE | COMPONENT | LOAD        |
|------|-----------|-------------|
| 10   | 3         | 0.10000E 01 |
| 20   | 3         | 0.10000E 01 |
| 30   | 3         | 0.10000E 01 |
| 40   | 3         | 0.10000E 01 |
| 50   | 3         | 0.10000E 01 |
| 60   | 3         | 0.10000E 01 |
| 70   | 3         | 0.10000E 01 |
| 80   | 3         | 0.10000E 01 |
| 90   | 3         | 0.10000E 01 |
| 100  | 3         | 0.10000E 01 |

14.2.19

ORIGINAL PAGE 11  
OF FOUR QUARTER

| ELEMENT |        | INTEGRATION |           |           |           |
|---------|--------|-------------|-----------|-----------|-----------|
| 1.0.    | LOCATE | POINT       | X1        | X2        | X3        |
| 10      | 0      | 1           | 1.092E-01 | 1.130E-01 | 6.057E-01 |
|         |        | 2           | 7.867E-02 | 4.017E-01 | 6.057E-01 |
|         |        | 3           | 4.074E-01 | 1.330E-01 | 6.057E-01 |
|         |        | 4           | 2.936E-01 | 4.217E-01 | 6.057E-01 |
|         |        | 5           | 1.092E-01 | 1.130E-01 | 8.943E-01 |
|         |        | 6           | 7.867E-02 | 4.017E-01 | 8.943E-01 |
|         |        | 7           | 4.074E-01 | 1.330E-01 | 8.943E-01 |
|         |        | 8           | 2.936E-01 | 4.217E-01 | 8.943E-01 |
| 20      | 0      | 1           | 6.188E-01 | 1.330E-01 | 6.057E-01 |
|         |        | 2           | 5.049E-01 | 4.217E-01 | 6.057E-01 |
|         |        | 3           | 8.978E-01 | 1.130E-01 | 6.057E-01 |
|         |        | 4           | 8.673E-01 | 4.017E-01 | 6.057E-01 |
|         |        | 5           | 6.188E-01 | 1.330E-01 | 8.943E-01 |
|         |        | 6           | 5.049E-01 | 4.217E-01 | 8.943E-01 |
|         |        | 7           | 8.978E-01 | 1.130E-01 | 8.943E-01 |
|         |        | 8           | 8.673E-01 | 4.017E-01 | 8.943E-01 |
| 30      | 0      | 1           | 6.400E-02 | 6.057E-01 | 6.057E-01 |
|         |        | 2           | 9.450E-02 | 8.943E-01 | 6.057E-01 |
|         |        | 3           | 2.388E-01 | 6.057E-01 | 6.057E-01 |
|         |        | 4           | 3.527E-01 | 8.943E-01 | 6.057E-01 |
|         |        | 5           | 6.400E-02 | 6.057E-01 | 8.943E-01 |
|         |        | 6           | 9.450E-02 | 8.943E-01 | 8.943E-01 |
|         |        | 7           | 2.388E-01 | 6.057E-01 | 8.943E-01 |
|         |        | 8           | 3.527E-01 | 8.943E-01 | 8.943E-01 |
| 40      | 0      | 1           | 4.502E-01 | 6.057E-01 | 6.057E-01 |
|         |        | 2           | 5.640E-01 | 8.943E-01 | 6.057E-01 |
|         |        | 3           | 8.527E-01 | 6.057E-01 | 6.057E-01 |
|         |        | 4           | 8.832E-01 | 8.943E-01 | 6.057E-01 |
|         |        | 5           | 4.502E-01 | 6.057E-01 | 8.943E-01 |
|         |        | 6           | 5.640E-01 | 8.943E-01 | 8.943E-01 |
|         |        | 7           | 8.527E-01 | 6.057E-01 | 8.943E-01 |
|         |        | 8           | 8.832E-01 | 8.943E-01 | 8.943E-01 |
| 50      | 0      | 1           | 1.092E-01 | 1.130E-01 | 1.057E-01 |
|         |        | 2           | 7.867E-02 | 4.017E-01 | 1.057E-01 |
|         |        | 3           | 4.074E-01 | 1.330E-01 | 1.057E-01 |
|         |        | 4           | 2.936E-01 | 4.217E-01 | 1.057E-01 |
|         |        | 5           | 1.092E-01 | 1.130E-01 | 3.943E-01 |
|         |        | 6           | 7.867E-02 | 4.017E-01 | 3.943E-01 |
|         |        | 7           | 4.074E-01 | 1.330E-01 | 3.943E-01 |
|         |        | 8           | 2.936E-01 | 4.217E-01 | 3.943E-01 |

14.2.10



RESIDUAL URM = 0.40179E-00  
 RESIDUAL URM = 0.29384E-00  
 RESIDUAL URM = 0.40179E-00  
 RESIDUAL URM = 0.28751E-05

INCREMENT = 1

MECHANICAL LOAD CURVE FACTORS = 0.1300E 01, 0.0

CREEP TIME INCREMENT = 0.0

NO. ELASTIC INTEGRATION POINTS = 54, NO. PLASTIC INTEGRATION POINTS = 10

10 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 0

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 4

SPECIFIED MAX. UNBALANCED-FORCE ERROR = 0.1000E-04, ACTUAL ERROR = 0.2879E-05

# CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

4.2112

| ** NODE ** |      | ***** FORCES ***** |                |                | ***** DISPLACEMENTS ***** |                |                |
|------------|------|--------------------|----------------|----------------|---------------------------|----------------|----------------|
| NO.        | I.D. | U                  | V              | W              | U                         | V              | W              |
| 1          | 1J   | -0.1400407E-05     | -0.1250005E 00 | 0.4445023E-06  | 0.0                       | 0.0            | 0.0            |
| 2          | 2J   | 0.1699103E-06      | -0.2500004E 00 | 0.9841290E-07  | 0.6854534E-05             | 0.0            | 0.0            |
| 3          | 3J   | -0.2919136E-06     | -0.1249999E 00 | -0.2920557E-06 | -0.7212162E-05            | 0.0            | 0.0            |
| 4          | 4J   | 0.6586836E-06      | 0.5033810E-06  | 0.4395864E-06  | 0.7152557E-05             | 0.3249999E 00  | 0.0            |
| 5          | 5J   | 0.2668880E-06      | 0.2371036E-06  | -0.1784057E-06 | 0.8683346E-05             | 0.6500012E 00  | 0.0            |
| 6          | 6J   | 0.3095244E-06      | 0.1150552E-06  | 0.2357038E-07  | -0.7569790E-05            | 0.6500003E 00  | 0.0            |
| 7          | 7J   | 0.1742891E-06      | -0.2058789E-06 | 0.4384188E-06  | 0.7331371E-05             | 0.6499992E 00  | 0.0            |
| 8          | 8J   | 0.6743869E-07      | 0.1249996E 00  | -0.2377780E-06 | 0.7396651E-05             | 0.1249999E 01  | 0.0            |
| 9          | 9J   | 0.6164924E-06      | -0.2499993E 00 | -0.6047355E-07 | -0.8940697E-05            | -0.1249999E 01 | 0.0            |
| 10         | 10J  | -0.6505514E-07     | 0.1250000E 00  | 0.1297262E-06  | 0.7688594E-05             | 0.1299999E 01  | 0.0            |
| 11         | 11J  | 0.6755136E-07      | -0.2500002E 00 | -0.1192174E-06 | 0.7316470E-05             | 0.0            | 0.2626330E-06  |
| 12         | 12J  | 0.2610477E-06      | -0.4999995E 00 | -0.7382590E-07 | 0.5960464E-05             | 0.0            | -0.1434237E-06 |
| 13         | 13J  | 0.6942285E-06      | -0.2499998E 00 | 0.1244648E-06  | 0.6973743E-05             | 0.0            | -0.1136214E-06 |
| 14         | 14J  | -0.2377183E-06     | 0.2431558E-06  | -0.3060770E-06 | 0.5722046E-05             | 0.3250002E 00  | -0.2160668E-06 |
| 15         | 15J  | -0.3174032E-06     | -0.4371747E-06 | -0.3056603E-06 | -0.6075948E-05            | -0.6500009E 00 | -0.1806766E-06 |
| 16         | 16J  | -0.4203317E-06     | 0.1295525E-06  | -0.3811952E-06 | 0.6139278E-05             | 0.6500006E 00  | -0.1788139E-06 |
| 17         | 17J  | 0.1919924E-06      | 0.1215579E-06  | -0.2073656E-06 | 0.6495906E-05             | 0.6499996E 00  | -0.5394686E-06 |
| 18         | 18J  | -0.9478049E-07     | -0.2499989E 00 | -0.1771810E-06 | 0.6128335E-05             | 0.1299999E 01  | -0.1498961E-06 |
| 19         | 19J  | 0.7254494E-07      | 0.4999983E 00  | 0.5848308E-07  | 0.6675720E-05             | 0.1299999E 01  | 0.0            |
| 20         | 20J  | -0.2394361E-06     | 0.2499995E 00  | 0.1461347E-06  | 0.6616116E-05             | 0.1299999E 01  | -0.3539076E-07 |
| 21         | 21J  | -0.1128429E-05     | -0.1250002E 00 | -0.5264842E-06 | 0.0                       | 0.0            | 0.0            |
| 22         | 22J  | 0.1664316E-06      | -0.2500002E 00 | -0.4644028E-07 | 0.5066395E-05             | 0.0            | 0.0            |
| 23         | 23J  | 0.9761050E-07      | -0.1249999E 00 | 0.6176833E-07  | 0.5364418E-05             | 0.0            | 0.0            |
| 24         | 24J  | 0.2260155E-06      | 0.2121894E-06  | -0.8619355E-07 | -0.4887581E-05            | -0.3250000E 00 | 0.0            |
| 25         | 25J  | -0.4144554E-07     | 0.1637511E-06  | -0.2836778E-06 | 0.5722046E-05             | 0.6500006E 00  | 0.0            |
| 26         | 26J  | -0.4313935E-06     | 0.1363975E-06  | 0.2111813E-06  | 0.4887581E-05             | 0.6500004E 00  | 0.0            |
| 27         | 27J  | -0.3883366E-07     | -0.1544342E-06 | 0.2985502E-06  | 0.5662441E-05             | 0.6499997E 00  | 0.0            |
| 28         | 28J  | -0.1299312E-06     | 0.1249995E 00  | -0.3165309E-07 | 0.4718569E-05             | 0.1299999E 01  | 0.0            |
| 29         | 29J  | -0.1017249E-07     | 0.2499992E 00  | 0.1795678E-06  | 0.6079674E-05             | 0.1299999E 01  | 0.0            |
| 30         | 30J  | -0.4958994E-07     | 0.1249996E 00  | -0.1452057E-06 | -0.6377697E-05            | -0.1299999E 01 | 0.0            |

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OF POOR QUALITY

# THERMAL AND ELASTIC STRAINS

| ELEMENT<br>NO.                         | I.D. | INTEGR.<br>POINT              | INCREMENTAL<br>THERMAL STRAINS | ***** INCREMENTAL ELASTIC STRAINS ***** |             |             |             |             |             |
|--|------|-------------------------------|--------------------------------|---|-------------|-------------|-------------|-------------|-------------|
|  |      |                               |                                | XX                                      | YY          | ZZ          | XY          | XZ          | YZ          |
| 1                                      | 10   | 1                             | 0.3000E-00                     | 0.3000E-00                              | 0.1000E-01  | 0.3000E-00  | 0.4105E-06  | 0.4184E-05  | 0.7631E-07  |
|  |      | 2                             | 0.3000E-00                     | -0.3000E-00                             | 0.1000E-01  | -0.3000E-00 | 0.2488E-06  | 0.3056E-06  | -0.2676E-07 |
|  |      | 3                             | 0.3000E-00                     | -0.3000E-00                             | 0.1000E-01  | -0.3000E-00 | -0.1890E-06 | -0.4584E-06 | -0.2746E-06 |
|  |      | 4                             | 0.3000E-00                     | -0.3000E-00                             | 0.1000E-01  | -0.3000E-00 | 0.1793E-06  | 0.9187E-06  | 0.5883E-06  |
|  |      | 5                             | 0.3000E-00                     | -0.3000E-00                             | 0.1000E-01  | -0.3000E-00 | 0.4711E-05  | -0.3944E-05 | -0.1944E-07 |
|  |      | 6                             | 0.3000E-00                     | -0.3000E-00                             | 0.1000E-01  | -0.3000E-00 | 0.4556E-05  | 0.6148E-06  | 0.6656E-07  |
|  |      | 7                             | 0.3000E-00                     | -0.3000E-00                             | 0.1000E-01  | -0.3000E-00 | 0.1232E-05  | -0.2196E-06 | -0.1458E-06 |
|  |      | 8                             | 0.3000E-00                     | -0.3000E-00                             | 0.1000E-01  | -0.3000E-00 | 0.1660E-05  | 0.1215E-05  | -0.3526E-06 |
| ***** CUMULATIVE ELASTIC STRAINS ***** |      |                               |                                |   |             |             |             |             |             |
| INTEGR.<br>POINT                       |      | CUMULATIVE<br>THERMAL STRAINS | XX                             | YY                                      | ZZ          | XY          | XZ          | YZ          |             |
| 1                                      |      | 0.3000E-00                    | -0.3000E-00                    | 0.1000E-01                              | -0.3000E-00 | 0.4105E-06  | -0.4184E-05 | -0.7631E-07 |             |
| 2                                      |      | 0.3000E-00                    | -0.3000E-00                    | 0.1000E-01                              | -0.3000E-00 | 0.2488E-06  | 0.3056E-06  | -0.2676E-07 |             |
| 3                                      |      | 0.3000E-00                    | -0.3000E-00                    | 0.1000E-01                              | -0.3000E-00 | -0.1890E-06 | -0.4584E-06 | -0.2746E-06 |             |
| 4                                      |      | 0.3000E-00                    | -0.3000E-00                    | 0.1000E-01                              | -0.3000E-00 | 0.1793E-06  | 0.9187E-06  | 0.5883E-06  |             |
| 5                                      |      | 0.3000E-00                    | -0.3000E-00                    | 0.1000E-01                              | -0.3000E-00 | 0.4711E-05  | -0.3944E-05 | -0.1944E-07 |             |
| 6                                      |      | 0.3000E-00                    | -0.3000E-00                    | 0.1000E-01                              | -0.3000E-00 | 0.4556E-05  | 0.6148E-06  | 0.6656E-07  |             |
| 7                                      |      | 0.3000E-00                    | -0.3000E-00                    | 0.1000E-01                              | -0.3000E-00 | 0.1232E-05  | -0.2196E-06 | -0.1458E-06 |             |
| 8                                      |      | 0.3000E-00                    | -0.3000E-00                    | 0.1000E-01                              | -0.3000E-00 | 0.1660E-05  | 0.1215E-05  | -0.3526E-06 |             |

74.2-13

# PLASTIC WORK AND STRAINS

| ELEMENT<br>NO. | I.D.             | INTEGR.<br>POINT           | INCREMENTAL<br>PLASTIC WORK | ***** INCREMENTAL PLASTIC STRAINS ***** |     |     |     |     |     |
|----------------|------------------|----------------------------|-----------------------------|---|-----|-----|-----|-----|-----|
|                |                  |                            |                             | XX                                      | YY  | ZZ  | XY  | XZ  | YZ  |
| 1              | 10               | 1                          | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|                |                  | 2                          | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                |                  | 3                          | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                |                  | 4                          | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                |                  | 5                          | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                |                  | 6                          | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                |                  | 7                          | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                |                  | 8                          | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                |                  |                            |                             | ***** CUMULATIVE PLASTIC STRAINS *****  |     |     |     |     |     |
|                | INTEGR.<br>POINT | CUMULATIVE<br>PLASTIC WORK | XX                          | YY                                      | ZZ  | XY  | XZ  | YZ  |     |
|                | 1                | 0.0                        | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                | 2                | 0.0                        | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                | 3                | 0.0                        | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                | 4                | 0.0                        | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                | 5                | 0.0                        | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                | 6                | 0.0                        | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                | 7                | 0.0                        | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |
|                | 8                | 0.0                        | 0.0                         | 0.0                                     | 0.0 | 0.0 | 0.0 | 0.0 |     |



# CUMULATIVE STRESS QUANTITIES

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | EFFECTIVE<br>STRESS CENTER | ***** CUMULATIVE STRESS CENTER ***** |     |     |     |     |     |
|---------------------|------------------|----------------------------|--------------------------------------|-----|-----|-----|-----|-----|
|                     |                  |                            | XX                                   | YY  | ZZ  | KY  | KZ  | YZ  |
| 1 10                | 1                | 0.0                        | 0.0                                  | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|                     | 2                | 0.0                        | 0.0                                  | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|                     | 3                | 0.0                        | 0.0                                  | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|                     | 4                | 0.0                        | 0.0                                  | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|                     | 5                | 0.0                        | 0.0                                  | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|                     | 6                | 0.0                        | 0.0                                  | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|                     | 7                | 0.0                        | 0.0                                  | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|                     | 8                | 0.0                        | 0.0                                  | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| INTEGR.<br>POINT | EFFECTIVE<br>STRESS | ***** CUMULATIVE STRESSES ***** |            |            |             |             |             |
|------------------|---------------------|---------------------------------|------------|------------|-------------|-------------|-------------|
|                  |                     | XX                              | YY         | ZZ         | KY          | KZ          | YZ          |
| 1                | 0.2000E 01          | 0.1605E-05                      | 0.2000E 01 | 0.9170E-06 | 0.6315E-06  | -0.6438E-05 | -0.1174E-06 |
| 2                | 0.2000E 01          | 0.9170E-06                      | 0.2000E 01 | 0.4565E-06 | 0.3829E-06  | -0.4702E-06 | -0.4117E-07 |
| 3                | 0.2000E 01          | 0.1834E-05                      | 0.2000E 01 | 0.2063E-05 | -0.2908E-06 | -0.7052E-06 | -0.4224E-06 |
| 4                | 0.2000E 01          | -0.1605E-05                     | 0.2000E 01 | 0.9170E-06 | 0.2758E-06  | 0.1413E-05  | -0.9051E-06 |
| 5                | 0.2000E 01          | 0.1945E-04                      | 0.2000E 01 | 0.8711E-05 | 0.7247E-05  | 0.6068E-05  | 0.2591E-07  |
| 6                | 0.2000E 01          | 0.9170E-06                      | 0.2000E 01 | 0.1375E-05 | 0.7010E-05  | 0.9459E-06  | 0.1024E-06  |
| 7                | 0.2000E 01          | 0.1972E-04                      | 0.2000E 01 | 0.9628E-05 | 0.1895E-05  | -0.3378E-06 | -0.3012E-06 |
| 8                | 0.2000E 01          | 0.1605E-05                      | 0.2000E 01 | 0.2292E-05 | 0.2553E-05  | 0.1869E-05  | 0.5245E-06  |

## PLASTIC AND CREEP STRAINS

ELEMENT NO. = 1 ID = 10

| INT | E-P | SUM | INCREMENTAL |             | TOTAL      | SURFACE |      | *** EFFECTIVE PLASTIC STRAINS *** |           |            | *** EFFECTIVE CREEP STRAINS *** |           |            |
|-----|-----|-----|-------------|-------------|------------|---------|------|-----------------------------------|-----------|------------|---------------------------------|-----------|------------|
|     |     |     | CODE        | TEMPERATURE |            | YIELD   | SIZE | INCREMENTAL                       | SUM INCR. | CUMULATIVE | INCREMENTAL                     | SUM INCR. | CUMULATIVE |
| 1   | 1   | 2   | 0.1000E 01  | 0.2000E 01  | 0.2000E 01 | 0.0     | 0.0  | 0.0                               | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 2   | 1   | 2   | 0.1000E 01  | 0.2000E 01  | 0.2000E 01 | 0.0     | 0.0  | 0.0                               | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 3   | 0   | -2  | 0.1000E 01  | 0.2000E 01  | 0.2000E 01 | 0.0     | 0.0  | 0.0                               | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 4   | 1   | 2   | 0.1000E 01  | 0.2000E 01  | 0.2000E 01 | 0.0     | 0.0  | 0.0                               | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 5   | 0   | -2  | 0.1000E 01  | 0.2000E 01  | 0.2000E 01 | 0.0     | 0.0  | 0.0                               | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 6   | 1   | 2   | 0.1000E 01  | 0.2000E 01  | 0.2000E 01 | 0.0     | 0.0  | 0.0                               | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 7   | 0   | -2  | 0.1000E 01  | 0.2000E 01  | 0.2000E 01 | 0.0     | 0.0  | 0.0                               | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |
| 8   | 0   | -2  | 0.1000E 01  | 0.2000E 01  | 0.2000E 01 | 0.0     | 0.0  | 0.0                               | 0.0       | 0.0        | 0.0                             | 0.0       | 0.0        |

14.2-14

# CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 2

INCREMENT 2

| NODE I.D. | 10          | 20          | 30          | 40          | 50          | 60          | 70          | 80          | 90          | 100         |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| TEMP.     | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 |

| NODE I.D. | 110         | 120         | 130         | 140         | 150         | 160         | 170         | 180         | 190         | 200         |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| TEMP.     | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 |

| NODE I.D. | 210         | 220         | 230         | 240         | 250         | 260         | 270         | 280         | 290         | 300         |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| TEMP.     | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 | 0.30000E 01 |

14.2-15

|               |             |
|---------------|-------------|
| RESIDUAL URM  | 0.54049E 00 |
| RESIDUAL NURM | 0.40851E 00 |
| RESIDUAL URM  | 0.41035E 00 |
| RESIDUAL URM  | 0.39556E 00 |
| RESIDUAL NURM | 0.28146E 00 |
| RESIDUAL URM  | 0.47630E 00 |
| RESIDUAL URM  | 0.36845E 01 |
| RESIDUAL URM  | 0.49752E 01 |
| RESIDUAL URM  | 0.16748E 01 |
| RESIDUAL NURM | 0.18280E 01 |
| RESIDUAL URM  | 0.28256E 02 |
| RESIDUAL URM  | 0.71523E 04 |
| RESIDUAL URM  | 0.20494E 04 |
| RESIDUAL URM  | 0.68801E 05 |

END OF LOAD INCREMENT 2

INCREMENT 2

MECHANICAL LOAD CURVE FACTORS = 0.2800E 01, 0.0

CREEP TIME INCREMENT = 0.5000E 01

NO. ELASTIC INTEGRATION POINTS = 0, NO. PLASTIC INTEGRATION POINTS = 64

5% INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 1

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 4

SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.1000E 04, ACTUAL ERROR = 0.6880E 05

# CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** |      | ***** FORCES ***** |                |                | ***** DISPLACEMENTS ***** |                |                |
|------------|------|--------------------|----------------|----------------|---------------------------|----------------|----------------|
| NO.        | 1.0- | U                  | V              | W              | U                         | V              | W              |
| 1          | 10   | -0.5172569E-07     | -0.1562499E 00 | 0.9578247E-07  | 0.0                       | 0.0            | 0.0            |
| 2          | 20   | -0.4715711E-07     | -0.3124999E 00 | -0.1685789E-06 | -0.2155782E-05            | 0.0            | 0.0            |
| 3          | 30   | 0.1411092E-06      | -0.1562499E 00 | 0.2150952E-06  | 0.2832159E-05             | 0.0            | 0.0            |
| 4          | 40   | -0.2186835E-07     | -0.1535602E-06 | 0.5432671E-06  | 0.2464210E-05             | 0.6959998E 00  | 0.0            |
| 5          | 50   | -0.1514582E-06     | -0.1382062E-07 | -0.1273684E-06 | -0.3110370E-05            | -0.1400000E-01 | 0.0            |
| 6          | 60   | 0.1555258E-07      | 0.2528590E-06  | -0.3224132E-06 | 0.3009126E-05             | 0.1400000E 01  | 0.0            |
| 7          | 70   | 0.1472909E-06      | 0.8867227E-07  | -0.1990048E-06 | 0.3013801E-05             | 0.1399999E 01  | 0.0            |
| 8          | 80   | -0.7241965E-08     | -0.1562496E-00 | -0.1521822E-06 | -0.2058851E-05            | -0.2799999E-01 | 0.0            |
| 9          | 90   | -0.3091270E-07     | 0.3124990E 00  | -0.2363387E-06 | 0.3496011E-05             | 0.2799999E 01  | 0.0            |
| 10         | 100  | 0.3964390E-07      | 0.1562496E 00  | -0.1043930E-06 | 0.3873974E-05             | 0.2799999E 01  | 0.0            |
| 11         | 110  | -0.2280329E-06     | -0.3124998E 00 | 0.6311808E-07  | -0.3442918E-05            | 0.0            | -0.2815538E-06 |
| 12         | 120  | 0.1153526E-07      | -0.6249990E 00 | -0.7500660E-07 | -0.2427951E-05            | 0.0            | -0.1295992E-06 |
| 13         | 130  | 0.1125775E-06      | -0.3124995E 00 | -0.3301000E-06 | -0.1887667E-05            | 0.0            | -0.9446256E-06 |
| 14         | 140  | -0.2139647E-06     | -0.1745229E-06 | -0.3112950E-06 | -0.2061063E-05            | -0.7000000E-00 | -0.1249778E-06 |
| 15         | 150  | -0.5015519E-06     | 0.1311443E-06  | 0.1257699E-06  | 0.1381520E-05             | 0.1400000E 01  | 0.1570841E-06  |
| 16         | 160  | 0.2240512E-06      | -0.5609059E-06 | -0.1300162E-06 | 0.1838955E-05             | 0.1400001E 01  | -0.3830354E-08 |
| 17         | 170  | -0.1694046E-06     | -0.7134558E-07 | -0.1175575E-06 | -0.2962200E-05            | -0.1400000E-01 | -0.3136598E-06 |
| 18         | 180  | -0.1292663E-06     | 0.3124991E 00  | 0.6345938E-07  | 0.8754371E-06             | 0.2799999E 01  | 0.2396637E-06  |
| 19         | 190  | 0.2054375E-07      | 0.6249979E 00  | -0.1114295E-06 | 0.2183379E-05             | 0.2799999E 01  | 0.6129881E-08  |
| 20         | 200  | -0.1063862E-06     | -0.3124991E 00 | -0.4936070E-07 | -0.3459167E-05            | -0.2799999E-01 | -0.6049105E-06 |
| 21         | 210  | -0.6363555E-07     | -0.1562498E 00 | -0.1006614E-06 | 0.0                       | 0.0            | 0.0            |
| 22         | 220  | 0.2313636E-08      | -0.3125001E 00 | -0.6692170E-07 | 0.1457794E-05             | 0.0            | 0.0            |
| 23         | 230  | -0.3646676E-07     | -0.1562498E 00 | 0.4993487E-07  | -0.9632167E-06            | 0.0            | 0.0            |
| 24         | 240  | 0.2306959E-06      | 0.2866300E-06  | -0.1418189E-06 | 0.1503276E-05             | 0.7000000E 00  | 0.0            |
| 25         | 250  | -0.2378811E-06     | -0.2072557E-07 | -0.5052379E-07 | 0.1153480E-05             | 0.1399999E 01  | 0.0            |
| 26         | 260  | -0.1650878E-06     | -0.4303375E-06 | -0.4300003E-06 | -0.1132758E-05            | -0.1400000E-01 | 0.0            |
| 27         | 270  | -0.4314932E-07     | 0.1978265E-07  | 0.3071169E-06  | 0.2251932E-05             | 0.1400001E 01  | 0.0            |
| 28         | 280  | -0.2171015E-07     | 0.1562496E 00  | 0.7931328E-07  | 0.1943008E-06             | 0.2799999E 01  | 0.0            |
| 29         | 290  | -0.3496049E-08     | -0.3124993E 00 | -0.3519430E-06 | -0.1462942E-05            | -0.2799999E-01 | 0.0            |
| 30         | 300  | 0.3125238E-07      | 0.1562495E 00  | 0.1703901E-06  | 0.3472938E-05             | 0.2799999E 01  | 0.0            |

14.2-16

## THERMAL AND ELASTIC STRAINS

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | INCREMENTAL<br>THERMAL STRAINS | ***** INCREMENTAL ELASTIC STRAINS ***** |            |            |            |            |            |
|---------------------|------------------|--------------------------------|---|------------|------------|------------|------------|------------|
|                     |                  |                                | XX                                      | YY         | ZZ         | XY         | XZ         | YZ         |
| 1 10                | 1                | 0.5000E 00                     | 0.2384E-06                              | 0.1013E-05 | 0.5960E-07 | 0.9918E-06 | 0.4364E-05 | 0.5170E-07 |
|                     | 2                | 0.5000E 00                     | 0.2027E-05                              | 0.1013E-05 | 0.0        | 0.5706E-06 | 0.1367E-06 | 0.2389E-06 |
|                     | 3                | 0.5000E 00                     | 0.2580E-06                              | 0.2980E-06 | 0.1788E-06 | 0.4743E-08 | 0.1765E-06 | 0.7590E-07 |
|                     | 4                | 0.5000E 00                     | 0.2086E-05                              | 0.2980E-06 | 0.3576E-06 | 0.6879E-07 | 0.8619E-06 | 0.3846E-06 |
|                     | 5                | 0.5000E 00                     | 0.7212E-05                              | 0.1252E-05 | 0.5960E-07 | 0.5479E-05 | 0.4172E-05 | 0.9574E-07 |
|                     | 6                | 0.5000E 00                     | 0.1192E-05                              | 0.1013E-05 | 0.1192E-06 | 0.4985E-05 | 0.4439E-06 | 0.3207E-06 |
|                     | 7                | 0.5000E 00                     | 0.6414E-05                              | 0.9537E-06 | 0.6567E-06 | 0.1322E-05 | 0.1210E-07 | 0.2816E-07 |
|                     | 8                | 0.5000E 00                     | 0.8345E-06                              | 0.1632E-05 | 0.1788E-06 | 0.1357E-05 | 0.1151E-05 | 0.1051E-06 |

| INTEGR.<br>POINT | CUMULATIVE<br>THERMAL STRAINS | ***** CUMULATIVE ELASTIC STRAINS ***** |            |             |             |             |             |
|------------------|-------------------------------|--|------------|-------------|-------------|-------------|-------------|
|                  |                               | XX                                     | YY         | ZZ          | XY          | XZ          | YZ          |
| 1                | 0.8000E 00                    | -0.3000E 00                            | 0.1000E 01 | -0.3000E 00 | -0.5813E-06 | 0.1800E-06  | -0.1280E-06 |
| 2                | 0.8000E 00                    | -0.3000E 00                            | 0.1000E 01 | -0.3000E 00 | -0.5217E-06 | 0.1689E-06  | -0.2656E-06 |
| 3                | 0.8000E 00                    | -0.3000E 00                            | 0.1000E 01 | -0.3000E 00 | -0.1843E-06 | -0.2618E-06 | -0.1937E-06 |
| 4                | 0.8000E 00                    | -0.3000E 00                            | 0.1000E 01 | -0.3000E 00 | 0.1105E-06  | 0.5677E-07  | -0.2037E-06 |
| 5                | 0.8000E 00                    | -0.3000E 00                            | 0.1000E 01 | -0.3000E 00 | 0.7682E-06  | 0.2277E-06  | -0.1152E-06 |
| 6                | 0.8000E 00                    | -0.3000E 00                            | 0.1000E 01 | -0.3000E 00 | -0.4291E-06 | 0.1709E-06  | -0.2541E-06 |
| 7                | 0.8000E 00                    | -0.3000E 00                            | 0.1000E 01 | -0.3000E 00 | -0.9057E-07 | -0.2317E-06 | -0.2238E-06 |
| 8                | 0.8000E 00                    | -0.3000E 00                            | 0.1000E 01 | -0.3000E 00 | 0.3029E-06  | 0.6346E-07  | 0.2474E-06  |

## PLASTIC WORK AND STRAINS

| ELEMENT<br>NO. I.D. | INTEGR.<br>POINT | INCREMENTAL<br>PLASTIC WORK | ***** INCREMENTAL PLASTIC STRAINS ***** |            |             |             |             |             |
|---------------------|------------------|-----------------------------|---|------------|-------------|-------------|-------------|-------------|
|                     |                  |                             | XX                                      | YY         | ZZ          | XY          | XZ          | YZ          |
| 1 10                | 1                | 0.1125E 01                  | -0.2500E 00                             | 0.5000E 00 | -0.2500E 00 | -0.4842E-07 | -0.1152E-05 | -0.5736E-07 |
|                     | 2                | 0.1125E 01                  | -0.2500E 00                             | 0.5000E 00 | -0.2500E 00 | -0.1883E-07 | 0.1394E-06  | -0.7506E-07 |
|                     | 3                | 0.1125E 01                  | -0.2500E 00                             | 0.5000E 00 | -0.2500E 00 | -0.1079E-06 | 0.2160E-06  | -0.1394E-06 |
|                     | 4                | 0.1125E 01                  | -0.2500E 00                             | 0.5000E 00 | -0.2500E 00 | 0.8455E-07  | 0.2809E-06  | -0.2192E-06 |
|                     | 5                | 0.1125E 01                  | -0.2500E 00                             | 0.5000E 00 | -0.2500E 00 | 0.1135E-05  | -0.1069E-05 | -0.3732E-07 |
|                     | 6                | 0.1125E 01                  | -0.2500E 00                             | 0.5000E 00 | -0.2500E 00 | 0.1193E-05  | 0.2303E-06  | -0.4604E-07 |
|                     | 7                | 0.1125E 01                  | -0.2500E 00                             | 0.5000E 00 | -0.2500E 00 | 0.3247E-06  | -0.1324E-06 | -0.1218E-06 |
|                     | 8                | 0.1125E 01                  | -0.2500E 00                             | 0.5000E 00 | -0.2500E 00 | 0.5731E-06  | 0.3696E-06  | -0.1651E-06 |

| INTEGR.<br>POINT | CUMULATIVE<br>PLASTIC WORK | ***** CUMULATIVE PLASTIC STRAINS ***** |            |             |             |             |             |
|------------------|----------------------------|--|------------|-------------|-------------|-------------|-------------|
|                  |                            | XX                                     | YY         | ZZ          | XY          | XZ          | YZ          |
| 1                | 0.1125E 01                 | -0.2500E 00                            | 0.5000E 00 | -0.2500E 00 | -0.4842E-07 | 0.1152E-05  | -0.5736E-07 |
| 2                | 0.1125E 01                 | -0.2500E 00                            | 0.5000E 00 | -0.2500E 00 | -0.1883E-07 | 0.1394E-06  | -0.7506E-07 |
| 3                | 0.1125E 01                 | -0.2500E 00                            | 0.5000E 00 | -0.2500E 00 | -0.1079E-06 | 0.2160E-06  | -0.1394E-06 |
| 4                | 0.1125E 01                 | -0.2500E 00                            | 0.5000E 00 | -0.2500E 00 | 0.8455E-07  | 0.2809E-06  | -0.2192E-06 |
| 5                | 0.1125E 01                 | -0.2500E 00                            | 0.5000E 00 | -0.2500E 00 | 0.1135E-05  | -0.1069E-05 | -0.3732E-07 |
| 6                | 0.1125E 01                 | -0.2500E 00                            | 0.5000E 00 | -0.2500E 00 | 0.1193E-05  | 0.2303E-06  | -0.4604E-07 |
| 7                | 0.1125E 01                 | -0.2500E 00                            | 0.5000E 00 | -0.2500E 00 | 0.3247E-06  | -0.1324E-06 | -0.1218E-06 |
| 8                | 0.1125E 01                 | -0.2500E 00                            | 0.5000E 00 | -0.2500E 00 | 0.5731E-06  | 0.3696E-06  | -0.1651E-06 |

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OF POOR QUALITY

14.2+17

# CUMULATIVE STRESS QUANTITIES

| ELEMENT NO. | I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | ***** CUMULATIVE STRESS CENTER ***** |            |             |             |             |             |
|-------------|------|---------------|-------------------------|--------------------------------------|------------|-------------|-------------|-------------|-------------|
|             |      |               |                         | XX                                   | YY         | ZZ          | XY          | XZ          | YZ          |
| 1           | 10   | 1             | 0.5000E 00              | 0.1667E 00                           | 0.3333E 00 | 0.1667E 00  | 0.3228E-07  | 0.7603E-06  | 0.3824E-07  |
|             |      | 2             | 0.5000E 00              | -0.1667E 00                          | 0.3333E 00 | -0.1667E 00 | -0.1255E-07 | 0.9241E-07  | -0.5004E-07 |
|             |      | 3             | 0.5000E 00              | -0.1667E 00                          | 0.3333E 00 | -0.1667E 00 | -0.7192E-07 | -0.1440E-06 | -0.5276E-07 |
|             |      | 4             | 0.5000E 00              | -0.1667E 00                          | 0.3333E 00 | -0.1667E 00 | -0.5637E-07 | 0.1873E-06  | -0.1455E-06 |
|             |      | 5             | 0.5000E 00              | -0.1667E 00                          | 0.3333E 00 | -0.1667E 00 | 0.7570E-06  | -0.7127E-06 | -0.2488E-07 |
|             |      | 6             | 0.5000E 00              | -0.1667E 00                          | 0.3333E 00 | -0.1667E 00 | 0.7950E-06  | 0.1535E-06  | -0.3009E-07 |
|             |      | 7             | 0.5000E 00              | -0.1667E 00                          | 0.3333E 00 | -0.1667E 00 | 0.2165E-06  | 0.8029E-07  | -0.8123E-07 |
|             |      | 8             | 0.5000E 00              | -0.1667E 00                          | 0.3333E 00 | -0.1667E 00 | 0.3821E-06  | 0.2464E-06  | -0.1101E-06 |

| INTEGR. POINT | EFFECTIVE STRESS | ***** CUMULATIVE STRESSES ***** |            |            |             |             |             |
|---------------|------------------|---------------------------------|------------|------------|-------------|-------------|-------------|
|               |                  | XX                              | YY         | ZZ         | XY          | XZ          | YZ          |
| 1             | 0.2500E 01       | 0.5731E-06                      | 0.2500E 01 | 0.2866E-06 | -0.1118E-05 | 0.3463E-06  | -0.2462E-06 |
| 2             | 0.2500E 01       | 0.3725E-05                      | 0.2500E 01 | 0.1719E-05 | -0.6187E-06 | -0.3248E-06 | -0.5108E-06 |
| 3             | 0.2500E 01       | 0.2006E-05                      | 0.2500E 01 | 0.1719E-05 | -0.3544E-06 | -0.5419E-06 | -0.3820E-06 |
| 4             | 0.2500E 01       | 0.4585E-05                      | 0.2500E 01 | 0.2292E-05 | 0.2125E-06  | 0.1092E-06  | -0.3917E-06 |
| 5             | 0.2500E 01       | 0.8597E-06                      | 0.2500E 01 | 0.2866E-06 | 0.1477E-05  | -0.4378E-06 | -0.2215E-06 |
| 6             | 0.2500E 01       | 0.3439E-05                      | 0.2500E 01 | 0.1146E-05 | -0.0252E-06 | 0.3287E-06  | -0.4887E-05 |
| 7             | 0.2500E 01       | 0.2866E-05                      | 0.2500E 01 | 0.4872E-05 | -0.1742E-06 | -0.4455E-06 | -0.4305E-06 |
| 8             | 0.2500E 01       | 0.7164E-05                      | 0.2500E 01 | 0.4585E-05 | 0.5824E-06  | 0.1230E-06  | -0.4758E-06 |

## CREEP WORK AND STRAINS

| ELEMENT NO. | I.D. | INTEGR. POINT | INCREMENTAL CREEP WORK | ***** INCREMENTAL CREEP STRAINS ***** |            |             |             |             |             |
|-------------|------|---------------|------------------------|---------------------------------------|------------|-------------|-------------|-------------|-------------|
|             |      |               |                        | XX                                    | YY         | ZZ          | XY          | XZ          | YZ          |
| 1           | 10   | 1             | 0.1125E 01             | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | -0.4842E-07 | -0.1152E-05 | -0.5736E-07 |
|             |      | 2             | 0.1125E 01             | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | -0.1883E-07 | 0.1354E-06  | -0.7506E-07 |
|             |      | 3             | 0.1125E 01             | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | -0.1379E-06 | -0.2160E-06 | -0.1391E-06 |
|             |      | 4             | 0.1125E 01             | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | 0.0455E-07  | 0.2809E-06  | -0.2182E-06 |
|             |      | 5             | 0.1125E 01             | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | 0.1135E-05  | -0.1069E-05 | -0.3732E-07 |
|             |      | 6             | 0.1125E 01             | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | 0.1193E-05  | -0.2303E-06 | -0.4604E-07 |
|             |      | 7             | 0.1125E 01             | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | 0.3247E-06  | -0.1324E-06 | -0.1218E-06 |
|             |      | 8             | 0.1125E 01             | -0.2500E 00                           | 0.5000E 00 | -0.2500E 00 | 0.5731E-06  | 0.3696E-06  | -0.1651E-06 |

| INTEGR. POINT | CUMULATIVE CREEP WORK | ***** CUMULATIVE CREEP STRAINS ***** |            |             |             |             |             |
|---------------|-----------------------|--------------------------------------|------------|-------------|-------------|-------------|-------------|
|               |                       | XX                                   | YY         | ZZ          | XY          | XZ          | YZ          |
| 1             | 0.1125E 01            | -0.2500E 00                          | 0.5000E 00 | -0.2500E 00 | -0.4842E-07 | -0.1152E-05 | -0.5736E-07 |
| 2             | 0.1125E 01            | -0.2500E 00                          | 0.5000E 00 | -0.2500E 00 | -0.1883E-07 | 0.1394E-06  | -0.7506E-07 |
| 3             | 0.1125E 01            | -0.2500E 00                          | 0.5000E 00 | -0.2500E 00 | -0.1379E-06 | -0.2160E-06 | -0.1391E-06 |
| 4             | 0.1125E 01            | -0.2500E 00                          | 0.5000E 00 | -0.2500E 00 | 0.0455E-07  | 0.2809E-06  | -0.2182E-06 |
| 5             | 0.1125E 01            | -0.2500E 00                          | 0.5000E 00 | -0.2500E 00 | 0.1135E-05  | -0.1069E-05 | -0.3732E-07 |
| 6             | 0.1125E 01            | -0.2500E 00                          | 0.5000E 00 | -0.2500E 00 | 0.1193E-05  | -0.2303E-06 | -0.4604E-07 |
| 7             | 0.1125E 01            | -0.2500E 00                          | 0.5000E 00 | -0.2500E 00 | 0.3247E-06  | -0.1324E-06 | -0.1218E-06 |
| 8             | 0.1125E 01            | -0.2500E 00                          | 0.5000E 00 | -0.2500E 00 | 0.5731E-06  | 0.3696E-06  | -0.1651E-06 |

14.2.18

# PLASTIC AND CREEP STRAINS

| ELEMENT NO. = 1 |      |      | ID = 10     |             |            |             |             |             | **** EFFECTIVE PLASTIC STRAINS **** |             |             | **** EFFECTIVE CREEP STRAINS **** |             |             |
|-----------------|------|------|-------------|-------------|------------|-------------|-------------|-------------|-------------------------------------|-------------|-------------|-----------------------------------|-------------|-------------|
| INT             | E-P  | SUM  | INCREMENTAL | TOTAL       | SURFACE    | INCREMENTAL | SUM INCR.   | CUMULATIVE  | INCREMENTAL                         | SUM INCR.   | CUMULATIVE  | INCREMENTAL                       | SUM INCR.   | CUMULATIVE  |
| PNT             | CODE | CODE | TEMPERATURE | TEMPERATURE | YIELD SIZE | TEMPERATURE | TEMPERATURE | TEMPERATURE | TEMPERATURE                         | TEMPERATURE | TEMPERATURE | TEMPERATURE                       | TEMPERATURE | TEMPERATURE |
| 1               | 0    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00  | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                          | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                        | 0.5000E 00  | 0.5000E 00  |
| 2               | 0    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00  | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                          | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                        | 0.5000E 00  | 0.5000E 00  |
| 3               | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00  | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                          | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                        | 0.5000E 00  | 0.5000E 00  |
| 4               | 0    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00  | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                          | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                        | 0.5000E 00  | 0.5000E 00  |
| 5               | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00  | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                          | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                        | 0.5000E 00  | 0.5000E 00  |
| 6               | 0    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00  | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                          | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                        | 0.5000E 00  | 0.5000E 00  |
| 7               | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00  | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                          | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                        | 0.5000E 00  | 0.5000E 00  |
| 8               | 1    | 2    | 0.1000E 01  | 0.3000E 01  | 0.2000E 01 | 0.5000E 00  | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                          | 0.5000E 00  | 0.5000E 00  | 0.5000E 00                        | 0.5000E 00  | 0.5000E 00  |

14.2-19

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## 14.3 DISTRIBUTED LOADING PROBLEM

A 1.0 x 1.0 x 1.0 cube is again used, to analyze a problem with linear variation of stress and strain. The loading is a uniformly distributed vertical shear, applied on all four vertical sides of the cube. Midside nodes are used to demonstrate equivalent loading values, which are in the ratio of 1:4:1 for the bottom, middle and top nodes, respectively. The input data and results are listed at the end of this section, for an elastic situation.

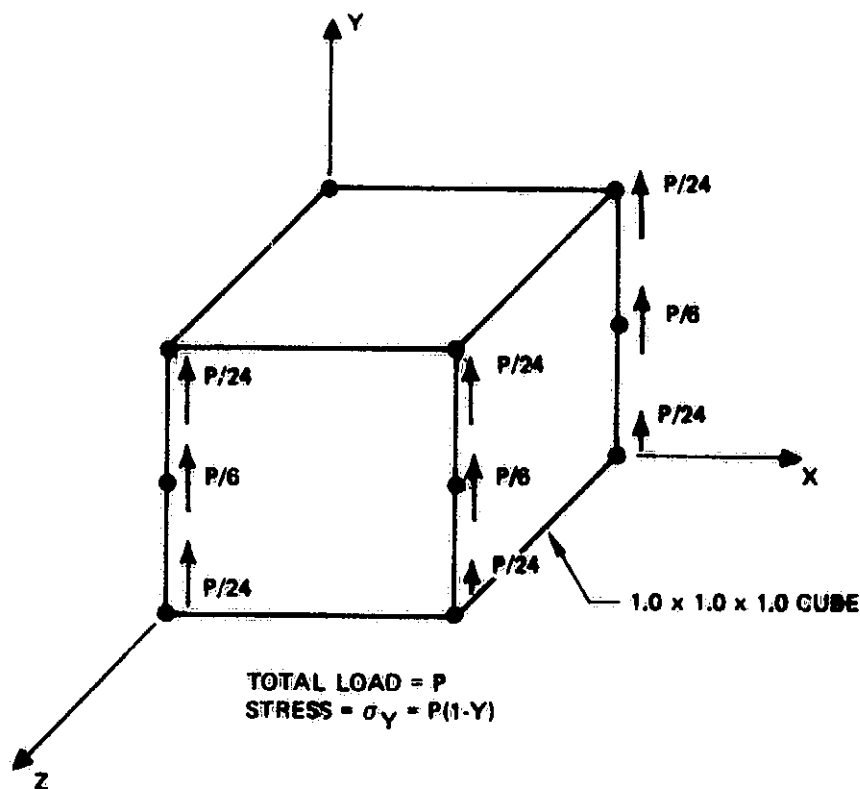


Figure 14.3-1. Distributed Loading Problem

**PRECEDING PAGE BLANK NOT FILMED**

START 5 5 6  
 BOPACE 3-C CHECK (VARIABLE STRESS-STRAIN, WITH ZERO POISSONS RATIO)

02/10/75

| 5    | 3       | 1    | 5    | 5    | 6      | 1    |
|------|---------|------|------|------|--------|------|
| 0.5  | .000001 |      |      |      |        |      |
|      | 1       | 1.0  |      |      |        |      |
| 1.0  | 0.0     | 2.0  | 0.3  | 3.0  | 0.8    | 4.0  |
| 4.5  | 1.5     | 5.0  | 1.8  | 6.0  | 2.3    | 7.0  |
| 8.0  | 1.0     | 9.0  | 0.5  | 10.0 | 1.15   | 12.0 |
| 13.0 | 2.5     | 14.0 | 2.8  | 15.0 | 5.1625 |      |
|      |         |      |      |      |        |      |
| 1.0  | 1.0     | 2.0  | 2.0  | 3.0  | 2.5    | 4.0  |
| 5.0  | 3.5     | 5.4  | 1.0  | 6.0  | 4.0    | 7.0  |
| 8.0  | 1.25    | 9.0  | 1.25 | 10.0 | 2.0    | 12.0 |
| 13.0 | 4.5     | 14.0 | 5.25 | 15.0 | 3.0    |      |
|      |         |      |      |      |        |      |
| 1.0  | 0.0     |      |      |      |        |      |
|      | 1       | 1    |      |      |        |      |
| 0.0  | 1 0.    | 3.0  | 2.0  | 9.0  | 3.5    |      |
|      | 2.0     |      |      |      |        |      |
| 0.0  | 0.0     | 1.0  | 1.0  | 3.0  | 2.0    | 9.0  |
|      |         |      |      |      |        | 3.5  |
|      |         |      |      |      |        |      |
| 0.0  | 1       | 2    |      |      |        |      |
|      | 0.0     | 10.0 | 1.0  | 30.0 | 2.0    |      |
|      |         |      |      |      |        |      |
| 0.0  | 1 0.0   |      |      |      |        |      |
|      | 1.0     | 3.0  | 1.0  | 11.0 | 9.0    |      |
|      |         |      |      |      |        |      |
| 1    | 3       | 2000 | 1010 | 1050 |        |      |
|      |         |      |      |      |        |      |
| 1000 | 0.0     | 0.0  | 1.0  |      |        |      |
| 10   | 1.0     | 0.0  | 1.0  |      |        |      |
| 20   | 0.0     | 0.5  | 1.0  |      |        |      |
| 30   | 1.0     | 0.5  | 1.0  |      |        |      |
| 40   | 0.0     | 1.0  | 1.0  |      |        |      |
| 50   | 1.0     | 1.0  | 1.0  |      |        |      |
| 2000 | 0.0     | 0.0  | 0.0  |      |        |      |

14.3-2



|      |     |     |     |
|------|-----|-----|-----|
| 1010 | 1.0 | 0.0 | 0.0 |
| 1020 | 0.0 | 0.5 | 0.0 |
| 1030 | 1.0 | 0.5 | 0.0 |
| 1040 | 0.0 | 1.0 | 0.0 |
| 1050 | 1.0 | 1.0 | 0.0 |

|      |    |      |    |    |      |      |      |      |      |   |   |
|------|----|------|----|----|------|------|------|------|------|---|---|
| 1000 | 1  | 1000 | 10 | 50 | 40   | 2000 | 1010 | 1050 | 1040 | 1 |   |
| 0    | 30 | 0    | 20 | 0  | 1030 | 0    | 1020 | 0    | 0    | 0 | C |

|      |        |      |        |      |        |        |      |        |
|------|--------|------|--------|------|--------|--------|------|--------|
| 1000 | 2-1000 | 10   | 2      | -10  | 2000   | 2-2000 | 1010 | 2-1010 |
| 2000 | 1-2000 | 2000 | 3-2000 | 1010 | 3-1010 |        |      |        |

2

|    |             |    |             |      |             |      |             |
|----|-------------|----|-------------|------|-------------|------|-------------|
| 20 | 2.166666667 | 30 | 2.166666667 | 1020 | 2.166666667 | 1030 | 2.166666667 |
| 40 | 2.041666667 | 50 | 2.041666667 | 1040 | 2.041666667 | 1050 | 2.041666667 |

1

0.0

1.0

1

INCREMENT

## STARTING PROBLEM

B0PAGE 3-D CHECK (VARIABLE STRESS-STRAIN, WITH ZERO POISSONS RATIO)

02/10/75

SOLUTION METHOD CODE = 5  
 MAXIMUM NO. STIFFNESS UPDATES PER INCREMENT = 3  
 MAXIMUM TOTAL ITERATIONS PER INCREMENT = 10  
 MAXIMUM ELASTIC ITERATIONS PER INCREMENT = 2  
 MAXIMUM MAGNITUDE FOR ELASTIC-PLASTIC SUM CODE = 2  
 MAXIMUM REDUCTIONS = 1  
 CONVERGENCE REDUCTION FACTOR = 0.50000E-00  
 MAXIMUM SPECIFIED ERROR NORM = 0.10000E-04  
 FRACTION FROM END OF INCREMENT TO EVALUATE SLOPE = 0.10000E 00

NO. OF MATERIALS = 1  
 FABRICATION TEMPERATURE = 0.10000E 01

## MATERIAL 01. 1 TEMPERATURE DEPENDENT PROPERTIES

TEMPERATURE THERMAL STRAIN  
 0.1000E 01 0.0  
 0.2000E 01 0.3000E 00  
 0.3000E 01 0.3000E 00  
 0.4000E 01 0.1050E 01  
 0.4500E 01 0.1500E 01  
 0.5000E 01 0.1800E 01  
 0.6000E 01 0.2300E 01  
 0.7000E 01 0.2300E 01  
 0.8000E 01 0.1000E 01  
 0.9000E 01 0.5000E 00  
 0.1000E 02 0.1150E 01  
 0.1200E 02 0.2050E 01  
 0.1300E 02 0.2500E 01  
 0.1400E 02 0.2300E 01  
 0.1500E 02 0.5152E 01

TEMPERATURE ELASTIC MOD.  
 0.1000E 01 0.1000E 01  
 0.2000E 01 0.2000E 01  
 0.3000E 01 0.2500E 01  
 0.4000E 01 0.3000E 01  
 0.5000E 01 0.3500E 01  
 0.6000E 01 0.4000E 01  
 0.7000E 01 0.2000E 01  
 0.8000E 01 0.1250E 01  
 0.9000E 01 0.1250E 01  
 0.1000E 02 0.2000E 01  
 0.1200E 02 0.3875E 01  
 0.1300E 02 0.4500E 01  
 0.1400E 02 0.5250E 01  
 0.1500E 02 0.3000E 01

TEMPERATURE POISSONS RATIO  
 0.1000E 01 0.0

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MATERIAL NO. 1, PLASTICITY TYPE 1, KINEMATIC CODE 0

MATERIAL NO. 1, TEMPERATURE = 0.0

PARAMETER ISOTROPIC STRESS

0.0 0.20000E 01

0.30000E 01 0.20000E 01

0.90000E 01 0.35000E 01

PARAMETER KINEMATIC SHAPE

0.0 0.0

0.10000E 01 0.10000E 01

0.30000E 01 0.20000E 01

0.90000E 01 0.35000E 01

TEMPERATURE = 0.0

PARAMETER ISOTROPIC STRESS

0.0 0.20000E 01

0.30000E 01 0.20000E 01

0.90000E 01 0.35000E 01

PARAMETER KINEMATIC SHAPE

0.0 0.0

0.10000E 01 0.10000E 01

0.30000E 01 0.20000E 01

0.90000E 01 0.35000E 01

MATERIAL NO. 1, CREEP TYPE 2

TIME CREEP STRAIN

0.0 0.0

0.1000E 02 0.1000E 01

0.3000E 02 0.2000E 01

MATERIAL NO. 1, TEMPERATURE = 0.0

STRESS CREEP FACTOR

0.0 0.1000E 01

0.3000E 01 0.1000E 01

0.1100E 02 0.9000E 01

MATERIAL NO. 1, TEMPERATURE = 0.0

STRESS CREEP FACTOR

0.0 0.1000E 01

0.3000E 01 0.1000E 01

0.1100E 02 0.9000E 01

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# CARTESIAN COORDINATE SYSTEMS DEFINED

| NUMBER | OPTION | REFERENCE<br>COORD SYS | .....ORIGIN..... | POINT ON<br>.....X-AXIS..... | POINT IN<br>.....XY-PLANE..... |
|--------|--------|------------------------|------------------|------------------------------|--------------------------------|
| 3      | 1      | N/A                    | NODE 2000        | NODE 1010                    | NODE 1050                      |

14.3-6

| NO. | I.D. | LOGATE | X1          | X2          | X3          | DISPLACE |
|-----|------|--------|-------------|-------------|-------------|----------|
| 1   | 1000 | 0      | 0.0         | 0.0         | 0.100000 01 | 0        |
| 2   | 10   | 0      | 0.100000 01 | 0.0         | 0.100000 01 | 0        |
| 3   | 20   | 0      | 0.0         | 0.500000 00 | 0.100000 01 | 0        |
| 4   | 30   | 0      | 0.100000 01 | 0.500000 00 | 0.100000 01 | 0        |
| 5   | 40   | 0      | 0.0         | 0.100000 01 | 0.100000 01 | 0        |
| 6   | 50   | 0      | 0.100000 01 | 0.100000 01 | 0.100000 01 | 0        |
| 7   | 2000 | 0      | 0.0         | 0.0         | 0.0         | 0        |
| 8   | 1010 | 0      | 0.100000 01 | 0.0         | 0.0         | 0        |
| 9   | 1020 | 0      | 0.0         | 0.500000 00 | 0.0         | 0        |
| 10  | 1030 | 0      | 0.100000 01 | 0.500000 00 | 0.0         | 0        |
| 11  | 1040 | 0      | 0.0         | 0.100000 01 | 0.0         | 0        |
| 12  | 1050 | 0      | 0.100000 01 | 0.100000 01 | 0.0         | 0        |

| ELEMENT | NO.  | I.D. | MATL | N1 | N2 | N3 | N4   | N5   | N6   | N7   | N8          | VOLUME | 1ST. LINE | INTERMEDIATE EDGE NODES    |
|---------|------|------|------|----|----|----|------|------|------|------|-------------|--------|-----------|----------------------------|
| 1       | 1000 | 1    | 1000 | 10 | 50 | 40 | 2000 | 1010 | 1050 | 1040 | 0.10000E 01 | 0      | 30        | 0 20 0 1030 0 1020 0 0 0 0 |

SUM OF ELEMENT VOLUMES = 0.1000E 01

## SPECIFIED FORCE-DISPLACEMENT-CONSTRAINT DOF

| NODE I.D. | COMPONENT | CODE  |
|-----------|-----------|-------|
| 1030      | 2         | -1000 |
| 10        | 2         | -10   |
| 2000      | 2         | -2000 |
| 1010      | 2         | -1010 |
| 2000      | 1         | -2000 |
| 2000      | 3         | -2000 |
| 1010      | 3         | -1010 |

NO. OF LOAD REFERENCE CURVES = 2

LOAD REFERENCE CURVE NO. 1  
NODE COMPONENT LOAD

LOAD REFERENCE CURVE NO. 2  
NODE COMPONENT LOAD

ORIGINAL PAGE IS  
OF POOR QUALITY

23 2 0.16667E-00  
30 2 0.16667E 00  
1020 2 0.16667E 00  
1030 2 0.16667E-00  
40 2 0.41667E-01  
50 2 0.41667E-01  
1040 2 0.41667E-01  
1050 2 0.41667E-01

ELEMENT LOCATE INTEGRATION POINT X1 X2 X3  
I.D.

1000 0 1 2.113E-01 2.113E-01 2.113E-01  
2 2.113E-01 7.887E-01 2.113E-01  
3 7.887E-01 2.113E-01 2.113E-01  
4 7.887E-01 7.867E-01 2.113E-01  
5 2.113E-01 2.113E-01 7.887E-01  
6 2.113E-01 7.887E-01 7.887E-01  
7 7.887E-01 2.113E-01 7.887E-01  
8 7.887E-01 7.887E-01 7.887E-01

14.3-7

NO. OF LOAD INCREMENTS = 1  
INCREMENT MAX. ITERATIONS MECHANICAL CURVE FACTORS CREEP TIME  
1 10 0.0 0.10000E 01 0.0

# CUMULATIVE THERMAL LOADS FOR LOAD INCREMENT 1

INCREMENT 1

| NODE I.D. | 1000        | 10          | 20          | 30          | 40          | 50          | 2000        | 1010        | 1020        | 1030        |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| TEMP.     | 0.10000E-01 | 0.10000E-01 | 0.10000E-01 | 0.10000E-01 | 0.10000E-01 | 0.10000E-01 | 0.10000E-01 | 0.10000E-01 | 0.10000E-01 | 0.10000E-01 |

| NODE I.D. | 1040        | 1050        |
|-----------|-------------|-------------|
| TEMP.     | 0.10000E-01 | 0.10000E-01 |

RESIDUAL URM = 0.14959E-05

END OF LOAD INCREMENT 1

INCREMENT 1

MECHANICAL LOAD CURVE FACTORS = 0.0 , 0.1000E 01

CREEP TIME INCREMENT = 0.0

NO. ELASTIC INTEGRATION POINTS = 8, NO. PLASTIC INTEGRATION POINTS = 0

0 INTEGRATION POINTS HAVE CHANGED ELASTIC TO PLASTIC, 0 INTEGRATION POINTS PLASTIC TO ELASTIC DURING THIS INCREMENT

SPECIFIED MAX. NO. STIFFNESS UPDATES = 3, NO. UPDATES PERFORMED = 0

SPECIFIED MAX. NO. ITERATIONS PER UPDATE = 10, NO. ITERATIONS PERFORMED SINCE LAST UPDATE = 1

SPECIFIED MAX. UNBALANCED FORCE ERROR = 0.1000E-04, ACTUAL ERROR = 0.1456E-05

## CUMULATIVE INTERNAL FORCES AND DISPLACEMENTS

| ** NODE ** | ** ** ** ** ** | FORCES         |                |                | DISPLACEMENTS  |                |  |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|--|
| NO. I.D.   | U              | V              | W              | U              | V              | W              |  |
| 1 1000     | 0.1194097E-07  | -0.2083329E 00 | -0.1279129E-07 | -0.5726237E-06 | 0.0            | -0.4165955E-07 |  |
| 2 10       | 0.1364723E-07  | -0.2083330E 00 | -0.3659454E-07 | -0.3164763E-06 | 0.0            | 0.1548393E-06  |  |
| 3 20       | -0.2973750E-08 | -0.1666662E 00 | -0.2958933E-07 | -0.6880555E-06 | -0.3749987E 00 | -0.2607703E-06 |  |
| 4 30       | -0.6744347E-07 | 0.1666663E 00  | 0.4372009E-07  | -0.8433662E-06 | 0.3749998E 00  | 0.4582107E-06  |  |
| 5 40       | -0.1083656E-07 | 0.4166665E-01  | -0.3658369E-07 | -0.1522758E-05 | 0.4999984E 00  | 0.8344650E-06  |  |
| 6 50       | -0.4253494E-07 | -0.4166673E-01 | -0.4109705E-07 | -0.1072884E-05 | -0.4999988E 00 | -0.9536743E-06 |  |
| 7 2000     | 0.4990113E-07  | -0.2083330E 00 | -0.2182122E-07 | 0.0            | 0.0            | 0.0            |  |
| 8 1010     | 0.1269245E-07  | -0.2033332E 00 | -0.6349069E-07 | -0.3526579E-06 | 0.0            | 0.0            |  |
| 9 1020     | -0.8071872E-08 | -0.1666663E 00 | -0.6329628E-07 | -0.5236750E-06 | -0.3749998E 00 | -0.2980232E-06 |  |
| 10 1030    | -0.2412356E-07 | 0.1666663E 00  | 0.8700215E-07  | -0.5103648E-06 | 0.3750007E 00  | 0.5364418E-06  |  |
| 11 1040    | -0.3052051E-07 | 0.4166676E-01  | -0.8558839E-08 | -0.1250934E-05 | 0.4999998E 00  | 0.9536743E-06  |  |
| 12 1050    | -0.2890356E-08 | -0.4166685E-01 | -0.2670725E-08 | -0.1311302E-05 | -0.5000014E 00 | -0.1136745E-06 |  |



# CUMULATIVE STRESS QUANTITIES

| ELEMENT NO. | I.D. | INTEGR. POINT | EFFECTIVE STRESS CENTER | *****CUMULATIVE STRESS CENTER***** |            |             |             |             |             |
|-------------|------|---------------|-------------------------|------------------------------------|------------|-------------|-------------|-------------|-------------|
|             |      |               |                         | XX                                 | YY         | ZZ          | XY          | XZ          | YZ          |
| 1           | 1000 | 1             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0         | 0.0         | 0.0         |
|             |      | 2             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0         | 0.0         | 0.0         |
|             |      | 3             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0         | 0.0         | 0.0         |
|             |      | 4             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0         | 0.0         | 0.0         |
|             |      | 5             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0         | 0.0         | 0.0         |
|             |      | 6             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0         | 0.0         | 0.0         |
|             |      | 7             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0         | 0.0         | 0.0         |
|             |      | 8             | 0.0                     | 0.0                                | 0.0        | 0.0         | 0.0         | 0.0         | 0.0         |
|             |      |               |                         | *****CUMULATIVE STRESSES*****      |            |             |             |             |             |
|             |      | INTEGR. POINT | EFFECTIVE STRESS        | XX                                 | YY         | ZZ          | XY          | XZ          | YZ          |
|             |      | 1             | 0.7807E 00              | -0.1214E-06                        | 0.7887E 00 | -0.1463E-07 | -0.1599E-06 | -0.8736E-07 | 0.7760E-07  |
|             |      | 2             | 0.2113E 00              | 0.3286E-07                         | 0.2113E 00 | -0.9045E-07 | -0.9415E-07 | -0.3862E-07 | -0.1978E-07 |
|             |      | 3             | 0.7887E 00              | -0.1214E-06                        | 0.7887E 00 | 0.2533E-07  | -0.3660E-07 | -0.5379E-07 | 0.2289E-06  |
|             |      | 4             | 0.2113E 00              | 0.3286E-07                         | 0.2113E 00 | -0.1353E-06 | -0.6311E-07 | 0.4744E-07  | -0.7694E-07 |
|             |      | 5             | 0.7887E-00              | -0.6427E-07                        | 0.7887E-00 | -0.1463E-07 | 0.2164E-07  | -0.6238E-07 | -0.5294E-07 |
|             |      | 6             | 0.2113E 00              | 0.5050E-07                         | 0.2113E 00 | -0.9045E-07 | -0.8196E-07 | 0.1620E-07  | -0.7094E-07 |
|             |      | 7             | 0.7887E 00              | -0.6427E-07                        | 0.7887E 00 | 0.2533E-07  | -0.1547E-06 | -0.3380E-07 | 0.1194E-06  |
|             |      | 8             | 0.2113E 00              | 0.5050E-07                         | 0.2113E 00 | -0.1353E-06 | -0.2052E-06 | 0.2502E-07  | -0.1281E-06 |

# PLASTIC AND CREEP STRAINS

ELEMENT NO. = 1 ID = 1000

| **** EFFECTIVE PLASTIC STRAINS **** |      |             |             |            |             |           |            |             |           |            |     |
|-------------------------------------|------|-------------|-------------|------------|-------------|-----------|------------|-------------|-----------|------------|-----|
| PNT                                 | CODE | TEMPERATURE | TEMPERATURE | YIELD SIZE | INCREMENTAL | SUM INCR. | CUMULATIVE | INCREMENTAL | SUM INCR. | CUMULATIVE |     |
| 1                                   | 0    | -1          | -0.5960E-07 | 0.1000E 01 | 0.2000E 01  | 0.0       | 0.0        | 0.0         | 0.0       | 0.0        | 0.0 |
| 2                                   | 0    | -1          | -0.5960E-07 | 0.1000E 01 | 0.2000E 01  | 0.0       | 0.0        | 0.0         | 0.0       | 0.0        | 0.0 |
| 3                                   | 0    | -1          | -0.5960E-07 | 0.1000E 01 | 0.2000E 01  | 0.0       | 0.0        | 0.0         | 0.0       | 0.0        | 0.0 |
| 4                                   | 0    | -1          | -0.5960E-07 | 0.1000E 01 | 0.2000E 01  | 0.0       | 0.0        | 0.0         | 0.0       | 0.0        | 0.0 |
| 5                                   | 0    | -1          | -0.5960E-07 | 0.1000E 01 | 0.2000E 01  | 0.0       | 0.0        | 0.0         | 0.0       | 0.0        | 0.0 |
| 6                                   | 0    | -1          | -0.5960E-07 | 0.1000E 01 | 0.2000E 01  | 0.0       | 0.0        | 0.0         | 0.0       | 0.0        | 0.0 |
| 7                                   | 0    | -1          | -0.5960E-07 | 0.1000E 01 | 0.2000E 01  | 0.0       | 0.0        | 0.0         | 0.0       | 0.0        | 0.0 |
| 8                                   | 0    | -1          | -0.5960E-07 | 0.1000E 01 | 0.2000E 01  | 0.0       | 0.0        | 0.0         | 0.0       | 0.0        | 0.0 |



BOPACE 3-D

PART III. PROGRAMMER MANUAL

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## 15.0 BASIC CONTENTS

The following sections in this manual describe the BOPACE program from a programming viewpoint. Section 16 defines all the labeled common. Except for JLB, common is used for the storage of permanent type data. Common JLB is a large scratch area used by many of the routines of BOPACE. In a sense, problem size capability increases as JLB increases.

Section 17 defines all the BOPACE subroutines except the main program (MAIN) and the linear equation solves routines. MAIN is discussed in section 11 and the linear equation solver routines in section 18. The linear equation solver routines cover the merging and decomposition of matrices and the solution of matrix equations.

Cross reference tables for subroutines and common are in Section 19. These tables are useful in determining how a program modification would affect the routines and common of BOPACE.

File usage is discussed in Section 20. This section covers the files required by BOPACE and where they are defined in the program.

A schematic of the overlay can be found in Section 21.

## 16.0      LABELED COMMON BLOCKS

This section contains a description of all labeled common in the BOPACE program.

### 16.1      /BRICKC/IC00(3,8), NCEDGE(2,12), NEDPOI(12)

BRICKC is used by subroutines COSHAP, EDSHAP AND KFORM to generate an elemental stiffness matrix.

- IC00(1,I)      -      coordinates  $\xi$ ,  $\eta$ ,  $\zeta$  of corner node I are in the I'th column of IC00.
- NCEDGE(1,I)      -      the end nodes for the I'th edge are stored in column I of NCEDGE.
- NEDPOI          -      an edge pointer array to the two local arrays NEDGE and XSI in subroutines COSHAP and KFORM. The I'th edge of an element has NEDGE(NEDPOI(I)) interior nodes and the coordinates for these nodes are in the NEDPOI(I) column of XSI.

### 16.2      /ELDAT0/IEDAT, IEDIN, IEDOUT, LASTEM, INTOT

ELADT0 contains the logical units where element and integration point data is stored plus the total number of elements and integration points.

- IEDAT - logical unit containing element data. The element data consists only of the data stored in common block ELDAT1.
  
- IEDIN - logical unit containing the integration point data for the current time in the problem solution. IEDIN contains only the data stored in common block ELDAT2.
  
- IEDOUT - logical unit where updated integration point data is stored. When all integration point data is updated, IEDIN and IEDOUT are swapped.
  
- LASTEM = 0. Shape functions have not been generated for the entire structure.
  
- = NEL (number of elements). The shape functions are stored on logical unit IEDIN.
  
- INTOT - total number of integration points in the structure.

16.3      /ELDAT1/NELE,IM,NELNO,ELNO(44),NIP,GCOS(9,20),COORD(3,20)

Element data for the current element being processed is stored in ELDAT1.

NELE            -    external ID of element.

IM              -    material number referenced by element NELE.

NELNO           -    NELNO mod 100 gives number of nodes for element  
NELE(including zero edge nodes).

NELNO divided by 100 gives a location in common  
block STOR where element nodes are stored before  
being transferred to ELDAT1. Subroutine BIGS does  
this only for a cold start.

ELNO           -    element definition nodes (including zero edge  
nodes) for element NELE.

NIP            -    number of integration points for element NELE.

GCOS(1,I)      -    direction cosines for element definition node I  
are in column I of GCOS.

COORD(1,I)     -    coordinates of element definition node I are in  
column I of COORD. The coordinates are in the  
global XYZ system.

16.4 /ELDAT2/PARTAL(3,20),SHAPE(20),WEI,INT,YCODE0,YCODE1,T0,T1  
 ,SIGMA0(6),SIGMA1(6),DE(6),TBASE0,TBASE1,EBASE0  
 ,EBASE1,DETEMP,YIELD0,TIELD1,TWORK0,TWORK1,ALPHA0(5)  
 ,ALPHA1(5),DEPO(5),DEP1(5),EET(6),EPT(5),DECO(5)  
 ,DEC1(5),CWORK0,CWORK1,CBASE0,CBASE1,ECT(5),SUMTS  
 ,SUMPS,SUMCS

ELDAT2 contains integration point data for the current element and integration point being processed. Variables ending in a zero and one pertain to the beginning and end of the current load increment, respectively.

- PARTAL(1,I) - derivatives of the shape function at the I'th element definition node are stored in the I'th column of PARTAL.
- SHAPE(I) - value of the shape function at the I'th definition node.
- WEI - weighting factor for this integration point.
- INT - integration point number.
- YCODE0,YCODE1 -  $\leq 0$  integration point is elastic.  
 $> 0$  integration point is plastic.
- T0,T1 - temperature at integration point INT.

- SIGMA0,SIGMA1 - stress values  $\sigma_{XX}$ ,  $\sigma_{YY}$ ,  $\sigma_{ZZ}$ ,  $\sigma_{XY}$ ,  $\sigma_{XZ}$  and  $\sigma_{YZ}$ .
- TBASE0,TBASE1 - abscissa  $\kappa$  to determine isotropic hardening.
- EBASE0,EBASE1 - abscissa  $\kappa^K$  to determine kinematic hardening.
- DETEMP - thermal strain change due to changing the temperature from T0 to T1.
- YIELD0,YIELD1 - uniaxial yield stress (yield surface size).
- TWORK0,TWORK1 - cumulative plastic work density.
- ALPHA0,ALPHA1 - stress center values  $\sigma_{XX}$ ,  $\sigma_{YY}$ ,  $\sigma_{XY}$ ,  $\sigma_{XZ}$  and  $\sigma_{YZ}$ .
- DEPO,DEP1 - incremental plastic strains  $\Delta\epsilon_{XX}^P$ ,  $\Delta\epsilon_{YY}^P$ ,  $\Delta\epsilon_{XY}^P$ ,  $\Delta\epsilon_{XZ}^P$  and  $\Delta\epsilon_{YZ}^P$ .
- EET - cumulative elastic strains  $\epsilon_{XX}^e$ ,  $\epsilon_{YY}^e$ ,  $\epsilon_{ZZ}^e$ ,  $\epsilon_{XY}^e$ ,  $\epsilon_{XZ}^e$  and  $\epsilon_{YZ}^e$ .
- EPT - cumulative plastic strains  $\epsilon_{XX}^P$ ,  $\epsilon_{YY}^P$ ,  $\epsilon_{XY}^P$ ,  $\epsilon_{XZ}^P$  and  $\epsilon_{YZ}^P$ .
- DECO,DEC1 - incremental creep strains  $\Delta\epsilon_{XX}^C$ ,  $\Delta\epsilon_{YY}^C$ ,  $\Delta\epsilon_{XY}^C$ ,  $\Delta\epsilon_{XZ}^C$  and  $\Delta\epsilon_{YZ}^C$ .

- CWORK0,CWORK1 - cumulative creep work density.
- CBASE0,CBASE1 - abscissa  $\kappa^C$  to determine creep hardening.
- ECT - cumulative creep strains  $\epsilon_{XX}^C, \epsilon_{YY}^C, \epsilon_{XY}^C, \epsilon_{XZ}^C$  and  $\epsilon_{YZ}^C$ .
- SUMTS - cumulative thermal strain from fabrication temperature.
- SUMPS - sum of increments of effective plastic strain.
- SUMCS - sum of increments of effective creep strain.

16.5 /GENC1/CTIME(60),PFACT(2,60),NITER(60)

GENC1 contains the incremental load data read by subroutine READ4.

- CTIME(I) - creep time increment for load increment I.
- PFACT(J,I) - mechanical load factor for load curve J and increment I.
- NITER(I) - maximum number of iterations for increment I.



16.6      /GENC2/CTYPE(5),NCREEP(5),CREEPX(10,5)CREEPY(10,5),CBASEX(10,5)  
                  ,NCTABX(5),NCTABY(5),CTABX(10,5),CTABY(6,5),CTAB(6,10,5)

GENC2 contains creep data read by READTC.

- CTYPE(I)      -      creep hardening type for material I. CTYPE(I) equals 1,2 or 3 for age, strain or work hardening, respectively.
- NCREEP(I)      -      number of points in reference creep curves CREEPX, CREEPY and CBASEX for material I.
- CREEPX(J,I)      -      time at point J for material I.
- CREEPY(J,I)      -      creep strain at point J for material I.
- CBASEX(J,I)      -      quantity to determine hardening at point J for material I.
- NCTABX(I)      -      number of stress values (abscissa) in table CTABX for material I.
- NCTABY(I)      -      number of temperature values (ordinate) in table CTABY for material I.
- CTABX(J,I)      -      stress value at point J for material I.

CTABY(K,I) - temperature value at point K for material I.

CTAB(K,J,I) - creep factor for temperature value at point K  
and stress value at point J for material I.

#### 16.7 /GENPO/AFACT,KTYPE(5),PTYPE(5)

The GENPO variables are stored by the subroutines READO and READTP.

AFACT - fraction from end of increment to evaluate plastic  
slope.

KTYPE(I) - kinematic hardening code for material I.  
(= 0 for 1 parameter hardening or  
= 1 for 2 parameter hardening)

PTYPE(J) - plastic hardening code for material I.  
(= 1 or 2 for strain or work hardening, respectively)

#### 16.8 /GENP7/NITABX(5)

Common blocks GENP7 through GENP17 contain temperature-plastic-hardening  
data. This data is read by subroutine READTP.

NITABX(I) - number of isotropic abscissa values in table  
ITABX for material I.

16.9 /GENP8/NKTABX(5)

NKTABX(I) - number of kinematic shape abscissa values in table KTABX for material I.

16.10 /GENP9/NFTABX(5)

NFTABX(I) - number of kinematic factor abscissa values in table FTABX for material I.

16.11 /GENP10/NTABY(5)

NTABY(I) - number of temperature ordinate values in table TABY for material I.

16.12 /GENP11/ITABX(30,5)

ITABX(J,I) - isotropic value at point J for material I.

16.13 /GENP12/KTABX(20,5)

KTABX(J,I) - kinematic shape value at point J for material I.

16.14 /GENP13/FTABX(30,5)

FTABX(J,I) - kinematic factor value at point J for material I.

16.15 /GENP14/TABY(6,5)

TABY(J,I) - temperature value at point J for material I.

16.16 /GENP15/ISTAB(6,30,5)

ISTAB(J,K,I) - isotropic stress (yield surface size) for  
temperature at point J and isotropic value  
at point K for material I.

16.17 /GENP16/KSTAB(6,20,5)

KSTAB(J,K,I) - kinematic stress shape (yield surface translation)  
for temperature at point J and kinematic shape  
value at point K for material I.

16.18 /GENP17/FSTAB(6,30,5)

FSTAB(J,K,I) - kinematic stress factor (yield surface  
translation) for temperature at point J and  
kinematic factor value at point K for material I.

16.19 /GEN1/KKODE

KKODE is a flag controlled by MAIN to direct routine KFORM to generate an elastic or plastic stiffness matrix.

KKODE - set to -1 or +1 to generate elastic or plastic matrix, respectively.

16.20 /GEN7/NEMOD(5),EMODX(20,5),EMODY(20,5),NPRAT(5),PRATX(20,5)  
,PRATY(20,5)

GEN7 contains the elastic modulus and Poissons ratio as a function of temperature for several materials. Subroutine READTM stores these functions in GEN7.

NEMOD(I) - number of points in the arrays EMODX and EMODY for material I.

EMODX(J,I) - temperature at point J for material I.

EMODY(J,I) - elastic modulus at point J for material I.

NPRAT(I) - number of points in the arrays PRATX and PRATY for material I.

PRATX(J,I) - temperature at point J for material I.

PRATY(J,I) - Poissons ratio at point J for material I.

16.21 /GEN8/NTHERM(5),THERMX(20,5),THERMY(20,5)

GEN8 contains thermal strain as a function of temperature for several materials. Subroutine READTM stores this function in GEN8.

NTHERM(I) - number of points in the arrays THERMX and THERMY for material I.

THERMX(J,I) - temperature at point J for material I.

THERMY(J,I) - thermal strain at point J for material I.

16.22 /IOUNIT/WIN1,WIN2,UOUT

The logical unit numbers for input and output of data are stored in IOUNIT. Subroutine READRS sets the values in IOUNIT.

WIN1 - logical unit for type I input data.

WIN2 - logical unit for type II input data.

UOUT - logical unit for output data.

16.23 /JLB/UNITS1,UNITS2,MBW,NSTOR,STOR(NSTOR)

JLB is a large area of core used for scratch purposes by many subroutines of BOPACE. These subroutines are BIGS,MERGE,DECOMP,SRTAPE,READ5,SOLN,MRTAPE and OUTEL.

- UNITS1 - logical unit for scratch use only.
- UNITS2 - logical unit for scratch use only
- MBW - maximum bandwidth expected during decomposition.
- NSTOR - number of words in the array STOR.
- STOR - array used for scratch purposes by many of the BOPACE subroutines.

16.24 /SIZES/NMAX1,NMAX2,NMAX3,NMAX4,NMAX5,NMAX6,NMAX7,NMAX8A,NMAX8B,NMAX8C,NMAX9,NMAX10,NMAX11,NMAX12,NMAX13,NMAX14,MNNPE

SIZES contains upper limits for the BOPACE program. SIZES is set by a block data program.

- NMAX1 - maximum number of materials.
- NMAX2 - maximum number of nodes.

- NMAX3 - maximum number of elements.
- NMAX4 - maximum node ID number.
- NMAX5 - maximum element ID number.
- NMAX6 - maximum number of points in a material property curve.
- NMAX7 - maximum number of temperature plasticity curves per material.
- NMAX8A - maximum number of isotropic curve points.
- NMAX8B - maximum number of kinematic slope points.
- NMAX8C - maximum number of kinematic factor points.
- NMAX9 - maximum number of points in creep reference curve.
- NMAX10 - maximum number of temperature creep factors per material.
- NMAX11 - maximum number of stress creep factors per material.



- NMAX12 - maximum number of special cartesian coordinate systems.
- NMAX13 - required number of mechanical load reference vectors.
- NMAX14 - maximum number of load increments.
- MNNPE - maximum number of nodes per element.

## 17.0 SUBROUTINES

All the subroutines which make up the BOPACE program, except the main program and the linear equation solver routines, are described in this section. The main program is described in Section 11.

The linear equation solver routines are described in Section 18. These routines are:

| <u>Subroutine</u> | <u>Entry Point(s)</u> |
|-------------------|-----------------------|
| BLOCK             | BLOCK,IBLOCK          |
| DECOMP            | DECOMP                |
| DELETE            | DELETE                |
| DIAG              | DIAG                  |
| DIC               | DIC                   |
| DOTHER            | DOTHER                |
| DROW              | DROW                  |
| DUMMY             | DUMMY                 |
| GENR8             | GENR8                 |
| INTER             | INTER                 |
| MERGE             | MERGE                 |
| MERSOR            | MERSOR                |
| MRTAPE            | MRTAPE                |
| RDFRWD            | RDFRWD,RDBACK,FROPEN  |
| SAVER             | SAVER,OPEN,CLOSE      |
| SEARCH            | SEARCH                |

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| <u>Subroutine</u> | <u>Entry Point(s)</u> |
|-------------------|-----------------------|
| SOLN              | SOLN                  |
| SRTAPE            | SRTAPE                |
| SUBFR             | SUBR                  |

## 17.1 BIGS

Subroutine BIGS performs the functions cold starting, restarting and checkpointing. For a cold start, BIGS reads the type I data from unit UIN and initializes some of the pertinent program variables. Element and nodal data is stored temporarily in common block JLB before it is transferred one element and one integration point at a time to disk files IEDAT and IEDIN via common blocks ELDAT1 and ELDAT2.

When restarting, BIGS reads type I data from a restart tape. Disk files IEDAT and IEDIN plus some nodal variables are also initialized from the restart start.

When checkpointing, BIGS writes type I data, a set of nodal arrays and the disk files IEDAT and IEDIN on the restart tape for load increment 0. For all other load increments only the load dependent variables on IEDIN are transferred to the checkpoint tape.

```
CALL BIGS(KODE,I1,I2,NMAT,TEMPO,NOD,NEL,NODE,KFD,PREF,TEM,P1,PP,QQ,PAR)
```

**KODE** = 1,2 or 3 for cold start, read restart tape or checkpoint, respectively.

**I1** - logical unit for restart tape or checkpoint tape.

**I2** - load increment that is to be found on the restart tape or the load increment that is to be checkpointed.

**NMAT** - number of materials.

**TEMPO** - fabrication temperature.

**NOD** - number of nodes.

**NEL** - number of elements

**NODE** - the ID for the I'th defined node is stored in **NODE(I)**.

**KFD** - the force-displacement-constraint code for the J'th freedom and I'th node is stored in **KFD(J,I)**.

**PREF** - load value on the J'th freedom and I'th node for load reference curve K is stored in **PREF(K,J,I)**.

- TEM - temperature for node I is stored in TEM(I).
- P1 - P1(J,I) contains the specified mechanical load (force or displacement) for freedom J and node I.
- PP - PP(J,I) contains the cumulative internal force for freedom J and node I.
- QQ - QQ(J,I) contains the cumulative displacement for freedom J and node I.
- PAR - PAR(J,I) contains the residual force at start of increment for freedom J and node I.

## 17.2 BLKDTA

BLKDTA is a block data program for initializing variables in common blocks BRICKC, SIZES and ELDAT0

## 17.3 COSHAP

This subroutine generates the shape function and derivatives of the shape function for the corner nodes of a brick element.

CALL COSHAP(ID,EVAL,IC00,NEDGE,XSI,NEDPOI,EDCORN,FUNC,PARTAL)

- ID = 3 for a brick element.
  
- EVAL - an array containing the local coordinates of the current integration point.
  
- IC00 - local coordinates of the I'th corner node are stored in the I'th column of IC00.
  
- NEDGE - the number of interior nodes on edge J are stored in NEDGE(J).
  
- XSI - the local edge coordinate of node I on edge J is stored in XSI(I,J).
  
- NEDPOI - an array of pointers. The I'th edge of a brick has NEDGE(NEDPOI(I)) interior nodes and coordinates in the NEDPOI(I)'th column of XSI.
  
- EDCORN - corner nodes for edge I are stored in column I of EDCORN.
  
- FUNC - FUNC(1,I) contains the value of the shape function at node I and the corresponding normalization value is stored in FUNC(2,I).

PARTAL - The values of the derivatives ( $\frac{\partial}{\partial \xi}$ ,  $\frac{\partial}{\partial \eta}$  and  $\frac{\partial}{\partial r_i}$ ) of the shape function at node I is stored in the I'th column of PARTAL.

#### 17.4 CSYS

CSYS calculates the coordinate transformation between the user's coordinates and the basic global XYZ system for each node.

CALL CSYS(GCOS,COORDA,COORD,IX,NOD,NMAX12)

GCOS - the I'th column of GCOS contains the transformation matrix for the I'th node.

COORDA - the I'th column of COORDA contains the definition for the I'th special cartesian coordinate system.

If COORDA(1,I) equals 1, then

COORDA(2,I) = node ID for origin,  
COORDA(3,I) = node ID on X-axis and  
COORDA(4,I) = node ID in X-Y plane.

If COORDA(1,I) equals 2, then

COORDA(2,I) = coordinate system ID for measuring  
coordinates

COORDA(3 to 5,I) = coordinates of origin.  
COORDA(6 to 8,I) = coordinates of point on X-axis.  
COORDA(9 to 11,I) = coordinates of point in X-Y  
plane.

- COORD - The I'th column of COORD contains the coordinates of node I in the basic cartesian system.
- IX - the coordinate system ID for node I is stored in IX(I).
- NOD - number of nodes.
- NMAX12 - maximum number of special cartesian coordinate systems.

#### 17.5 CSYS1

This subroutine calculates the coordinates in the basic cartesian system for all special cartesian systems. On input the COORDA array is as defined in subroutine CSYS. After returning from CSYS1, the COORDA array is as defined in this section.

CALL CSYS1(COORDA,COORD,NMAX12)

- COORDA - the I'th column of COORDA contains the definition for the I'th special cartesian coordinate system.  
 COORDA(1,I) = 1  
 COORDA(2 to 4,I) = coordinates in basic system for the origin.



COORDA(5 to 7,I) = coordinates in basic system  
for a point on the X-axis.

COORDA(8 to 10,I) = coordinates in basic system for  
a point in the X-Y plane.

|        |   |   |                         |
|--------|---|---|-------------------------|
| COORD  | — | } | same as subroutine CSYS |
| NMAX12 | — |   |                         |

## 17.6 CSYS2

CSYS2 calculates coordinate transformations for special cartesian systems via vector cross products.

CALL CSYS2(T,XZZ)

- T                    -    array where the coordinate transformation is to be stored for the current node being processed.
  
- XYZ                  -    array containing the coordinates in the basic cartesian system of the nodes that define the special cartesian system.

**17.7      DEFORM**

DEFORM forms the 6X6 elastic material stress-strain matrix.

CALL DEFORM(E,NU,D)

E                    -    elastic modulus

NU                   -    Poissons ratio

D                    -    material matrix

**17.8      DPFORM**

DPFORM forms the 6X6 plastic material stress-strain matrix.

CALL DPFORM(E,NU,D)

E                    -    elastic modulus

NU                   -    Poissons ratio

D                    -    material matrix

## 17.9 DYVAL

This is an incremental interpolation function routine, similar to the total interpolation function routine YVAL. It provides an incremental ordinate value equal to YVAL(NP,AX,AY,X1) minus YVAL(NP,AX,AY,X0).

DYVAL(NP,AX,AY,X0,X1)

|    |   |                               |
|----|---|-------------------------------|
| NP | - | number of points in curve     |
| AX | - | vector of abscissas           |
| AY | - | vector of ordinates           |
| X0 | - | first desired abscissa value  |
| X1 | - | second desired abscissa value |

## 17.10 EDSHAP

EDSHAP computes the shape function and the derivatives of the shape function for the edge nodes of a brick element.

CALL EDSHAP(ID,EVAL,COO,PARTAL,FUNC)

|       |   |  |
|-------|---|--|
| ID    | - | an array of indicators                               |
| ID(1) | = | 3  |
| ID(2) | = | number of edge nodes on the edge<br>being processed. |

ID(3 to 5) - coordinates of edge in  $\xi, \eta, \zeta$  system  
(a zero means the edge is parallel to  
this coordinate).

EVAL - an array containing the local coordinates of the  
current integration point.

COO - an array containing the coordinates of the nodes  
along an edge.

|        |   |                             |
|--------|---|-----------------------------|
| PARTAL | - | } same as subroutine COSHAP |
| FUNC   | - |                             |

## 17.11 ERCOMP

This routine computes the residual norm.

CALL ERCOMP(NN,KFD,DP,P,ERR,UO)

|     |   |  |
|-----|---|--|
| NN  | - | number of freedoms                                       |
| KFD | - | array containing force-displacement-constraint<br>codes. |
| DP  | - | incremental nodal force array.                           |
| P   | - | residual nodal force array.                              |

## 17.12 FORCE

Subroutine FORCE computes the incremental nodal forces from the difference in elemental stresses at the start and end of the current load increment for each integration point.

CALL FORCE(DP)

DP                    -    array containing incremental nodal forces.

## 17.13 GENER8

GENER8 is an interface subroutine between the linear equation solution routines defined in section 18.0 and the elemental stiffness matrix generation routine KFORM.

CALL GENER8(II,NUM,NODES,K,NK)

|       |   |                                  |
|-------|---|----------------------------------|
| II    | - | element number.                  |
| NUM   | - | number of nodes on element II.   |
| NODES | - | definition nodes for element II. |
| K     | - | stiffness matrix for element II. |
| NK    | - | column dimension of K.           |

## 17.14 HEAD

Subroutine HEAD writes summary type information at the end of each load increment.

```
CALL HEAD(UO,NEL,INCR,F1,F2,CTIME,TIDENT,NP,NE,NP1,NE1,NITER,NI,MAXUP
          ,NUP,ERRMAX,ERR)
```

|        |   |   |
|--------|---|---|
| UO     | - | logical unit for output.  |
| NEL    | - | number of elements.   |
| INCR   | - | load increment number.  |
| F1,F2  | - | load factors on mechanical reference curves 1 and 2, respectively.      |
| CTIME  | - | creep time increment.   |
| TIDENT | - | thermal load increment identification array.                            |
| NP     | - | number of plastic integration points.                                   |
| NE     | - | number of elastic integration points.                                   |
| NP1    | - | number of integration points that have changed from elastic to plastic. |
| NE1    | - | number of integration points that have changed from plastic to elastic. |
| NITER  | - | maximum number of iterations allowed.                                   |
| NI     | - | actual number of iterations performed.                                  |
| MAXUP  | - | maximum number of stiffness matrix updates allowed.                     |

- NUP - actual number of stiffness matrix updates performed.
- ERRMAX - maximum allowed residual norm.
- ERR - actual residual norm obtained.

#### 17.15 ITER

ITER is the major elastic-plastic-creep material-dependent routine. It takes the given array DE of incremental strains, and separates it into elastic, plastic, and creep portions. It also computes end-of-increment stresses SIGMA1, and updates many of the incremental parameters. The ITER routine used in BOPACE 3-D is based on the new "strain-space" algorithm described in Section 2.6.

CALL ITER(MAXYC,INCR)

- MAXYC - maximum allowable magnitude for the elastic-plastic sum code variable YCODE1.
- INCR - load increment number.

#### 17.16 ITER1

This routine is called from the elastic-plastic-creep routine ITER, to compute an improved value of the plastic proportionality variable LAMDA. The calculation for LAMDA uses a "linear intersection" method described in

Section 2.6. The BOPACE 3-D program would still operate if the call to ITER1 were removed, but convergence in general would be slowed due to the less accurate value for LAMDA in each iteration.

```
CALL ITER1(DEP1,ALPHA0,ALPHA1,EPSXX,EPSTY,EPSTY,EPSTY,EPSTY,EPSTY,AVE,BETXX0
          ,BETYY0,BETXY0,BETXZ0,BETYZ0,TS1,IS1,IE0,G0,G1,DC1,LAMDA)
```

- DEP1 - array of five plastic strain components.
- ALPHA0,ALPHA1 - arrays of five stress center components at start, end of increment
- EPSXX to EPSTY - five components of deviatoric strain vector (representing initial elastic strain + total strain increment, and denoted by  $e'$  in section 2.6).
- AVE - hydrostatic (average) part of EPSXX etc. variables.
- BETXX0 to BETYZ0 - five components of deviatoric strain center at start of increment.
- TS1 - yield stress corresponding to start-of-increment plastic deformation and end-of-increment temperature.
- IS1 - yield stress corresponding to end-of-increment plastic deformation and temperature.



- IE0 - yield strain quantity at start of increment.
- G0,G1 - shear modulus at start, end of increment.
- DC1 - creep strain occurring during plastic portion of increment.

## 17.17 KFORM

KFORM generates the elemental stiffness matrix in the basic cartesian system for each brick element. The stiffness matrix is a function of the stress-strain material matrix and the shape function derivatives. The material matrix is generated by calling DEFORM or DPFORM for an elastic or plastic integration point, respectively. The shape function derivatives are computed by COSHAP and EDSHAP and are saved on a disk file for only the first pass through the generation routines on a cold start. For all succeeding calls to the generation routines, the derivatives are read from a disk file.

CALL KFORM(II,NUM,NODES,K,NK)

|       |   |                  |
|-------|---|------------------|
| IT    | - | } same as GENER8 |
| NUM   | - |                  |
| NODES | - |                  |
| K     | - |                  |
| NK    | - |                  |

## 17.18 OUTC

OUTC writes the incremental and cumulative creep work and creep strains for one element on the output file.

CALL OUTC(UO,II,NELE,CWORK0,CWORK1,DEC1,ECT)

|        |   |                                |
|--------|---|--------------------------------|
| UO     | - | logical unit number for output |
| II     | - | element number (internal).     |
| NELE   | - | element number (external).     |
| CWORK0 | - | } see common ELDAT1            |
| CWORK1 | - |                                |
| DEC1   | - |                                |
| ECT    | - |                                |

## 17.19 OUTE

OUTE writes the incremental and cumulative thermal and elastic strains for one element on the output file.

CALL OUTE(UO,II,NELE,DETEMP,SUMTS,DE,EET)

|      |   |                           |
|------|---|---------------------------|
| UO   | - | } same as subroutine OUTC |
| II   | - |                           |
| NELE | - |                           |

|        |   |                     |
|--------|---|---------------------|
| DETEMP | - | } see common ELDAT1 |
| SUMTS  | - |                     |
| DE     | - |                     |
| EET    | - |                     |

## 17.20 OUTEL

OUTEL collects data by element from the integration point type data stored in common ELDAT2 for the purpose of printing on the output file. OUTEL also updates some of the data stored in ELDAT2.

CALL OUTEL(UOUT,NEL,CTIME,INCR)

|       |   |                                 |
|-------|---|---------------------------------|
| UOUT  | - | logical unit for output.        |
| NEL   | - | number of elements.             |
| CTIME | - | array of creep time increments. |
| INCR  | - | current load increment number.  |

## 17.21 OUTG

OUTG writes elastic-plastic codes, temperatures, and effective plastic and creep strains for one element on the output file.

CALL OUTG(UO,II,NELE,YCODE0,YCODE1,TO,T1,YIELD1,DEP1,SUMPS,EPT,DEC1,SUMCS,  
ECT)

|        |   |   |                          |
|--------|---|---|--------------------------|
| UO     | - | } | same as subroutine OUTC. |
| II     | - |   |                          |
| NELE   | - |   |                          |
| YCODE0 | - | } | see common ELDAT2.       |
| YCODE1 | - |   |                          |
| T0     | - |   |                          |
| T1     | - | } | see common ELDAT2.       |
| YIELD1 | - |   |                          |
| DEP1   | - |   |                          |
| SUMPS  | - |   |                          |
| EPT    | - |   |                          |
| DEC1   | - |   |                          |
| SUMCS  | - |   |                          |
| ECT    | - |   |                          |

## 17.22      OUTP

Subroutine OUTP writes incremental and cumulative plastic work and plastic strains for one element on the output file.

CALL OUTP(UO,II,NELE,TWORK0,TWORK1,DEP1,EPT)

|      |   |   |                          |
|------|---|---|--------------------------|
| UO   | - | } | same as subroutine OUTC. |
| II   | - |   |                          |
| NELE | - |   |                          |

|        |   |                      |
|--------|---|----------------------|
| TWORK0 | - | } see common ELDAT2. |
| TWORK1 | - |                      |
| DEP1   | - |                      |
| EPT    | - |                      |

## 17.23 OUTPQ

OUTPQ writes the nodal displacements and forces on the output file.

CALL OUTPQ(UO,NOD,NODE,P,Q)

|      |   |                                     |
|------|---|-------------------------------------|
| UO   | - | logical unit for output.            |
| NOD  | - | number of nodes.                    |
| NODE | - | array containing user node numbers. |
| P    | - | nodal forces.                       |
| Q    | - | nodal displacements.                |

## 17.24 OUTS

OUTS writes cumulative stress center and stress data for one element on the output file.

CALL OUTS(UO,II,NELE,ALPHA1,SIGMA1)

|        |   |                        |
|--------|---|------------------------|
| UO     | - | } see subroutine OUTC. |
| II     | - |                        |
| NELE   | - |                        |
| ALPHA1 | - | } see common ELDAT2.   |
| SIGMA1 | - |                        |

## 17.25 READC

READC reads the user's data which define special cartesian coordinate systems and stores the data in the COORDA array.

CALL READC(UI,UO,NMAX12,COORDA)

|        |   |                               |
|--------|---|-------------------------------|
| UI     | - | logical unit for input data.  |
| UO     | - | logical unit for output data. |
| NMAX12 | - | } see subroutine CSYS.        |
| COORDA | - |                               |

## 17.26 READM

Routine READM reads the user's data which define nodes and elements, transforms all coordinates to the basic XYZ system, generates coordinate transformations for each node with a call to CSYS and computes the volume of each element by calling VTET.

CALL READM(UI,UO,NMAT,NMAX2,NMAX3,NMAX4,NMAX5,NMAX12,MNNPE,NOD,NEL,COORDA,  
COORD,GCOS,IMAT,ELNO,NODE,NELE,NODI,NELI,NELNO)

|        |   |  |
|--------|---|--|
| UI     | - | logical unit for input data.   |
| UO     | - | logical unit for output data.  |
| NMAT   | - | number of material properties.   |
| NMAX2  | - | } see common SIZES.  |
| NMAX3  | - |  |
| NMAX4  | - | } see common SIZES.  |
| NMAX5  | - |  |
| NMAX12 | - |  |
| NOD    | - | number of nodes.   |
| NEL    | - | number of elements.  |
| COORDA | - | } see subroutine CSYS.   |
| COORD  | - |  |
| GCOS   | - |  |
| IMAT   | - | the material number for element I is stored IMAT(I).                               |
| ELNO   | - | an array containing the definition nodes for all elements (See NELNO description). |
| NODE   | - | the user ID (external) for the I'th defined node is stored in NODE(I).             |
| NELE   | - | the user ID (external) for the I'th defined element is stored in NELE(I).          |
| NODI   | - | the internal node ID number for external ID I is stored in NODI(I).                |

- NELI - the internal element ID number for external ID I is stored in NELI(I).
- NELNO - a packed array containing the number of nodes for each element and pointers to the ELNO array where the definition nodes are stored. For the I'th element,  $NELNO(I) = 100 * LLL + NNODES$  where LLL is the starting location in ELNO where the definition nodes are stored and NNODES is the number of nodes.

#### 17.27 READRS

READRS reads the first card on the card file unit to determine the start-restart code, the files to be used for the input and output of data, and checkpoint-restart parameters.

CALL READRS(UIN1,UIN2,UOUT,INCR,UINRS,UOUTRS)

- UIN1 - logical file number for the input of type I data.
- UIN2 - logical file number for the input of type II data.
- UOUT - logical file number for the output of data (e.g., printer).
- INCR - previous load increment number from the end of which a restart is to be made (If INCR = 0, then problem is restarted from initial conditions).



- UINRS - logical file number for the input of the restart tape.
- UOUTRS - logical file number for writing a checkpoint tape  
(If UOUTRS = 0, then BOPACE does not checkpoint).

## 17.28 READTC

READTC reads the reference creep curve data (times CREEPX and stresses CREEPY), and the creep factor data (stress levels CTABX, temperatures CTABY, and table of creep factors CTAB). Although for a given material, different abscissas CTABX may be input for each of its temperature values CTABY, READTC interpolates abscissas for all temperatures of the material to those of the first temperature input. This allows later use of an efficient table lookup procedure. Both the input and interpolated tables are output during echo check of the input data.

```
CALL READTC(UI,UO,NMAX9,NMAX10,NMAX11,IMAT,CTYPE,NCREEP,CREEPX,CREEPY,
           CBASEX,NCTABX,NCTABY,CTABX,CTABY,CTAB)
```

- UI,UO - input, output file unit numbers.
- NMAX9 - maximum number of points in a creep reference curve.
- NMAX10 - maximum number of temperatures for creep factor table.
- NMAX11 - maximum number of stresses for creep factor table.
- IMAT - material number.

|        |   |   |
|--------|---|---|
| CTYPE  | - | creep type = 1, 2, 3 for age, strain, work hardening. |
| NCREEP | - | number of points in reference creep curve.            |
| CREEPX | - | vector of time values.                                |
| CREEPY | - | vector of creep strain values.                        |
| CBASEX | - | vector of creep hardening parameter values.           |
| NCTABX | - | number of abscissa (stress) values for table.         |
| NCTABY | - | number of ordinate (temperature) values for table.    |
| CTABX  | - | abscissa values for table.                            |
| CTABY  | - | ordinate values for table.                            |
| CTAB   | - | table of creep factors.                               |

## 17.29 READTM

READTM reads the temperature-dependent material properties (thermal strain, elastic modulus, Poisson's ratio).

CALL READTM(UI,UO,NMAX6,IMAT,NTHERM,THERMX,THERMY,NEMOD,EMODX,EMODY,NPRAT,  
PRATX,PRATY)

|        |   |  |
|--------|---|--|
| UI,UO  | - | input, output file unit numbers.                       |
| NMAX6  | - | maximum number of points in a material property curve. |
| IMAT   | - | material number.                                       |
| NTHERM | - | number of values in thermal strain curves.             |
| THERMX | - | temperatures for thermal strain curves.                |

|        |   |
|--------|---|
| THERMY | - thermal strain values.                      |
| NEMOD  | - number of values in elastic modulus curves. |
| EMODX  | - temperatures for elastic modulus curves.    |
| EMODY  | - elastic modulus values.                     |
| NPRAT  | - number of values in Poisson's ratio curves. |
| PRATX  | - temperatures for Poisson's ratio curves.    |
| PRATY  | - Poisson's ratio values.                     |

### 17.30 READTP

READTP reads the plastic hardening data, in the form of tables (ISTAB,KSTAB and FSTAB) of yield surface size, and shape and factor for yield surface translation, given as a function of temperature and hardening parameters. READTP logic is similar to that in READTC.

```
CALL READTP(UI,UO,NMAX7,NMAX8A,NMAX8B,NMAX8C,IMAT,PTYPE,KTYPE,NITABX,NKTABX,
            NFTABX,NTABY,ITABX,KTABX,FTABX,TABY,ISTAB,KSTAB,FSTAB)
```

|        |   |
|--------|---|
| UI,UO  | - input, output file unit numbers.                          |
| NMAX7  | - maximum number of temperatures per material.              |
| NMAX8A | - maximum number of isotropic hardening points.             |
| NMAX8B | - maximum number of kinematic shape points.                 |
| NMAX8C | - maximum number of kinematic factor points.                |
| IMAT   | - material number.  |
| PTYPE  | - plastic hardening type = 1, 2 for strain, work hardening. |

|        |   |   |
|--------|---|---|
| KTYPE  | - | kinematic hardening code = 0, 1 for 1-parameter, 2-parameter kinematic hardening. |
| NITABX | - | number of isotropic abscissa values.  |
| NKTABX | - | number of kinematic shape abscissa values.  |
| NFTABX | - | number of kinematic factor abscissa values.                                       |
| NTABY  | - | number of temperature ordinate values.  |
| ITABX  | - | isotropic abscissa values.  |
| KTABX  | - | kinematic shape abscissa values.  |
| FTABX  | - | kinematic factor abscissa values.   |
| TABY   | - | temperature ordinate values.  |
| ISTAB  | - | table of isotropic hardening (yield surface size) values.                         |
| KSTAB  | - | table of kinematic shape values.  |
| FSTAB  | - | table of kinematic factor values.   |

### 17.31 READO

READO reads the problem identification title, and various incremental and iterative constants. For constants not read (left blank by the user) default values are assigned.

CALL READO(UI,UO,SCODE,MAXUP,MAXIT,MAXIE,MAXYC,MAXCUT,CUT,ERRMAX,AFACT)

|       |   |   |
|-------|---|---|
| UI,UO | - | input, output file unit numbers.                |
| SCODE | - | code (1 to 5) for solution of system equations. |

|        |   |   |
|--------|---|---|
| MAXUP  | - | maximum number of stiffness updates.                      |
| MAXIT  | - | maximum number of residual-force iterations.              |
| MAXIE  | - | maximum number of initial elastic iterations.             |
| MAXYC  | - | maximum value for elastic-plastic sum code.               |
| MAXCUT | - | maximum number of solution adjustments.                   |
| CUT    | - | solution adjustment fraction.                             |
| ERRMAX | - | maximum allowable residual norm.                          |
| AFACT  | - | fraction from end of increment to evaluate plastic slope. |

#### 17.32 READ1

READ1 reads basic program codes and constants.

CALL READ1(UI,UO,NMAX1,NMAT,TEMPO)

|       |   |                                  |
|-------|---|----------------------------------|
| UI,UO | - | input, output file unit numbers. |
| NMAX1 | - | maximum number of materials.     |
| NMAT  | - | number of materials.             |
| TEMPO | - | fabrication temperature.         |

#### 17.33 READ2

The specified force-displacement-constraint degrees of freedom are read by READ2.

CALL READ2(UI,UO,NMAX4,MOD,MODI,MEL,NELNO,ELNO,KFD)

|       |   |   |
|-------|---|---|
| UI,UO | - | input, output file unit numbers.                        |
| NMAX4 | - | maximum node ID number.                                 |
| NOD   | - | number of nodes.  |
| NODI  | - | array of internal node numbers.                         |
| NEL   | - | number of elements.                                     |
| NELNO | - | } see subroutine READM.                                 |
| ELNO  | - |   |
| KFD   | - | vector of force-displacement-constraint specifications. |

#### 17.34 READ3

READ3 reads the two load reference curves and places their values in the PREF array.

CALL READ3(UI,UO,NMAX4,NMAX13,NOD,NODI,KFD,PREF)

|        |   |   |
|--------|---|---|
| UI,UO  | - | input, output file unit numbers.                      |
| NMAX4  | - | maximum node ID number.                               |
| NMAX13 | - | required number of mechanical load reference vectors. |
| NOD    | - | number of nodes.                                      |
| NODI   | - | vector of internal node numbers.                      |

- KFD - vector of force-displacement-constraint specifications.
- PREF - array of load reference values.

#### 17.35 READ4

READ4 reads incremental data.

CALL READ4(UI,UO,NMAX14,MAXIT,NINCR,NITER,PFACT,CTIME)

- UI, UO - input, output file unit numbers.
- NMAX14 - maximum number of load increments.
- MAXIT - default value for maximum number of iterations.
- NINCR - number of load increments.
- NITER - maximum number of iterations for each increment.
- PFACT - mechanical load factors on reference curves for each increment.
- CTIME - creep time increment.

#### 17.36 READ5

Routine READ5 reads the thermal nodal temperatures for the current load increment and transforms these temperatures to the integration points by using the shape function for each element. If a thermal temperature is not input for anode, then it is assumed to be the same as for the last increment.

CALL READ5(UI,UO,INCR,NMAX4,NOD,NODE,NODI,NEL,TEM,TIDENT,II)

|        |   |  |
|--------|---|--|
| UI     | - | logical unit number for input.                         |
| UO     | - | logical unit number for output.                        |
| INCR   | - | load increment number.                                 |
| NMAX4  | - | maximum allowed node ID.                               |
| NOD    | - | } same as subroutine READM.                            |
| NODE   | - |  |
| NODI   | - |  |
| NEL    | - |  |
| TEM    | - | the temperature for the I'th node is stored in TEM(I). |
| TIDENT | - | title for the thermal load increment.                  |
| II     | - | element number.  |

### 17.37 ROTK

ROTK transforms an elemental stiffness matrix from the basic XYZ system to the user's global coordinate system.

CALL ROTK(NUM,GCOS,K,KDIM)

|      |   |  |
|------|---|--|
| NUM  | - | number of non-zero nodes for this element.         |
| GCOS | - | coordinate transformation matrix for this element. |
| K    | - | stiffness matrix.                                  |
| KDIM | - | column dimension of K.                             |



## 17.38 ROTQ

ROTQ transforms nodal forces and displacements for an element from the basic XYZ system to the user's global coordinate system or from the user's system to the basic system.

CALL ROTQ(NN,GCOS,KODE,Q)

|      |   |  |
|------|---|--|
| NN   | - | number of non-zero nodes for this element.                                 |
| GCOS | - | coordinate transformation matrix for this element.                         |
| KODE | - | 0 or 1 for transforming from user to basic or basic to user, respectively. |
| Q    | - | nodal forces or displacements for this element.                            |

## 17.39 STRAIN

This subroutine computes element strains from nodal displacements for integration point INT as defined in common ELDAT2.

CALL STRAIN(Q,ET)

|    |   |  |
|----|---|--|
| Q  | - | nodal displacements                        |
| ET | - | element strains for integration point INT. |

## 17.40 VTET

VTET calculates the volume of a tetrahedron assuming the edges are defined by straight lines connecting the corner nodes. If the four nodes of the tetrahedron are a, b, c and d, then the volume =  $\frac{1}{6} (\overline{ab} \times \overline{ac} \cdot \overline{ad})$ . The brick element is composed of five tetradrons, thus READM calls VTET five times to compute the volume of the brick.

CALL VTET(VOL,XYZ,N1,N2,N3,N4,IUO)

|             |   |  |
|-------------|---|--|
| VOL         | - | current volume of brick element.                                   |
| XYZ         | - | the basic XYZ coordinates of node I are in the I'th column of XYZ. |
| N1,N2,N3,N4 | - | corner node numbers of the tetrahedron.                            |
| IUO         | - | logical unit number for output.                                    |

## 17.41 YVAL

This is a linear interpolation function routine, which takes on the interpolated ordinate value.

YVAL(NP,AX,AY,X)

|    |   |                            |
|----|---|----------------------------|
| NP | - | number of points in curve. |
| AX | - | vector of abscissas        |

- AY - vector of ordinates.
- X - desired abscissa value.

#### 17.42 ZVAL

This is a linear table interpolation function routine, which is a 2-dimensional version of YVAL.

ZVAL(NTABX,NTABY,TABX,TABY,TABLE,X,Y)

- NTABX - number of X values.
- NTABY - number of Y values.
- TABX - vector of X values.
- TABY - vector of Y values.
- TABLE - 2-dimensional array of Z values.
- X,Y - desired abscissa, ordinate values.

## 18.0 LINEAR EQUATION SOLVER

### 18.1 INTRODUCTION

These routines are written in FORTRAN IV for use on IBM 360 and 370, and UNIVAC 1108 computers.

#### 18.1.1 PURPOSE

These routines are used to: 1) generate elemental matrices and merge them into the gross matrix; 2) decompose the gross matrix; 3) forward and back substitute to find the unknowns (see Section 5 of the theory document); and 4) two special routines, one to merge two gross matrices, and one to read and/or write a checkpoint tape containing the gross matrix and the decomposed matrix. The routines are separated into the above logical sections in order to give the user complete flexibility in their use based on the type of problem to be solved.

The linear equation solver routines have been written as an independent package. As such, they can be used in any program without recoding. The user must supply routines to generate the elemental matrices (details given in Section 18.2), which enhances their independence and use.

#### 18.1.2 NOMENCLATURE

NF - Number of freedoms per node

NN - Number of nodes in problem

F - Input - vector of combined known forces and displacements  
Output - vector of known and calculated (reactions) forces

D - Vector of known and calculated displacements

MBW - Maximum bandwidth

LDT - Load definition vector

### 18.1.3 METHOD

#### 18.1.3.1 MATRIX PARTITIONING

Figure 18.1-1 shows a two bar, three node structure; each bar is capable of carrying axial loads.

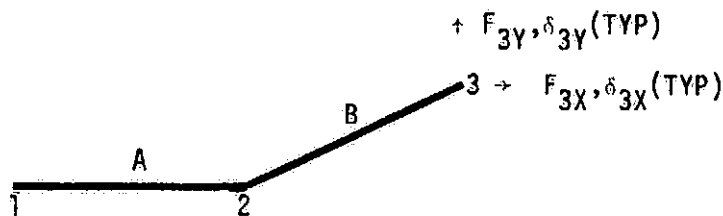


FIGURE 18.1-1: TWO BAR STRUCTURE

Bar A, using nodes 1 and 2, has the elemental matrix given in Equation (18.1-1). Note that bar A is parallel to the X axis and does not have Y freedoms.

$$\begin{Bmatrix} F_{1X} \\ F_{2X} \end{Bmatrix} = \begin{bmatrix} K & K \\ K & K \end{bmatrix} \begin{Bmatrix} \delta_{1X} \\ \delta_{2X} \end{Bmatrix} \quad (18.1-1)$$

where K represents the stiffness terms (for this discussion their value is immaterial).

Bar B, using nodes 2 and 3, has the elemental matrix given in Equation (18.1-2). Note that bar B has both X and Y freedoms.

$$\begin{Bmatrix} F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \end{Bmatrix} = \begin{bmatrix} K & K & K & K \\ K & K & K & K \\ K & K & K & K \\ K & K & K & K \end{bmatrix} \begin{Bmatrix} \delta_{2X} \\ \delta_{2Y} \\ \delta_{3X} \\ \delta_{3Y} \end{Bmatrix} \quad (18.1-2)$$

Merging these two elemental matrices yields the gross matrix

$$\begin{Bmatrix} F_{1X} \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \end{Bmatrix} = \begin{bmatrix} K & K & 0 & 0 & 0 \\ K & K & K & K & K \\ 0 & K & K & K & K \\ 0 & K & K & K & K \\ 0 & K & K & K & K \end{bmatrix} \begin{Bmatrix} \delta_{1X} \\ \delta_{2X} \\ \delta_{2Y} \\ \delta_{3X} \\ \delta_{3Y} \end{Bmatrix} \quad (18.1-3)$$

The program requires that each node has the same number of freedoms in the matrices. Therefore Equation (18.1-1) must be rewritten

$$\begin{Bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \end{Bmatrix} = \begin{bmatrix} K & 0 & K & 0 \\ 0 & 0 & 0 & 0 \\ K & 0 & K & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \delta_{1X} \\ \delta_{1Y} \\ \delta_{2X} \\ \delta_{2Y} \end{Bmatrix} \quad (18.1-4)$$

and the gross matrix becomes

$$\begin{Bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \end{Bmatrix} = \begin{bmatrix} K & 0 & K & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ K & 0 & K & K & K & K \\ 0 & 0 & K & K & K & K \\ 0 & 0 & K & K & K & K \\ 0 & 0 & K & K & K & K \end{bmatrix} \begin{Bmatrix} \delta_{1X} \\ \delta_{1Y} \\ \delta_{2X} \\ \delta_{2Y} \\ \delta_{3X} \\ \delta_{3Y} \end{Bmatrix} \quad (18.1-5)$$

The gross matrix of Equation (18.1-5) can be partitioned

$$\begin{pmatrix} \{F_{1X}\} \\ \{F_{1Y}\} \\ \{F_{2X}\} \\ \{F_{2Y}\} \\ \{F_{3X}\} \\ \{F_{3Y}\} \end{pmatrix} = \begin{bmatrix} \begin{bmatrix} K & 0 \end{bmatrix} & \begin{bmatrix} K & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \end{bmatrix} \\ \begin{bmatrix} K & 0 \end{bmatrix} & \begin{bmatrix} K & K \end{bmatrix} & \begin{bmatrix} K & K \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & \begin{bmatrix} K & K \end{bmatrix} & \begin{bmatrix} K & K \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & \begin{bmatrix} K & K \end{bmatrix} & \begin{bmatrix} K & K \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & \begin{bmatrix} K & K \end{bmatrix} & \begin{bmatrix} K & K \end{bmatrix} \end{bmatrix} \begin{pmatrix} \{\delta_{1X}\} \\ \{\delta_{1Y}\} \\ \{\delta_{2X}\} \\ \{\delta_{2Y}\} \\ \{\delta_{3X}\} \\ \{\delta_{3Y}\} \end{pmatrix} \quad (18.1-6)$$

The reason for the programs requirement that each node have the same number of freedoms becomes apparent when examining the partitions of Equation (18.1-6). Note that each partition has the same matrix order (in this case a 2X2), also that the freedoms within the partition are ordered identically. Further, the location of each partition is known by identifying the node number associated with its rows and the node number associated with its columns.

The program takes advantage of these conditions for several reasons. First, by generating the full elemental matrices (as shown in Equations (18.1-2) and (18.1-4)), and doing the partitioning on them, makes merging to form the gross matrix a simple addition of partitions having identical row/column node numbers. Second, the special equation generation process described in Section 5 of the theory manual becomes a matter of how the elemental matrices are partitioned. (And the row/column node numbers assigned.) For example, assume that freedom Y, node 3 of bar B (see Figure 18.1-1) is

constrained to be equal to freedom Y of node 4 (not shown). Then rewritten Equation (18.1-2) (with partitioning) yields

$$\begin{pmatrix} \{F_{2X}\} \\ \{F_{2Y}\} \\ \{F_{3X}\} \\ \{F_{3Y}\} \\ \{F_{4X}\} \\ \{F_{4Y}\} \end{pmatrix} = \begin{bmatrix} \begin{bmatrix} K & K \end{bmatrix} & \begin{bmatrix} K & 0 \end{bmatrix} & \begin{bmatrix} 0 & K \end{bmatrix} \\ \begin{bmatrix} K & K \end{bmatrix} & \begin{bmatrix} K & 0 \end{bmatrix} & \begin{bmatrix} 0 & K \end{bmatrix} \\ \begin{bmatrix} K & K \end{bmatrix} & \begin{bmatrix} K & 0 \end{bmatrix} & \begin{bmatrix} 0 & K \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \end{bmatrix} \\ \begin{bmatrix} K & K \end{bmatrix} & \begin{bmatrix} K & 0 \end{bmatrix} & \begin{bmatrix} 0 & K \end{bmatrix} \end{bmatrix} \begin{pmatrix} \{\delta_{2X}\} \\ \{\delta_{2Y}\} \\ \{\delta_{3X}\} \\ \{\delta_{3Y}\} \\ \{\delta_{4X}\} \\ \{\delta_{4Y}\} \end{pmatrix} \quad (18.1-7)$$

Third, the program needs to retain only those partitions which have at least one non-zero term. Further, due to symmetry of the stiffness matrix, it retains only the upper triangular partitions (those whose row node numbers are less than or equal to the column node numbers).

Fourth, and finally, the decomposition, and forward/backward substitution process becomes a matter of operating on the non-zero partitions. This in turn simplified the inherent bookkeeping necessary during these steps. It cuts down the core requirements, particularly during the decomposition for the matrix, and also for the bookkeeping arrays.

The row/column node numbers are packed into one FORTRAN integer. This partition ID number has the form

$$ID = f * I + J \quad (18.1-8)$$

where    ID = the ID number  
           f = a factor (currently f = 1001)  
           I = row node number  
           J = column node number.



The use of this ID number allows the merging routine to: 1) search only a one dimensional vector of ID numbers in order to merge the partitions, and 2) to order the partitions on the merged matrix tape (disk, drum, etc.) by low ID to high ID numbers. This means that the partitions are ordered on the tape low row node numbers to high row node numbers. And within each group of row node numbered partitions, the partitions are ordered low column node number to high column node numbers. The value of a sorted merged matrix tape is used to great advantage in calculating the maximum bandwidth (Subroutine DUMMY) and the incore decomposition (Subroutine DIC).

#### 18.1.3.2 NODF NUMBERING

These routines require that the node numbers are FORTRAN integers in the range 1 to NN, where NN is the number of nodes in the problem. This requirement allows the program to use the node number as a FORTRAN index in locating necessary data in the vectors (F, D, and LDT).

For example, assume that it is necessary to obtain the value of the  $j^{\text{th}}$  freedom of the  $i^{\text{th}}$  node of vector V. Then the following FORTRAN code can be used.

```
DIMENSION V(NF,NN), W(1)
EQUIVALENCE (V(1,1), W(1))
1  W = V(J,I)
2  W = W(NF*(I-1) + J)
```

These routines use the second form (statement 2).

### 18.1.3.3 LOAD DEFINITION VECTOR

The load definition vector contains one value for each freedom ( $NF \cdot MN$  values) in the problem. Its purpose is to describe the constraints of the freedoms. It is a FORTRAN integer vector, setup with node numbering and freedom storage as described in Section 18.1.3.2. The values assigned for each freedom must be assigned according to the following rules. (Let K equal the assigned value.)

- 1) +K, a positive value describes a freedom with a known force and the displacement is to be calculated
- 2) -K, a negative value describes a freedom with a known displacement and the reaction force is to be calculated.
- 3) The value K is a node number, and in general is the node number of the freedom, i.e.,  $LDT(J,I) = \pm I$  where J = freedom and I = node number. For the special equation generation case (Section 5 of the theory document) the value of K at node I, freedom J, becomes the connecting node to tie the freedoms together. To clarify, consider the structure shown in Figure (18.1-2), consisting of three bars, capable of carrying axial loads only, and five nodes.

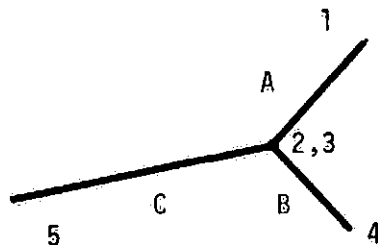


FIGURE 18.1-2: THREE BAR STRUCTURE

Bar A on nodes 1 and 2, bar B on nodes 3 and 4, and bar C on nodes 3 and 5. Further assume that the X freedom at nodes 2 and 3 is constrained to displace equally, and the Y freedoms are to be allowed to displace unequally. The load definition vector (LDT) for this problem is given in Table 18.1-1.

| NODE | FREEDOM | LDT | NODE | FREEDOM | LDT |
|------|---------|-----|------|---------|-----|
| 1    | X       | 1   | 4    | X       | -4  |
|      | Y       | 1   |      | Y       | -4  |
| 2    | X       | 2   | 5    | X       | -5  |
|      | Y       | 2   |      | Y       | 5   |
| 3    | X       | 2   |      |         |     |
|      | Y       | 3   |      |         |     |

TABLE 18.1-1: LOAD DEFINITION VECTOR

One essential point - the cross connection is on an identical freedom basis, a freedom X to a freedom X, etc., but not a freedom Y to a freedom X.

Two powerful uses of the load definition vector are available. First, in the case of known displacements, the nodes do not need to be attached, (see nodes 4 and 5 above). Second, the known forces at the connected nodes are additive (an input force at 2X and an input force at 3X are added in the above example).

#### 18.1.4 LIMITATIONS

The maximum core usage occurs during the incore decomposition. The available core is defined by the user in a named common block. One value in this common block is the length of the common block. (Details of this common

block are given in Section 18.2.1).

Let MBW = maximum bandwidth (# of nodes)

NF = number of freedoms per node

$N = ((MBW)*(MBW+1)) / (2)$

$L = (MBW) + (MBW)*(NF^2) + (4)$

C = required core (words)

Then

$C = (MBW) + (N) + (2)*(NF^2)*(N) + (6)*(NF^2) + (L)$

Currently (APR 1975) the only available decomposition routine requires in-core storage having the above storage requirement. An out-of-core or large decomposition routine was planned but not implemented (to date).

One other decomposition limitation is the maximum size of L. This maximum is hard coded in the program (see "MAXLT" in subroutine DECOMP) and is set to 1000 words.

## 18.2 USER INTERFACE

### 18.2.1 USAGE

While there are some twenty-four routines in the Linear Equation Solver package, the full capability is available via five "main" entry routines.

- 1) MERGE - to generate and merge the elemental matrices to obtain the gross matrix
- 2) DECOMP - to decompose the gross matrix
- 3) SOLN - to calculate the unknown forces and displacements
- 4) MRTAPE - to merge two gross matrices (from MERGE) using the equation

$$[K_M] = A*[K_1] + B*[K_2]$$

where

$[K_M]$  is the new merged matrix

$[K_1]$  is the first input gross matrix

$[K_2]$  is the second input gross matrix

A is an user supplied scalar

B is an user supplied scalar

- 5) SRTAPE - to write or read a checkpoint tape

Details of the calling sequences is given in Section 18.2.2 .

The gross matrix (or matrices) and the decomposed matrix are stored on an I/O unit (tape, disk, drum, etc.). The user is responsible for assignment of the FORTRAN logical units.

The core storage used is a named common block. This allows the user complete control of core storage requirements for his program. The format of the common block is

```
COMMON/JLB/KT, LT, MBW, LA, A(LA)
```

where KT and LT are the FORTRAN logical unit numbers of two I/O units to be used as scratch

MBW is the maximum bandwidth to be expected during decomposition.

This value is calculated by the Generate/Merge section or the MRTAPE routine and posted here.

LA is the length in words (the dimension) of array A.

A is the scratch storage array of length LA.

The node numbers are required to be in the range 1 to NN as described in Section 18.1.3.2.

Three arrays are used, they must be in user storage (not in common block /JLB/). The first array is the Load Definition Vector described in Section 18.1.3.3. The two remaining vectors are FORTRAN single precision real (floating point) vectors setup with node numbering and freedom storage as described in Section 18.1.3.2. These two arrays are the Force vector (F) and the Displacement vector (D). They are used only by the Forward/Backward Substitution section (Subroutine SOLN).

The Force vector (F) has dual usage; on input to SOLN, it must contain the combined known forces and displacements. On return from SOLN, it will contain only forces (known and calculated reactions).

The Displacement vector (D) is not used for input to SOLN; on return, it will contain the displacements (known and calculated).

In order to give the user complete freedom of type of structural elements to be used, the Linear Equation Solver routines require the user to supply his own elemental matrix generation routines. Interface between the Generation/Merge section and the user routines is via the user written subroutine GENER8. Details are given in Section 18.2.3.

#### 18.2.2 CALLING SEQUENCE

The calling sequence to the entry routines follows. Unless otherwise noted, all calling sequence arguments are fullword integer values or arrays.

CALL MERGE (IT,NN,NE,NF,LEN,LDT)

where

- IT = FORTRAN logical unit number of the unit for the output gross matrix.
- NN = number of nodes
- NE = number of structural elements
- NF = number of freedoms per node
- LEN = the maximum number of nodes that can be generated for any one structural element
- LDT = the Load Definition Vector

CALL DECOMP (IT,JT,NF,LDT)

where

IT = FORTRAN logical unit number of the input gross matrix  
JT = FORTRAN logical unit number of the unit for the output  
decomposed matrix  
NF = number of freedoms per node  
LDT = the Load Definition Vector

CALL SOLN (JT,NF,NN,LDT,F,D)

where

JT = FORTRAN logical unit number of the input decomposed matrix  
NF = number of freedoms per node  
NN = number of nodes  
LDT = the Load Definition Vector  
F = single precision real (floating point) array of the input  
combined known force and displacement vector, and the output  
forces  
D = single precision real (floating point) array of the output  
displacement vector.

CALL MRTAPE (IT,JT,KT,A,B,NF)

where

IT = FORTRAN logical unit number of the first input gross matrix  
JT = FORTRAN logical unit number of the second input gross matrix  
KT = FORTRAN logical unit number of the output merged matrix  
A = single precision real (floating point) value, scale factor for  
the first input matrix on unit IT



B = single precision real (floating point) value, scale factor  
for the second input matrix on unit JT  
NF = number of freedoms per node

CALL SRTAPE (IT,JT,NF,IP,IS)

where

IT = FORTRAN logical unit number of the input gross matrix or the  
input decomposed matrix  
JT = FORTRAN logical unit number of the unit for the checkpoint tape  
NF = number of freedoms per node  
IP = 1 process a gross matrix  
= 2 process a decomposed matrix  
IS = 1 write the checkpoint tape  
= 2 read the checkpoint tape

SRTAPE must be called for each matrix to be read or written. It does not position the checkpoint tape other than the inherent positioning caused by the read/write operations. The user is required to position the tape as necessary. The no positioning concept allows the user to use the tape for checkpoint of his own data.

### 18.2.3 USER ROUTINES

The user is required to supply his own elemental matrix generator routines. Interface is via a user supplied routine with the specific name GENER8, which has the specific calling sequence.

CALL GENER8 (I,N,NODES,S,NS)

where

- I = fullword integer value containing the element number  
whose elemental matrix is to be supplied (I is in the range 1  
to NE where NE is the number of elements)
- N = fullword integer value to be returned containing the number  
of nodes of the element
- NODES = fullword integer array to be returned containing the N node  
numbers of the element. (Use the first N terms of this array.)
- S = double precision real (floating point) array to be returned  
containing the elemental matrix.
- NS = dimensions of array S.

Array S is dimensioned

DOUBLE PRECISION S(NS,NS)

The elemental matrix must be a full symmetric matrix (do not return a symmetric half). The nodes and freedoms are to be ordered such that the elemental matrix partitions can be formed directly from the elemental matrix, with no sorting or inserting of rows and columns to meet the nodal partition order.

The node ordering of the matrix in array S must match the node ordering in array NODES.

Subroutine GENER8 is called once for each element in the problem. Its content, logic, and programming is left to the user.

## 18.3 PROGRAMMING

### 18.3.1 TAPE FORMATS

Two tape formats are used. The first format is used for the matrix partitions including all scratch tapes and the final gross matrix tape. The second format is for the decomposed matrix tape.

Figure 18.3-1 is the tape format for matrix partitions

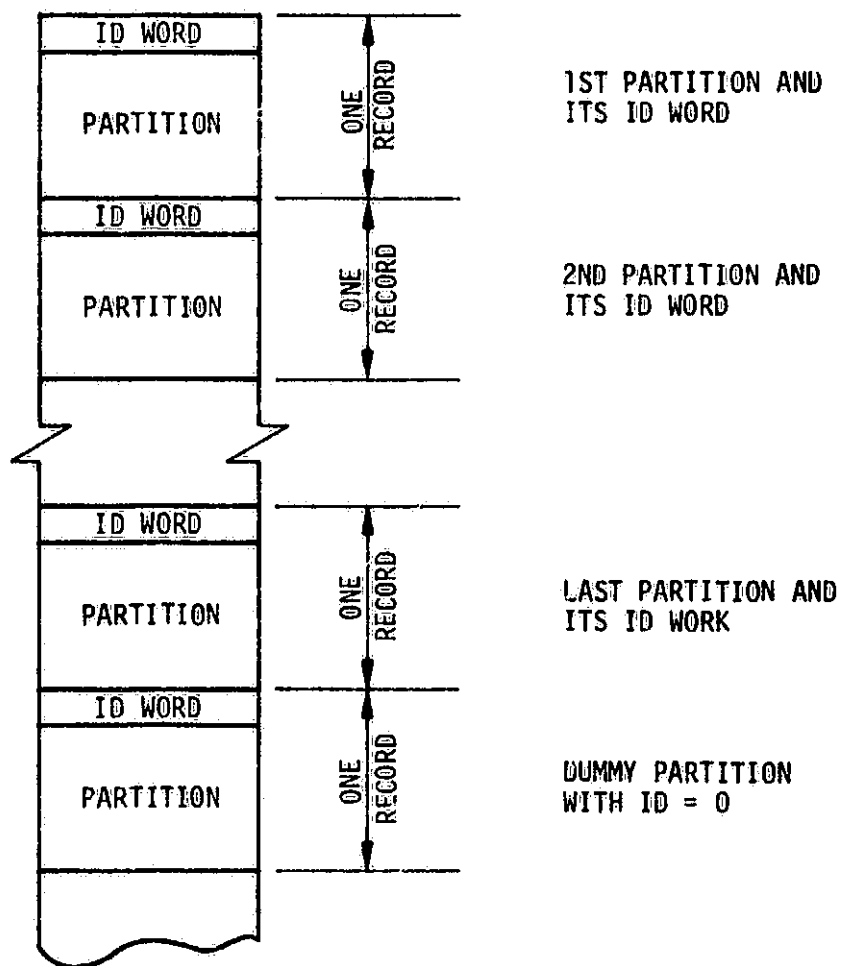


FIGURE 18.3-1: PARTITIONED MATRIX TAPE FORMAT

Let  $NF$  be the number of freedoms per node. Then each record consists of the partition's ID (row/column packed node numbers) FORTRAN integer value (one full word) followed by the  $NF^2$  double precision values of the elements of the partition. The elements of the partitions are written by rows, the  $NF$  elements of row one, followed by the  $NF$  elements of row two, etc. The physically last record on the tape must be a dummy partition with an ID number of zero.

Figure 18.3-2 shows the general tape format, and the format of one general data record. Each record on the tape, including the header and trailer records, has a FORTRAN fullword integer as its first word. This integer (NNW) is the number of words in the record.

The header and trailer records are each ten FORTRAN integer fullwords with the format shown in Figure 18.3-3.

Each data record contains one or more nodal records, each nodal record having the format shown in Figure 18.3-4.

Each nodal record contains all the data necessary from the decomposition step in order to perform the forward/backward substitution step. Each contains the data for one decomposition row. The partitions are single precision real (floating point) values, all other values are FORTRAN fullword integers. The nodes array contains the column node numbers of the partitions, with the first node also being the node number of the decomposition row.

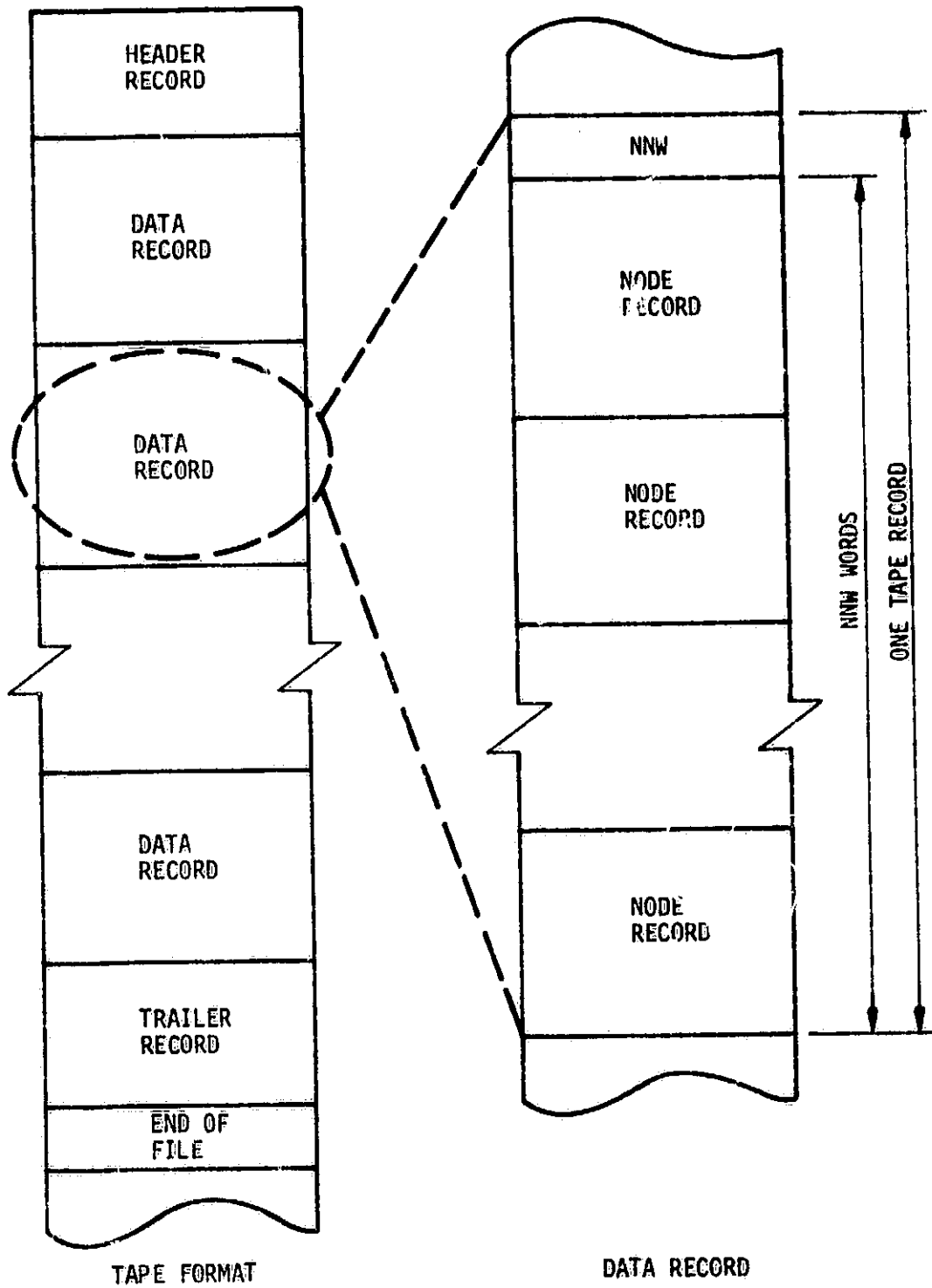
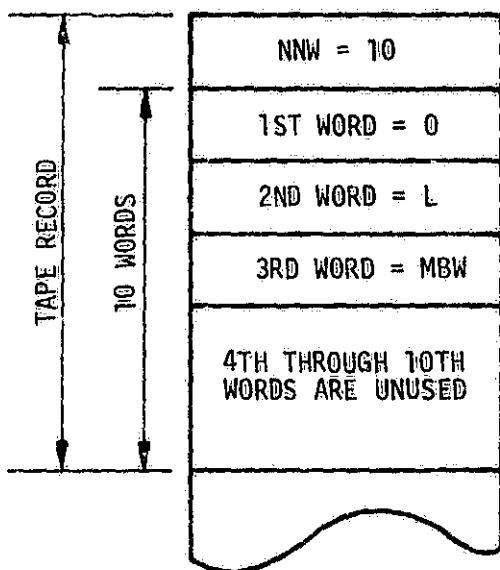


FIGURE 18.3-2: DECOMPOSITION TAPE FORMAT

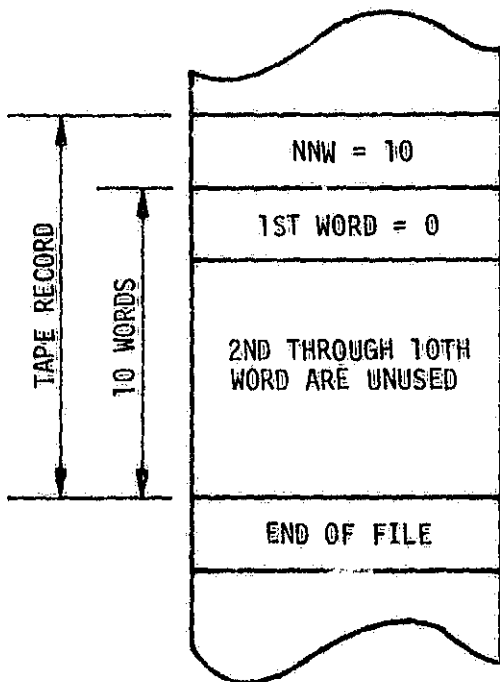
As many nodal records are placed in each data record as possible without exceeding the maximum length of a data record. Each nodal record is completely contained in its data record.



L = MAXIMUM LENGTH OF ANY ONE DATA  
RECORD (SEE MAXLT IN SUBROUTINE  
DECOMP)

MBW = MAXIMUM BANDWIDTH

HEADER RECORD



TRAILER RECORD

FIGURE 18.3-3: HEADER AND TRAILER RECORDS FORMAT

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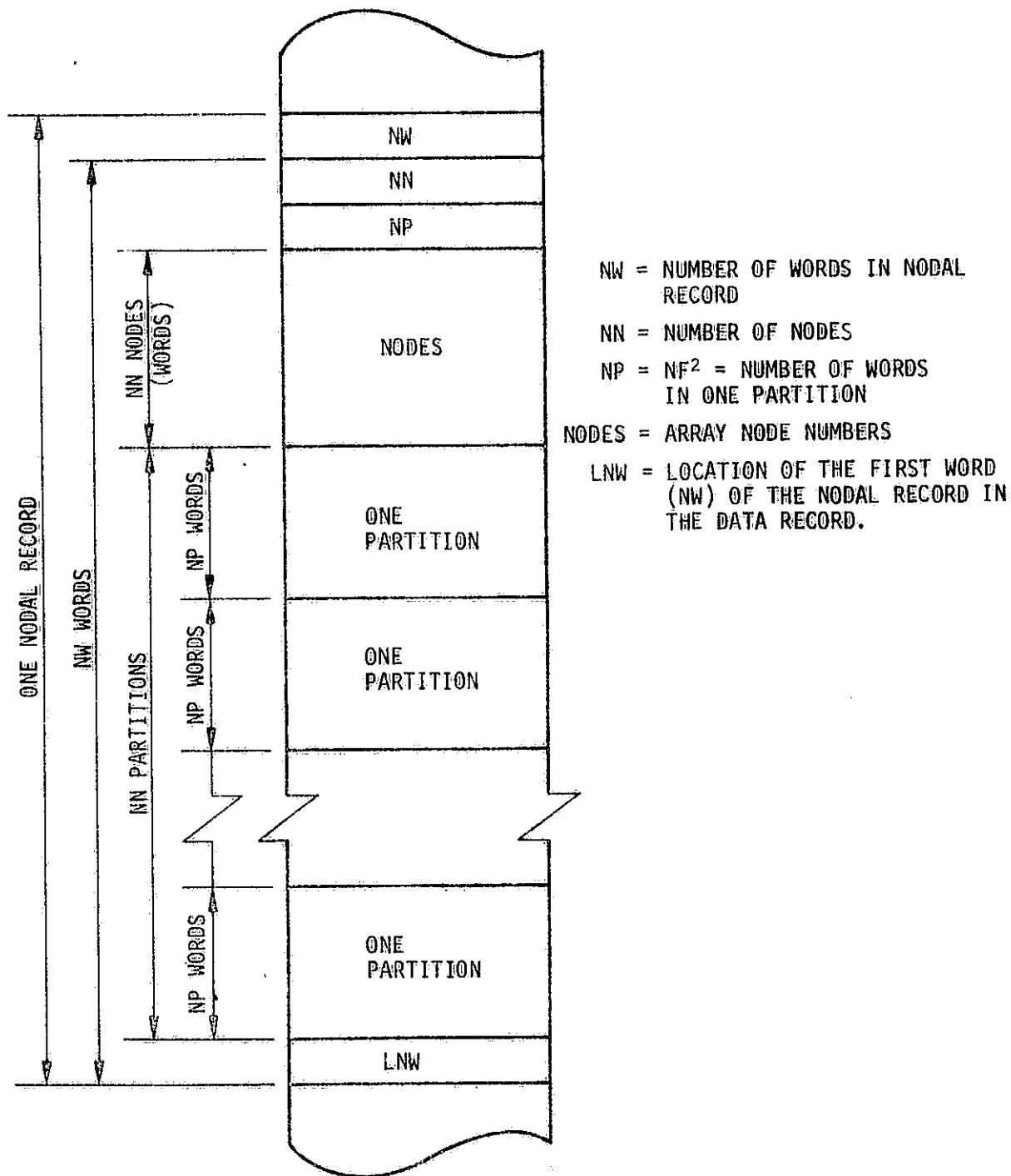


FIGURE 18.3-4: NODAL RECORD



### 18.3.2 GENERATE AND MERGE ROUTINES

The general flow of the GENERATE and MERGE routines are shown in Figure 18.3.5.

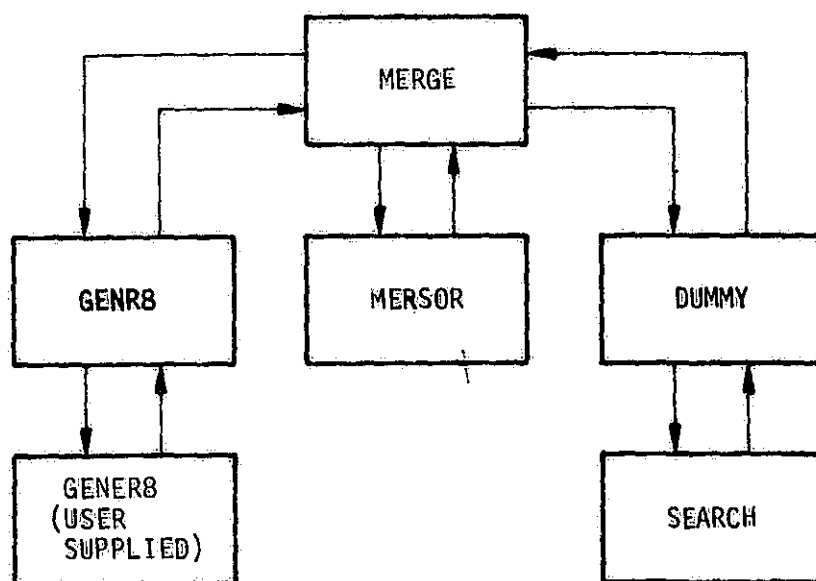


FIGURE 18.3-5: GENERAL FLOW GENERATE/MERGE ROUTINES

MERGE is the "main" entry subroutine called by the user. Its main function is to compute storage assignments for the arrays needed in the other routines. These storage assignments are in the common block /JLB/ and are passed via the calling sequence arguments.

GENR8 controls the generation of the elemental matrices. It partitions the elemental matrices, writing random ordered, unmerged partitions on a tape for later merging.

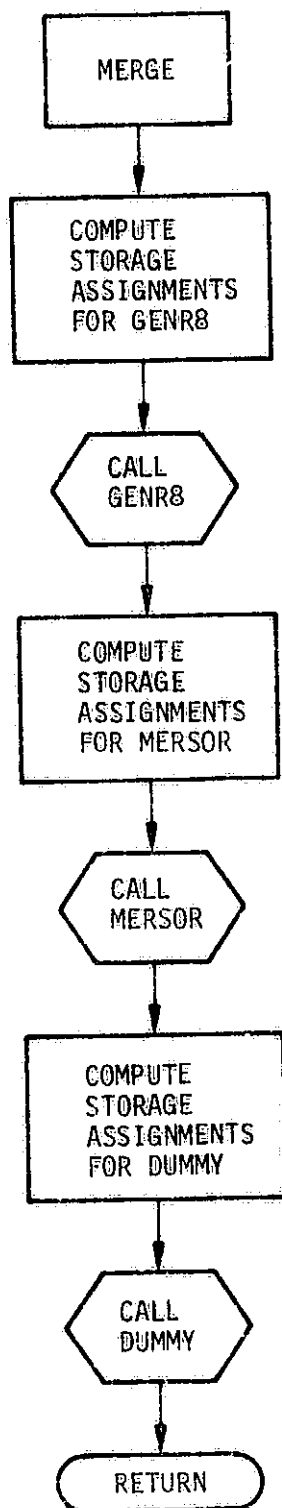
MERSOR merges the partitions generated by GENR8 and writes the sorted merged gross matrix on the gross matrix tape.

DUMMY computes the maximum bandwidth using the merged gross matrix tape from MERSOR.

SEARCH is a working routine used to locate a node number in an array of node numbers; it is also used by the decomposition step and the special routine MRTAPE.

Subroutine MERGE

Calling sequence - see Section 18.2.2.



### Subroutine GENR8

#### Calling Sequence

CALL GENR8(ITAPE,NE,NF,NSN,LDT,S,B,C,LTT,NODES,LDD)

when

ITAPE = FORTRAN logical unit number of the unit for the generated partitions

NE = number of structural elements

NF = number of freedoms per node

NSN = dimension of array S

= LEN\*NF where LEN = maximum number of nodes in any one element

(see calling sequence for MERGE, Section 18.2.2)

LDT = Load Definition vector

S = generated matrix (see calling sequence to GENER8, Section 18.2.3)

B = array for one partition

C = array for one partition

LTT = array for Load Definition vector for one element

NODES = array of elemental nodes

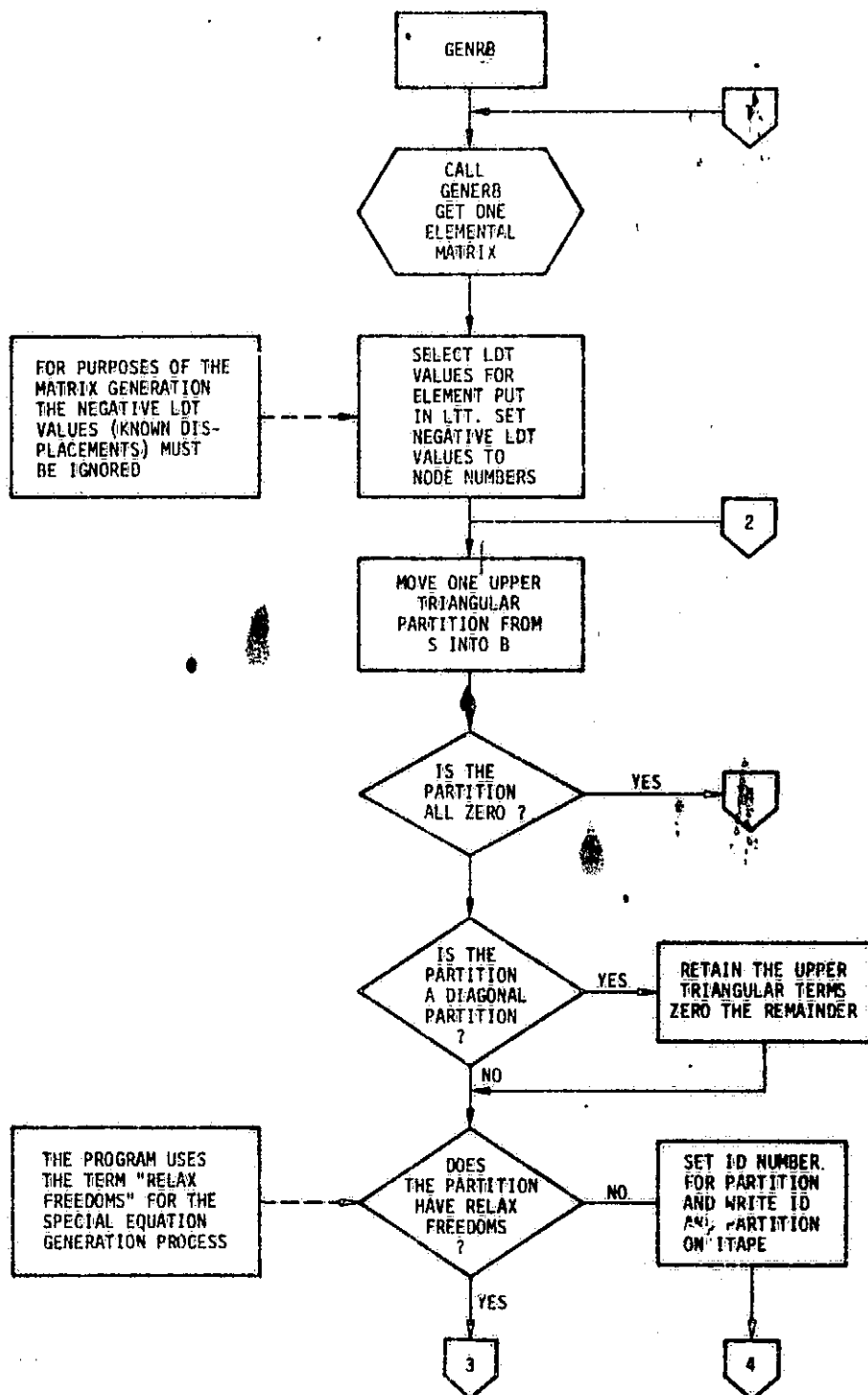
LDD = bookkeeping array

#### Array Dimensions

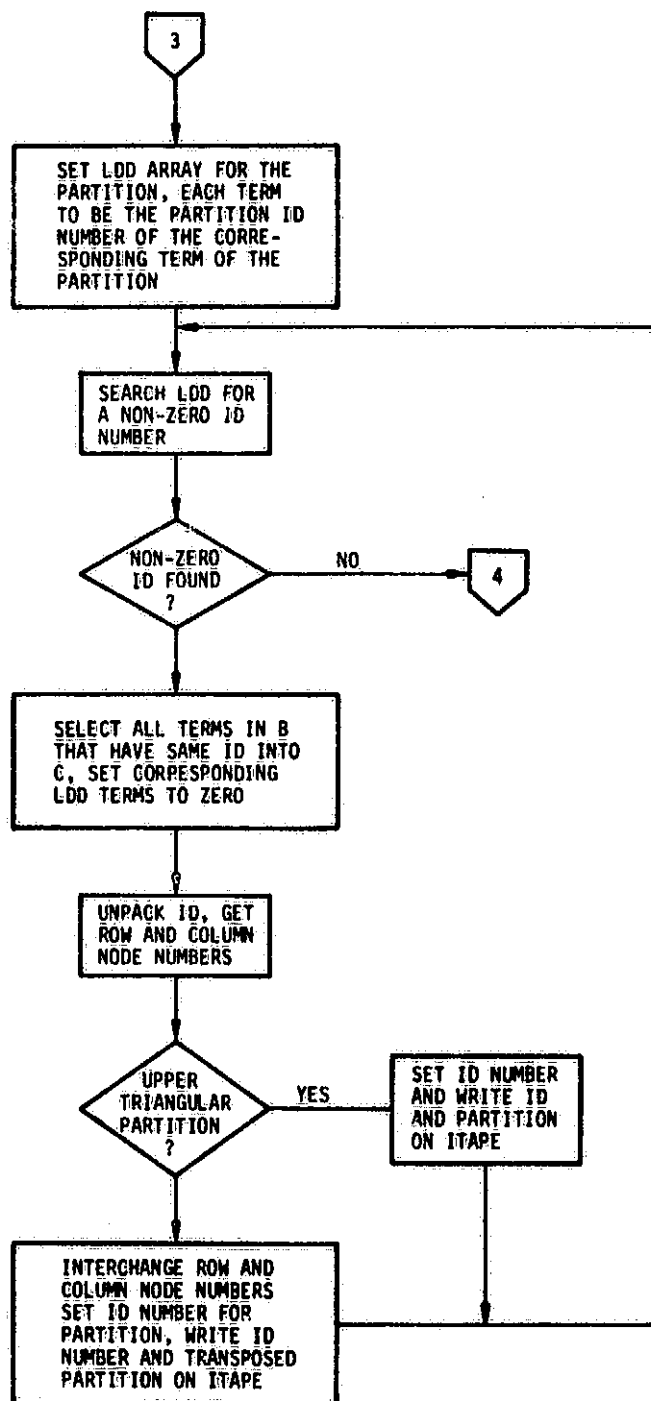
DOUBLE PRECISION S(NSN,NSN), B(NF,NF), C(NF,NF)

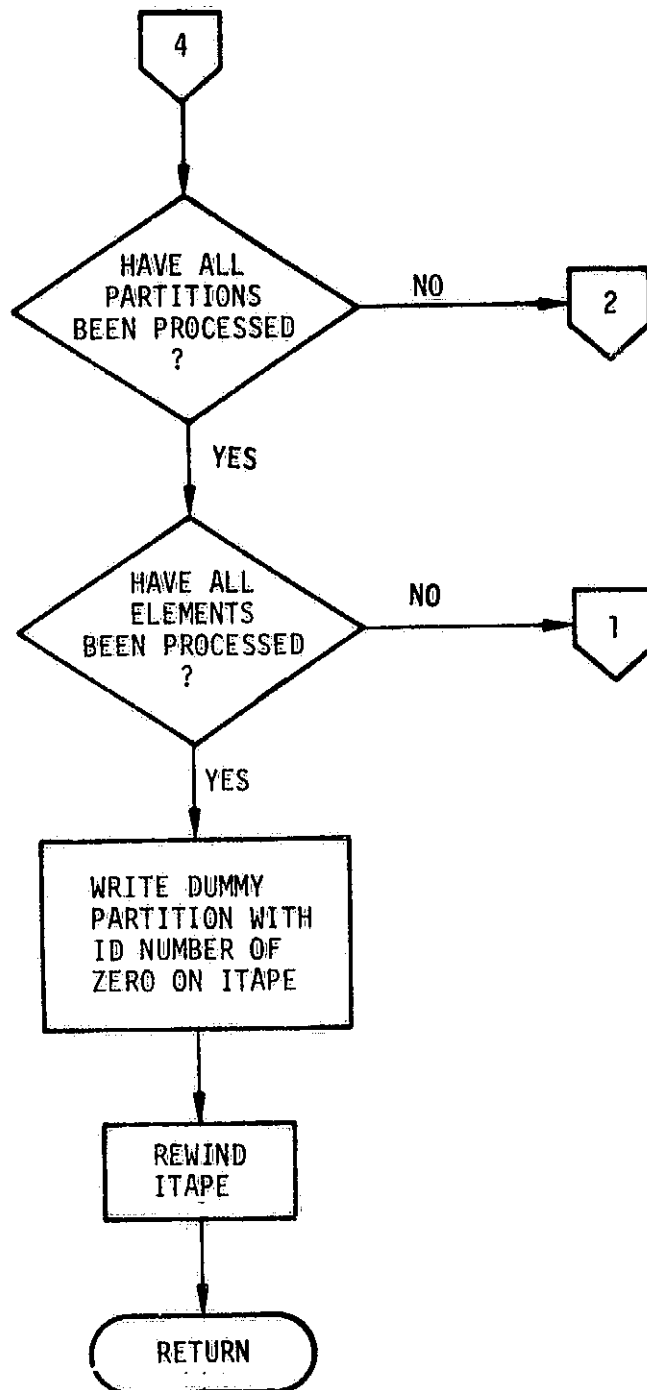
INTEGER NODES(LEN), LDT(NN\*NF), LTT(NSN), LDD(NF,NF)

where NN = number of nodes in problem



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### Subroutine MERSOR

#### Calling sequence

```
CALL MERSOR (NTAPE,KTAPE,LTAPE,NARG,MAX,DATA,TEMP,NDATA,LIST)
```

#### where

NTAPE = FORTRAN logical unit number of the input partitions to be merged  
and the output merged sorted partitions

KTAPE = FORTRAN logical unit numbers of a unit to be used as scratch

LTAPE = FORTRAN logical unit number of a unit to be used as scratch

NARG = number of words in one partition

MAX = number of partitions that can be kept incore

DATA = array for incore partition storage

TEMP = array for one partition

NDATA = array for incore partition ID numbers

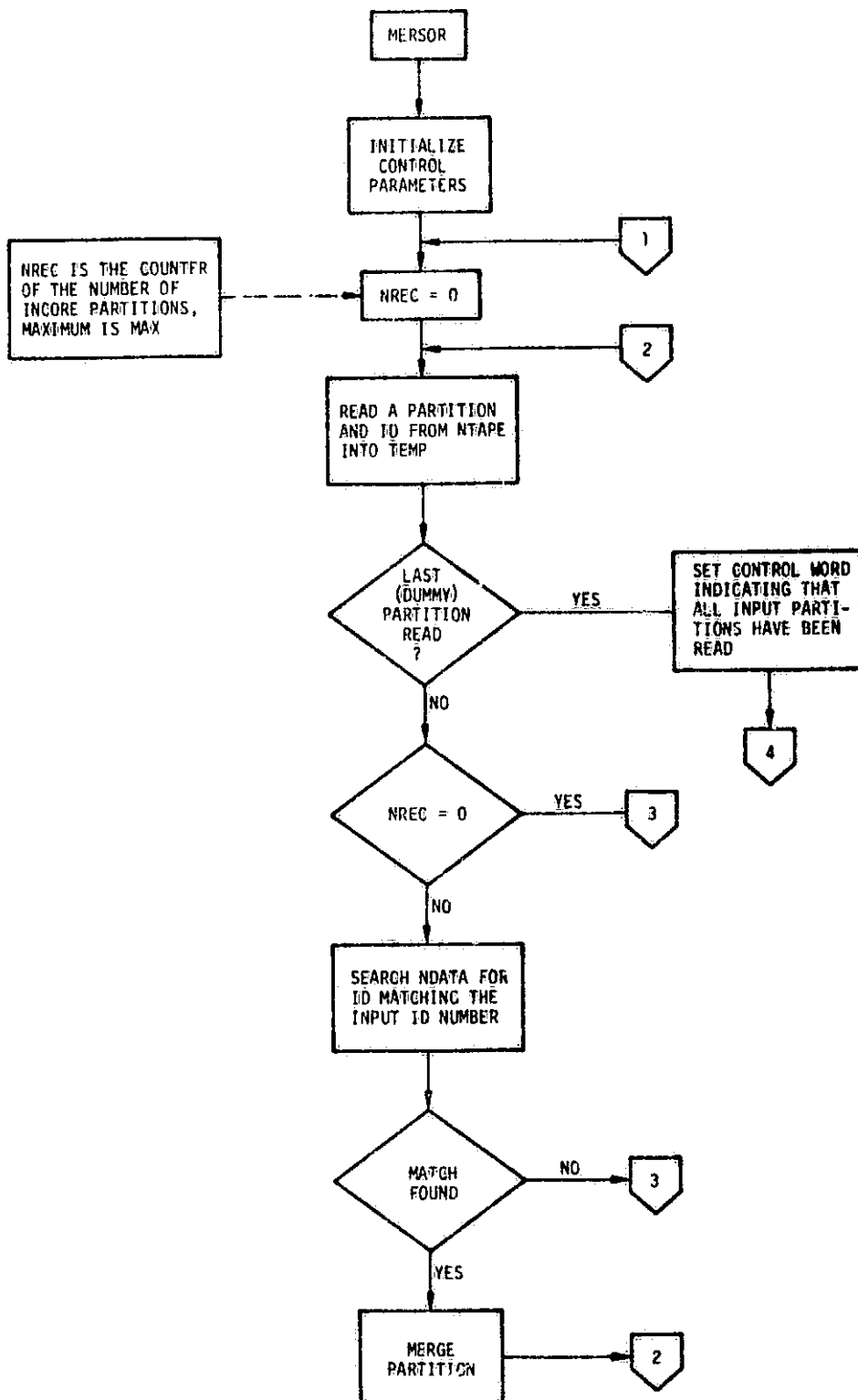
LIST = sorting array for ID numbers

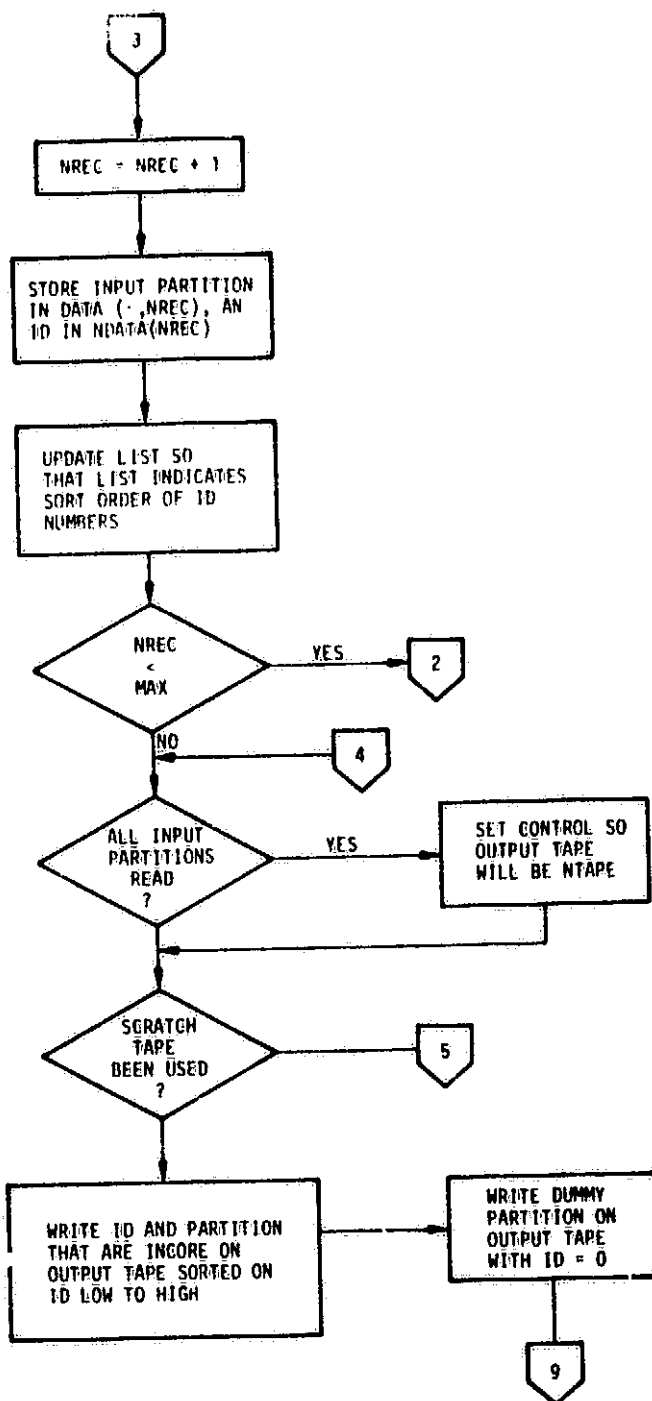
#### Array Dimensions

DOUBLE PRECISION DATA(NARG,MAX), TEMP(NARG)

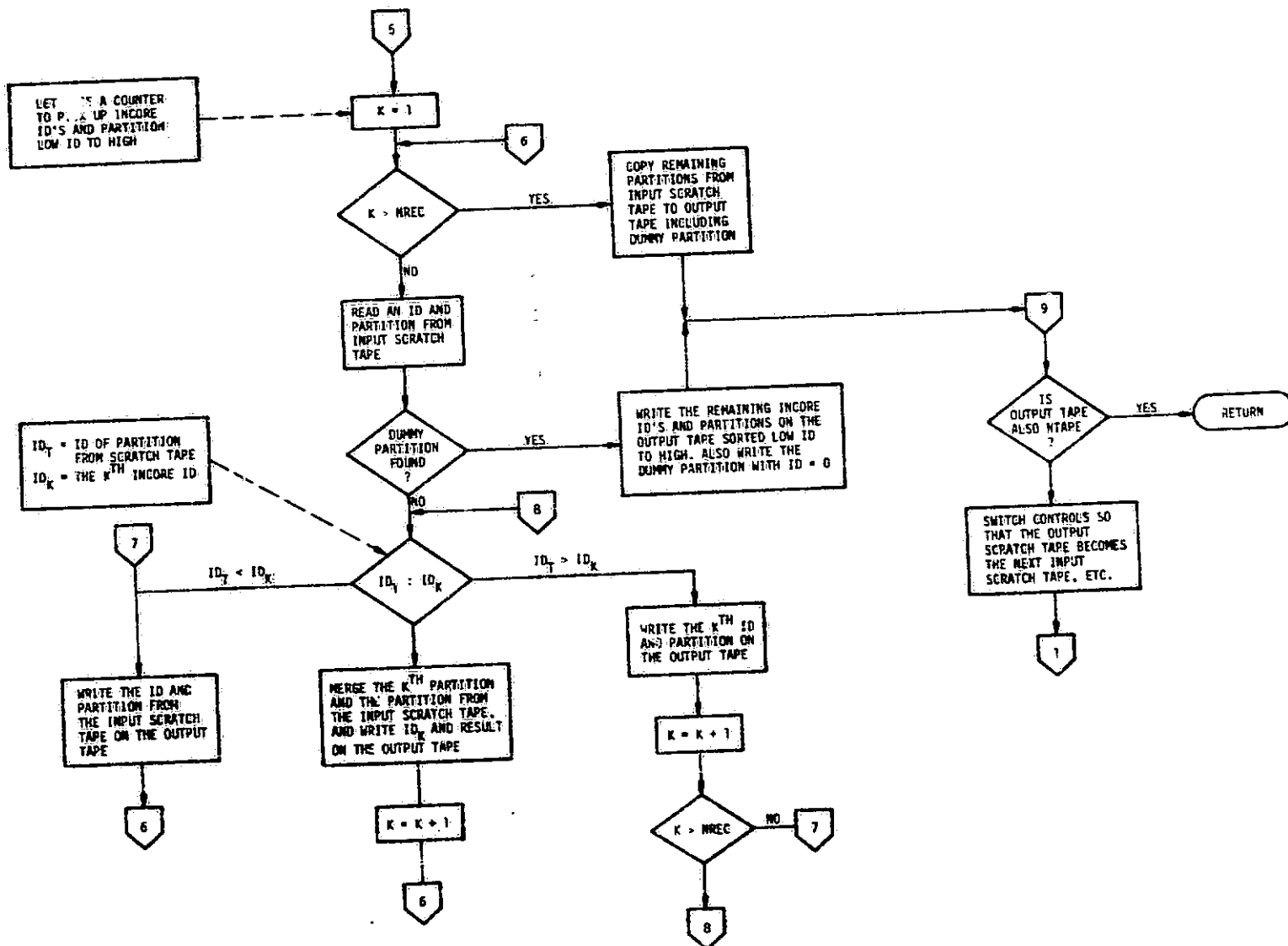
INTEGER NDATA(MAX), LIST(MAX)







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Subroutine DUMMY

CALL DUMMY (ITAPE,MBW,NFN,TEMP,MA)

where

ITAPE = FORTRAN logical unit number of the unit containing the merged  
ordered matrix

MBW = maximum bandwidth

NFN = number of words in one partition

TEMP = array for input partition

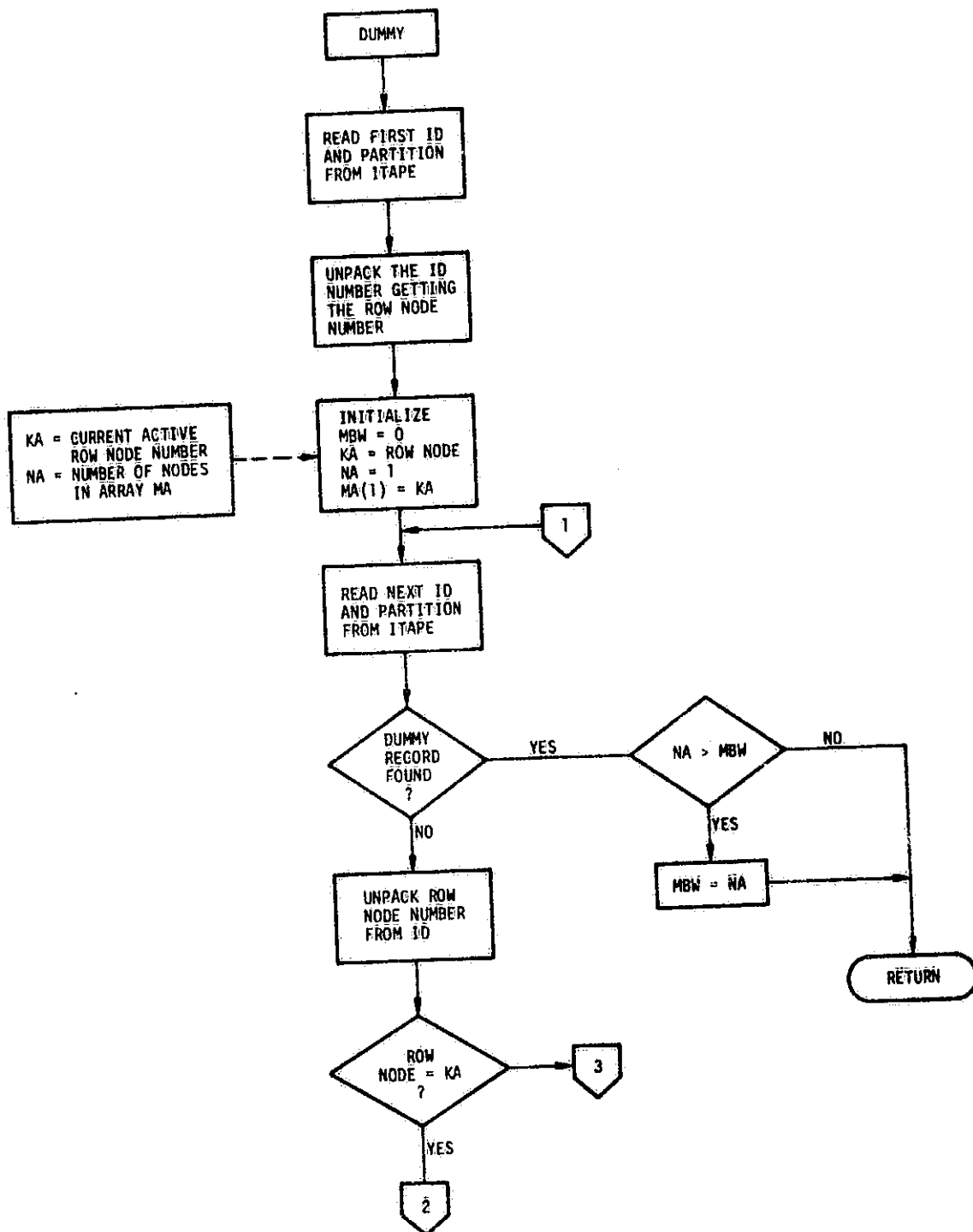
MA = array of active node numbers

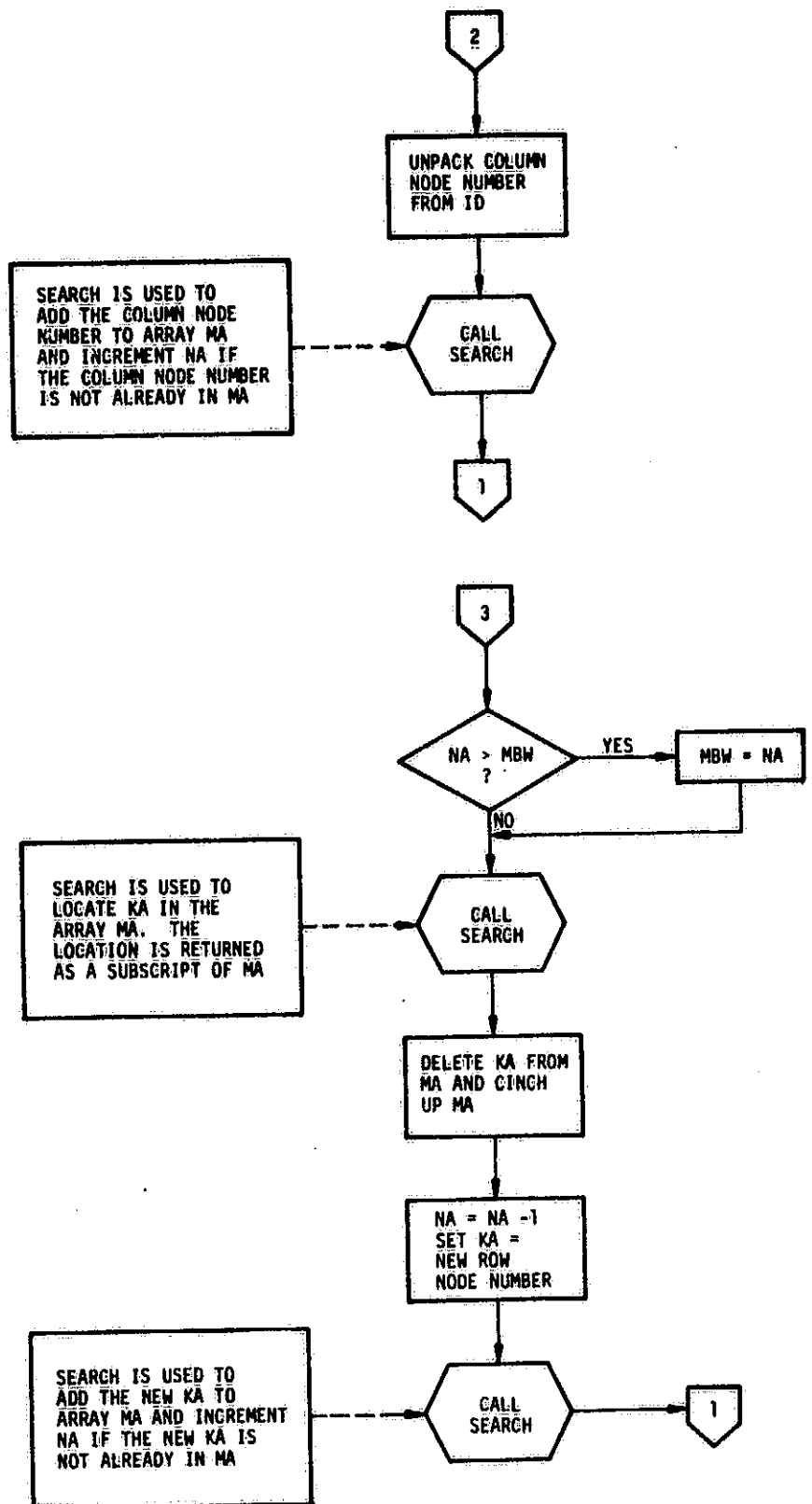
Array Dimensions

DOUBLE PRECISION TEMP(NFN)

INTEGER MS(NN)

where NN = number of nodes in problem





### Subroutine SEARCH

#### Calling Sequence

CALL SEARCH (N,K,MA,NA)

where

N is the node number to be located or added to array MA.

K is the location (subscript) of node N in array MA.

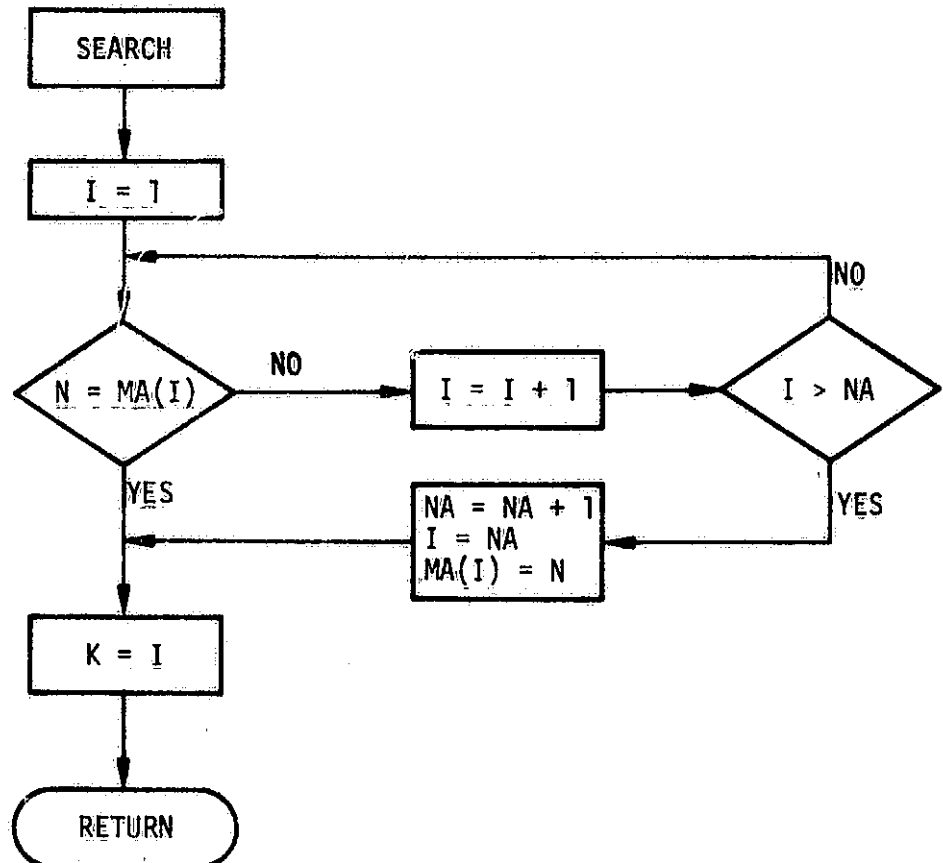
MA is an array of nodes.

NA is the number of nodes in array MA.

#### Array Dimensions

INTEGER MA (NN)

where NN is the number of nodes in the problem



### 18.3.3 DECOMPOSITION ROUTINES

The general flow of the Decomposition routine is shown on Figure 18.3-6.

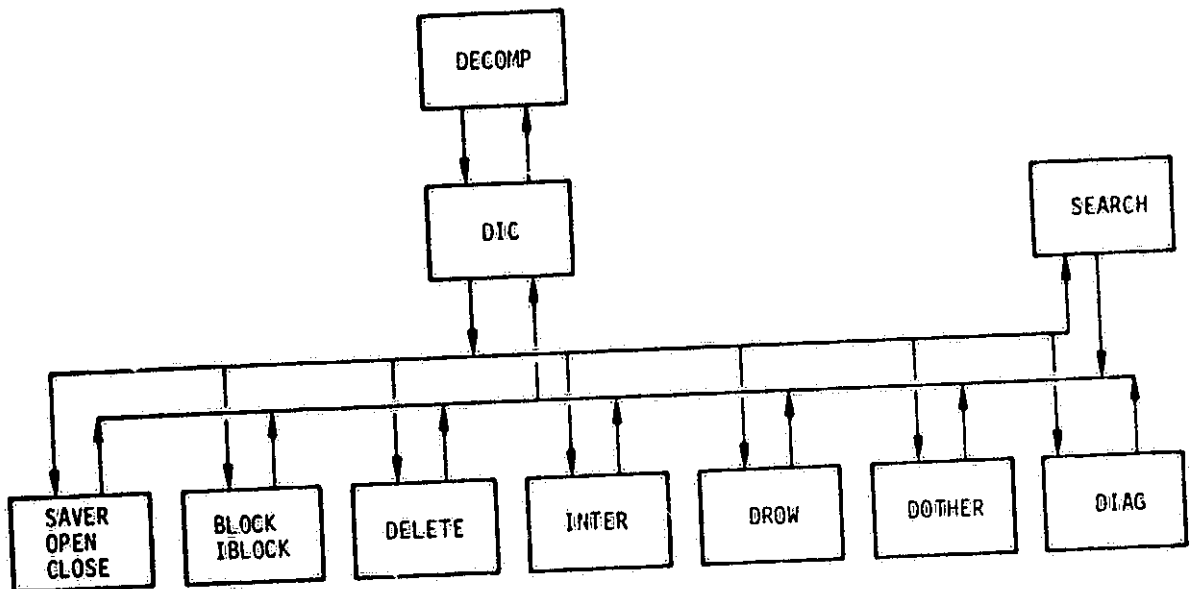


FIGURE 18.3-6: DECOMPOSITION FLOW

DECOMP is the "MAIN" entry subroutine called by the user. Its main functions are to 1) compute storage assignments for the arrays needed in the routines; and 2) to test the storage requirements against the amounts of storage available, and halt the run when insufficient storage is available. Storage assignments are in common block /JLB/ and are passed via the calling sequence arguments.

DIC is the incore decomposition routine performing the actual decomposition.

SAVER/OPEN/CLOSE is one routine with three entry points. OPEN initializes the decomposition tape and writes the header record. SAVER moves the nodal record data into the data records and writes the data records on the decomposition tape when they are full. CLOSE writes the trailer record on the decomposition tape.



BLOCK/IBLOCK is one routine with two entry points. Both operate on the partition storage usage bookkeeping array. IBLOCK initializes the array. BLOCK determines the next available partition storage block to be used.

DELETE operates on the partition storage bookkeeping array, releasing partition storage blocks for later use.

INTER operates on the partition storage bookkeeping array and the active node array, interchanging the items in these arrays so that the node of the row being decomposed is the last node in the active node bookkeeping array.

DIAG does the internal decomposition of the diagonal partition of the row being decomposed.

DROW does the decomposition of the off-diagonal partitions in the row being decomposed.

DOTHER does the decomposition of the row into the partitions not in the row.

#### STORAGE AND BOOKKEEPING CONCEPTS

The key concept used during decomposition is that the only partitions required incore are those that are active in the decomposition for the row being decomposed. The number of partitions requiring storage is

$$NBLKS = \left(\frac{1}{2}\right) (MBW)(MBW+1) \quad (18.3-1)$$

where MBW is the maximum bandwidth.

Three bookkeeping arrays are needed, an active node array, a partition row/column ID versus partition storage block number array, and a partition storage use array. In practice the storage for the last two bookkeeping arrays are combined by packing the values.

The partition storage blocks are a double dimensioned array, the first dimension for the terms in the partition, the second the partition storage block number (in the range 1 to NBLKS). There is a one to one correspondence between the partition storage block second index and the storage use bookkeeping array. The storage use array is simply a flag indicating whether or not its corresponding partition storage block is used by an active partition.

The active node bookkeeping array is used in conjunction with the partition row/column ID bookkeeping array to locate a specific partition in the partition storage blocks. This is done by thinking of the partition row/column ID bookkeeping array as an upper triangular matrix as shown in Figure 18.3-7.

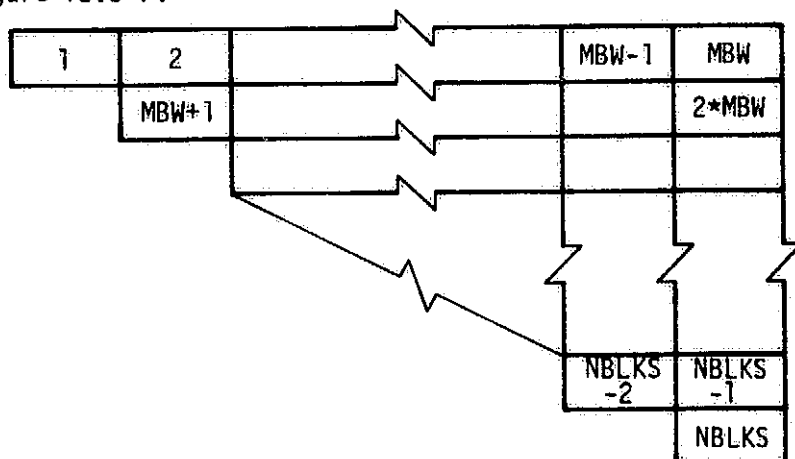
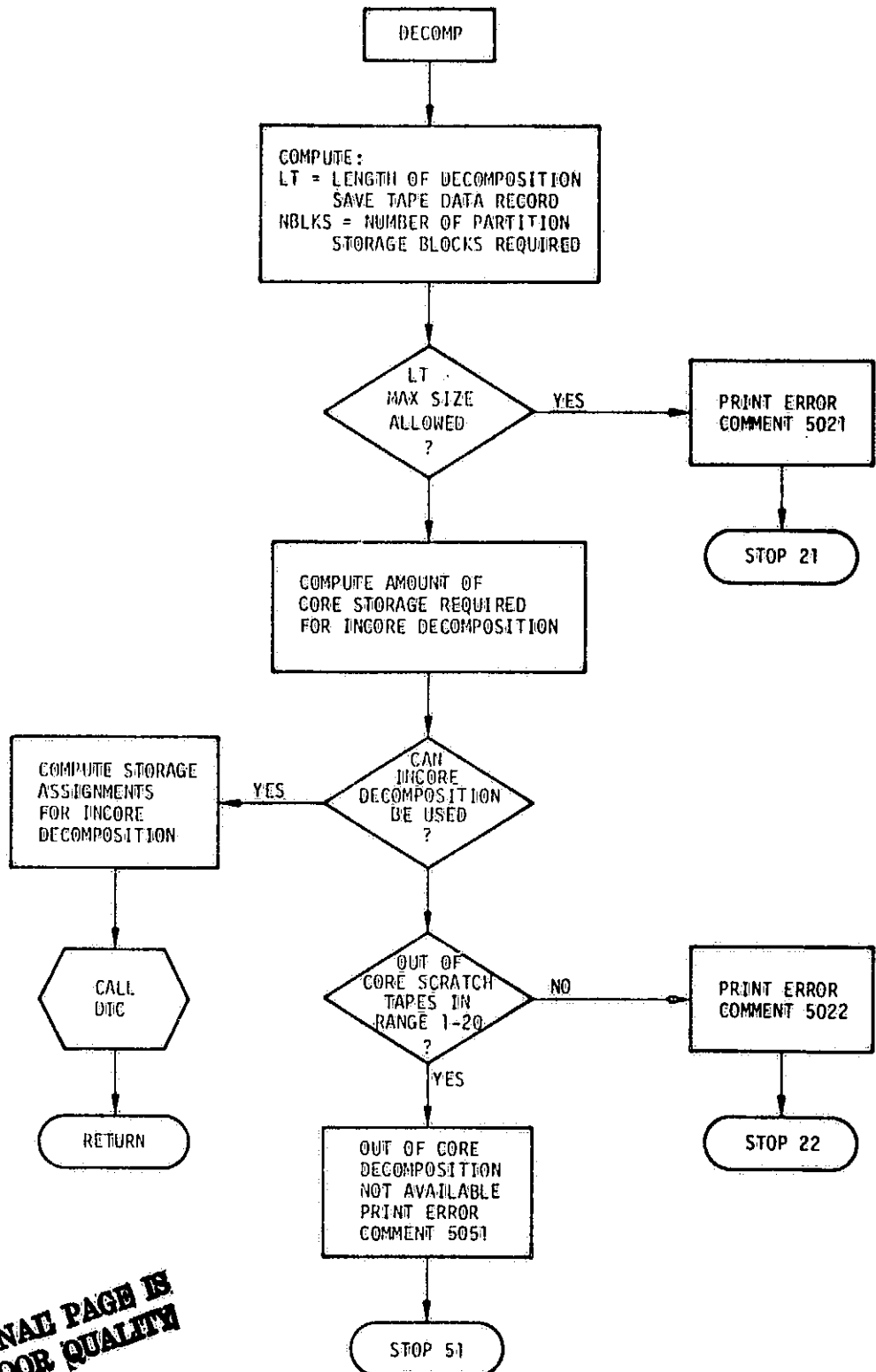


FIGURE 18.3-7: PARTITION ROW/COLUMN ID BOOKKEEPING ARRAY

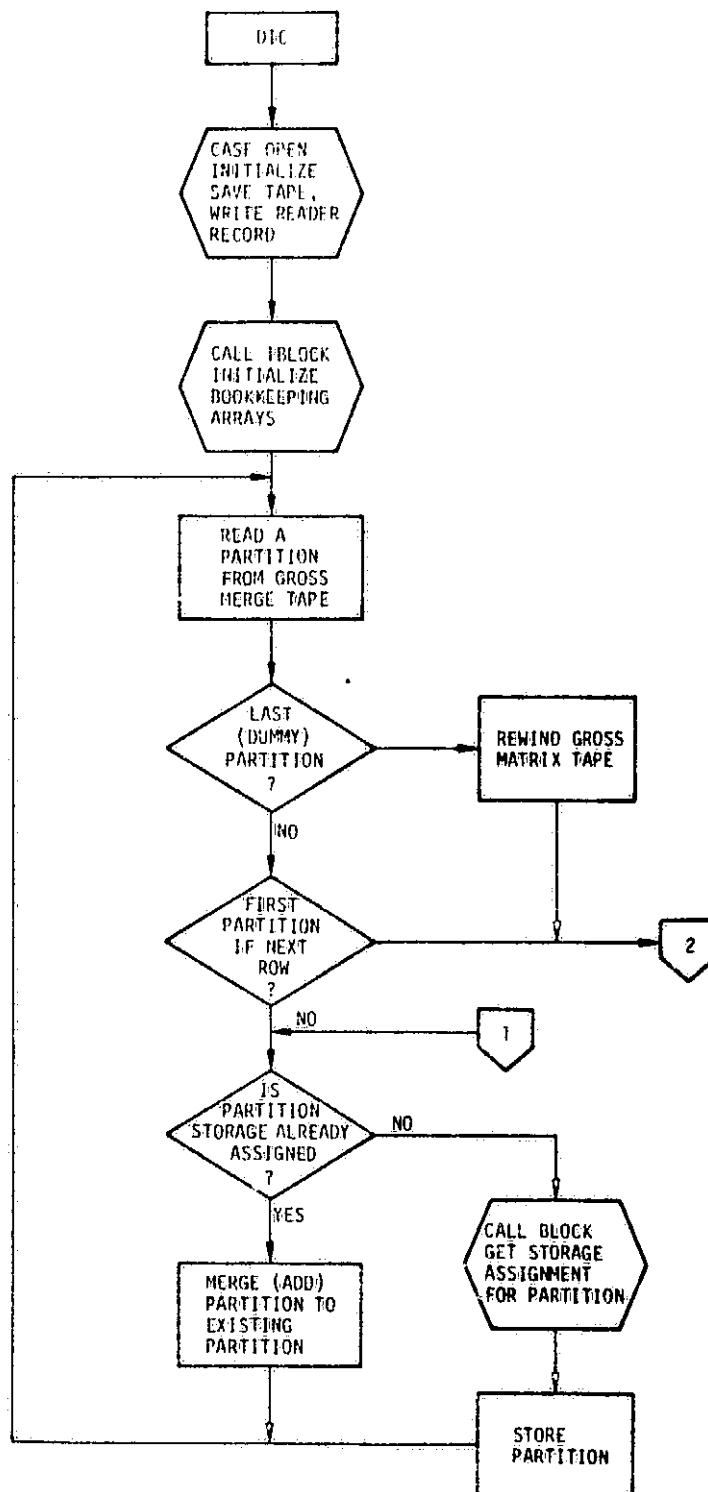
There is a one to one correspondence between the active nodes bookkeeping array and the rows and columns in this upper triangular matrix. To locate a specific partition, say partition (I,J) (row node I, column node J). First, locate the nodes I and J in the active node bookkeeping array (the node numbers are stored in the array), let i and j be their location (subscript) respectively. Second, if j is less than i ( $j < i$ ), interchange j and i. Finally, the location (second index of the partition storage blocks) of the partition is stored in the partition row/column ID bookkeeping array at location k of that bookkeeping array,

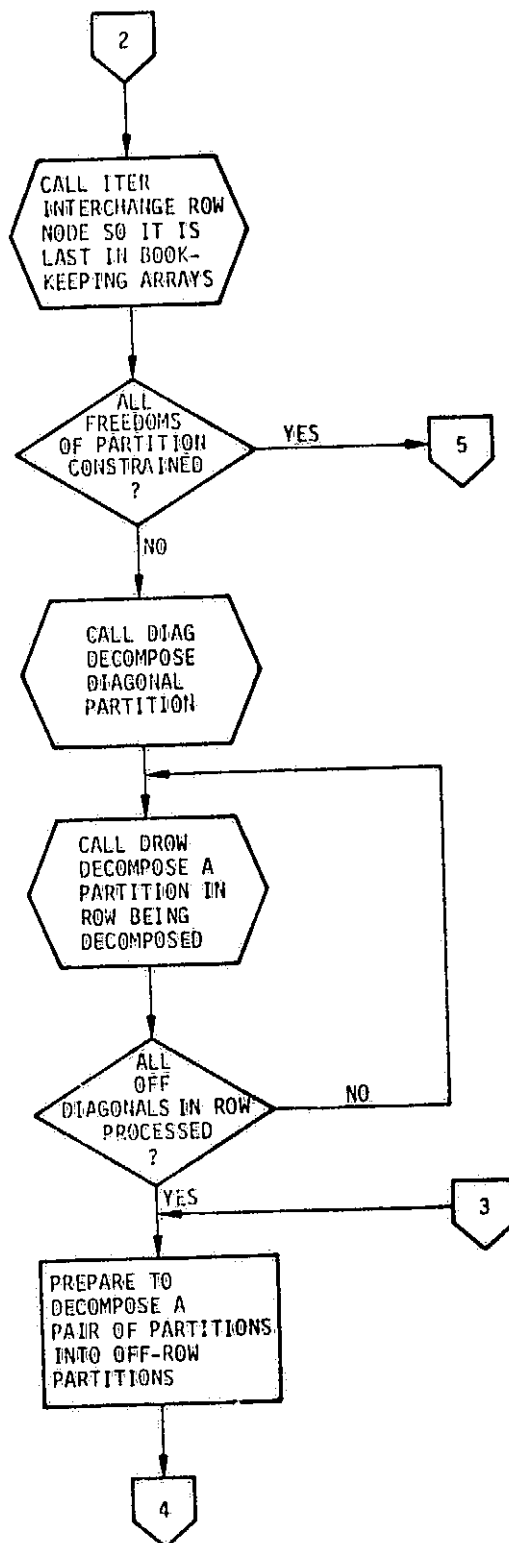
where

$$k = \left(\frac{1}{2}\right) * (i) * (2*MBW - i + 1) - MBW + j \quad (18.3-2)$$

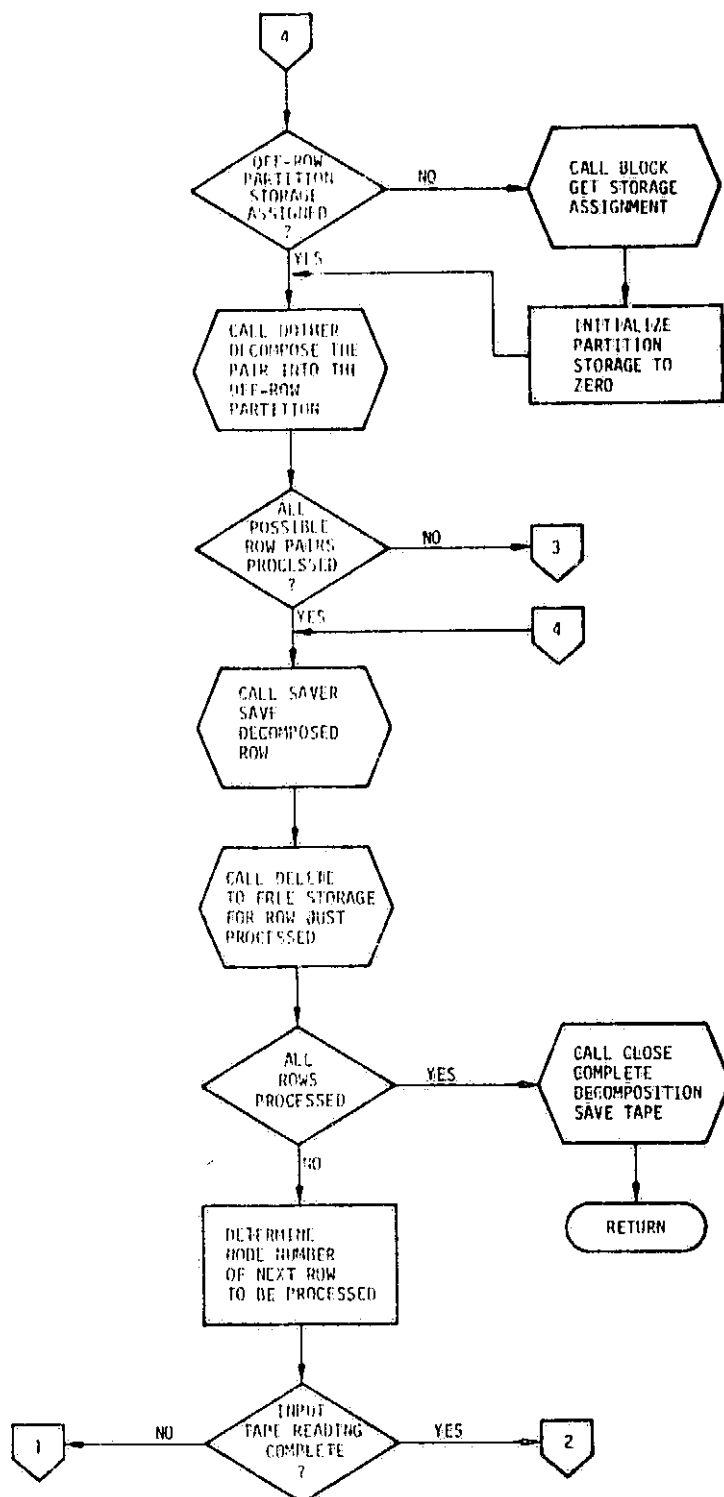


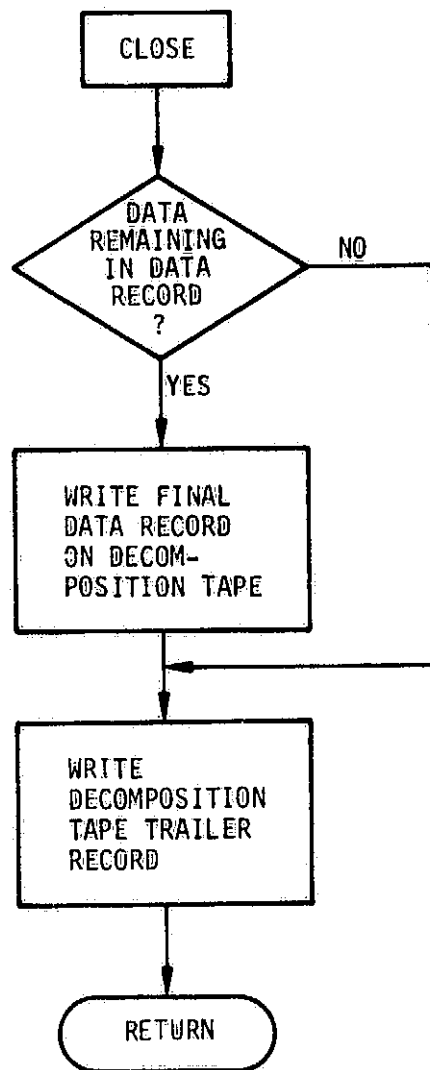
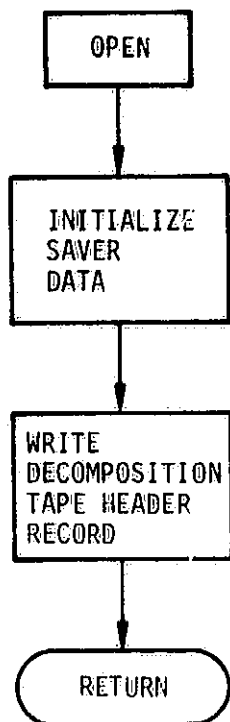
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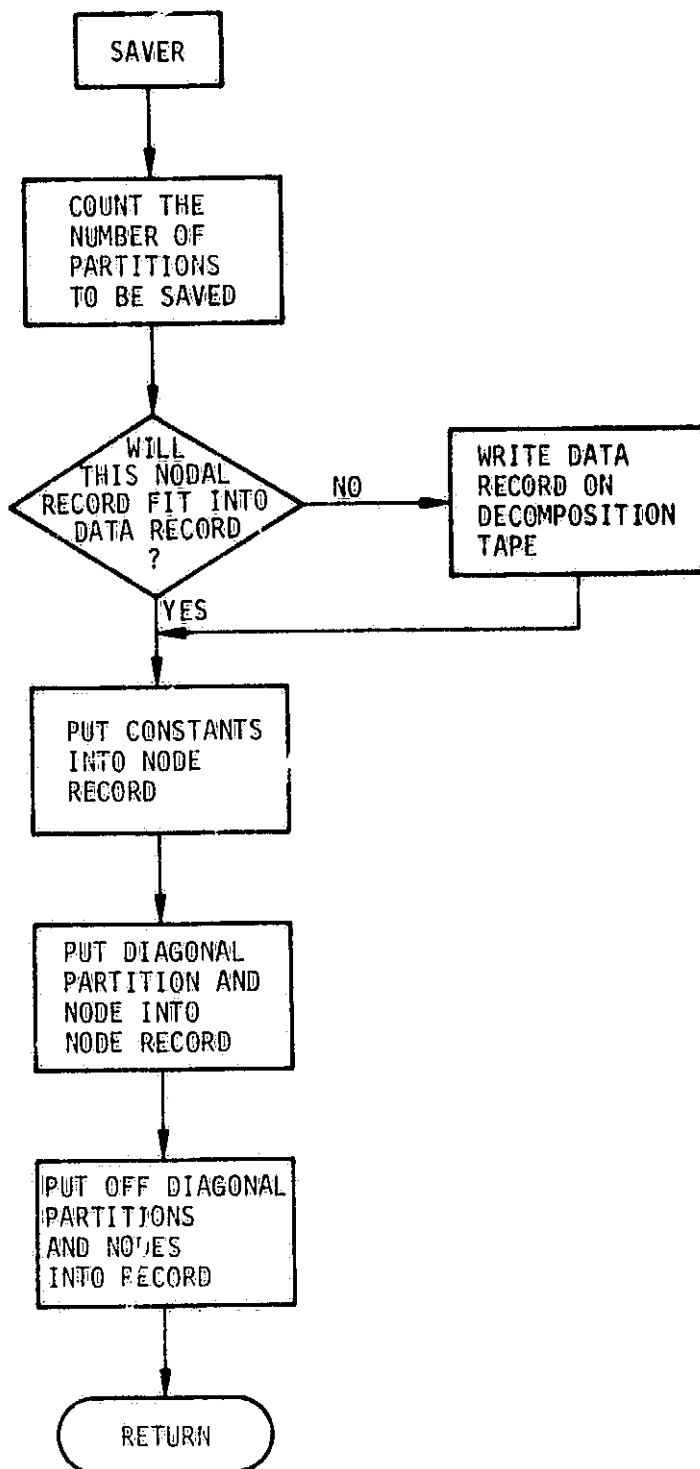
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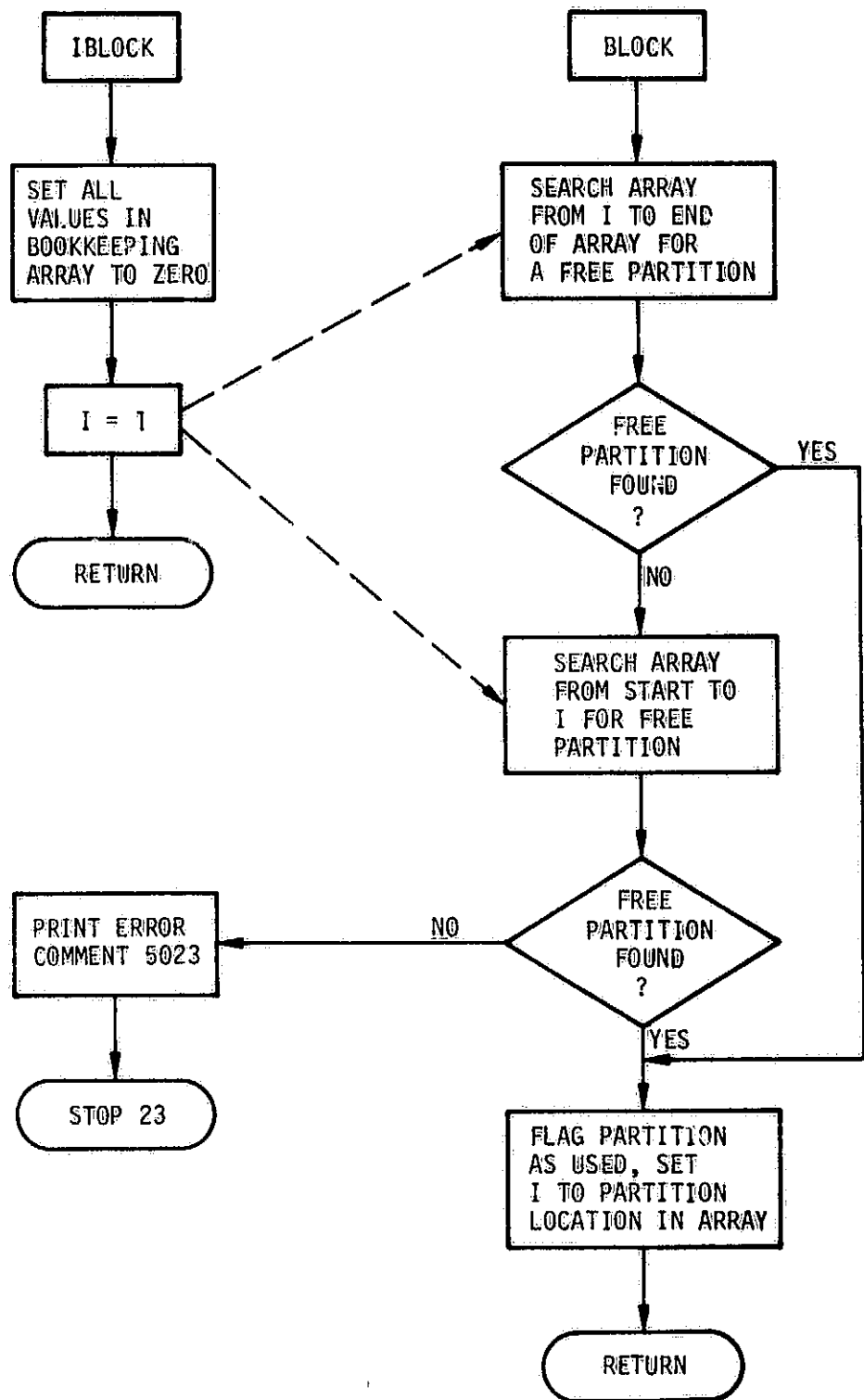


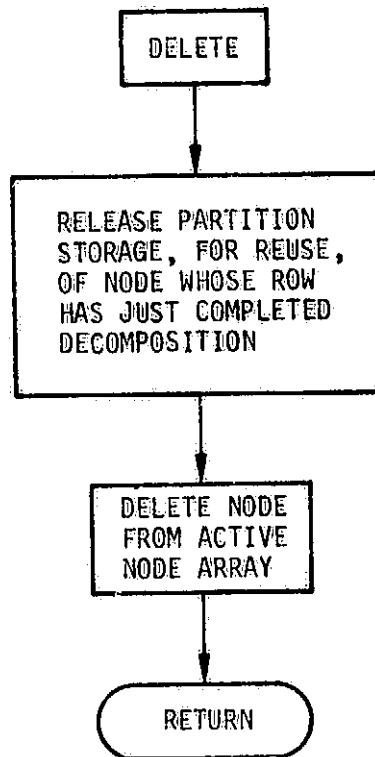


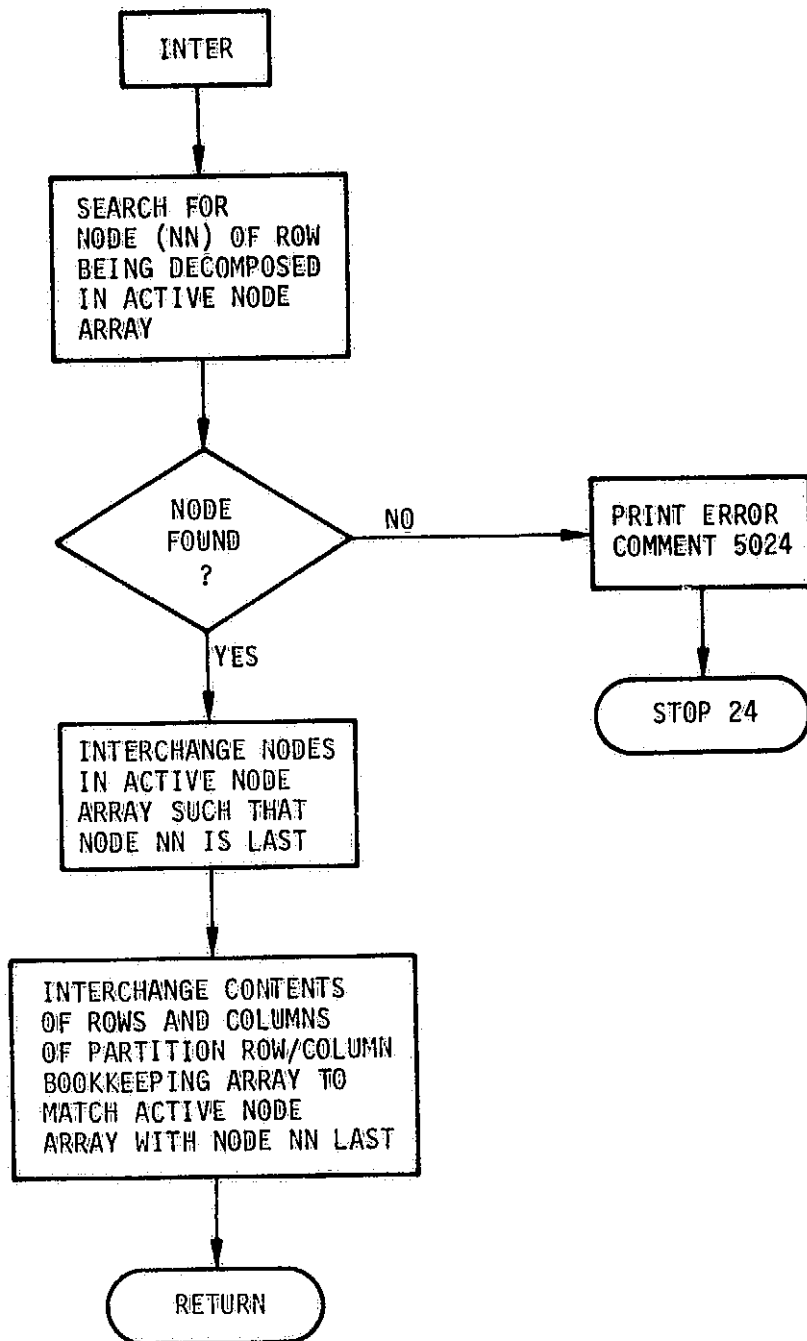
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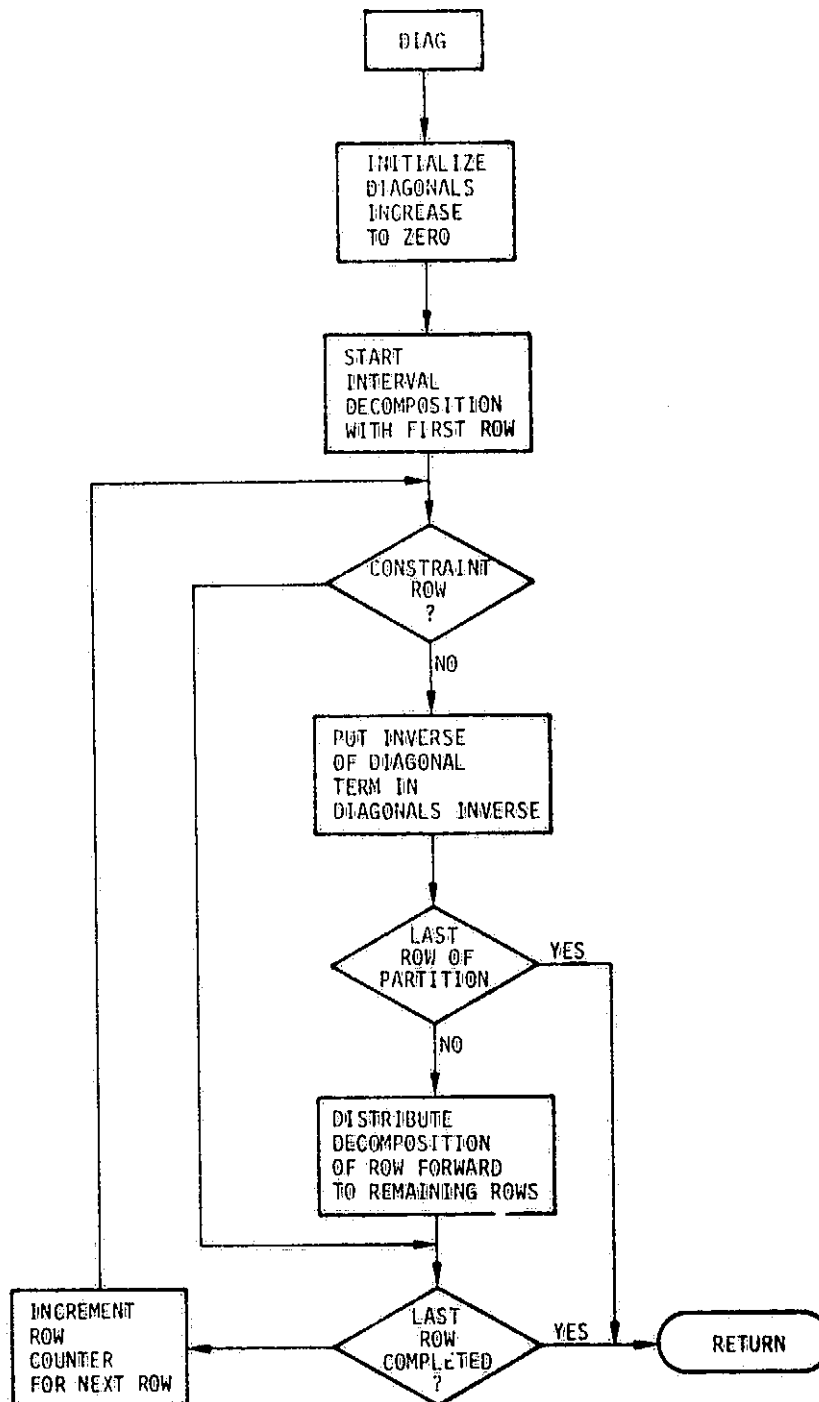




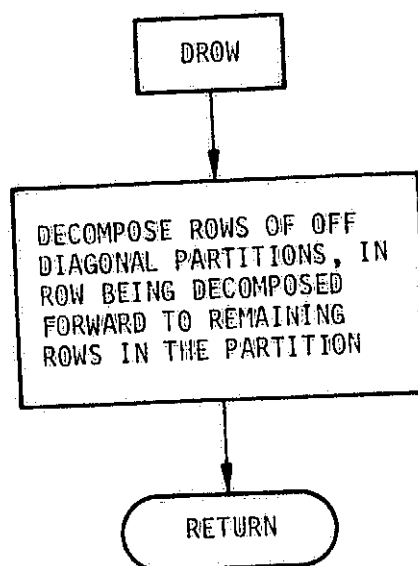
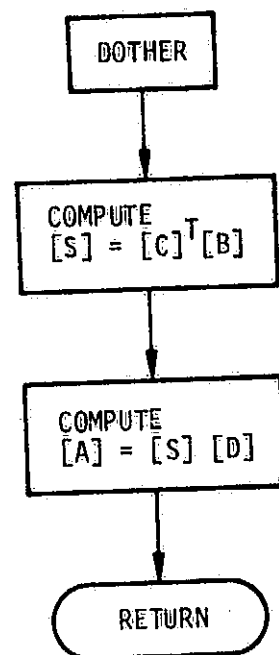








$[A] = [C]^T [B] [D]$   
WHERE  
[C] AND [D] ARE OFF DIAGONAL  
PARTITIONS IN THE ROW BEING  
DECOMPOSED  
[B] IS THE INVERSE OF THE  
DIAGONAL OF THE DIAGONAL  
PARTITION IN THE ROW BEING  
DECOMPOSED  
[A] IS THE RECEIVING PARTITION  
[S] IS A SCRATCH PARTITION



#### 18.3.4 FORWARD/BACKWARD SUBSTITUTION ROUTINES

The general flow of the Forward/Backward Substitution routines is shown in Figure 18.3-8

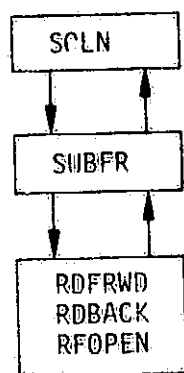
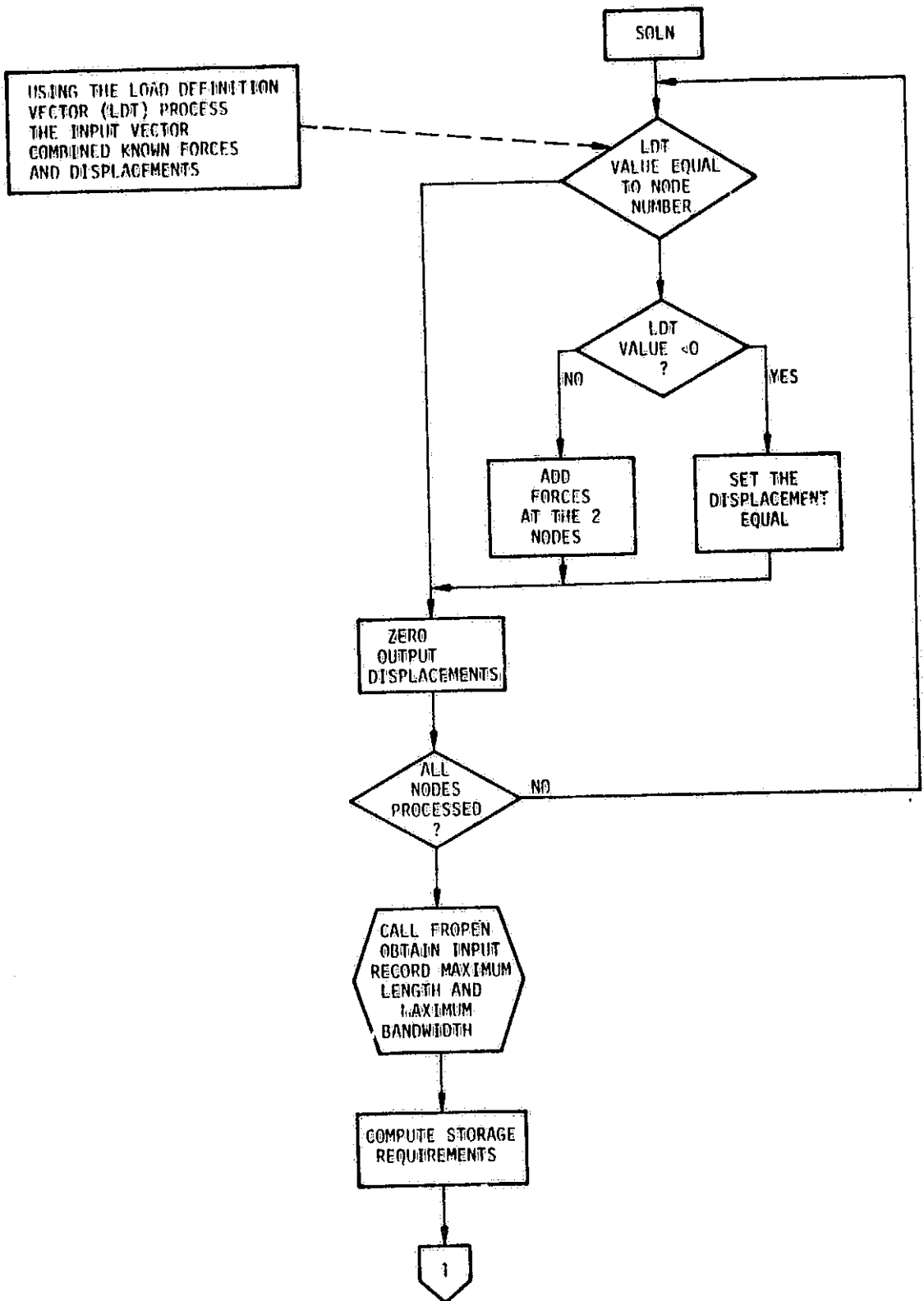


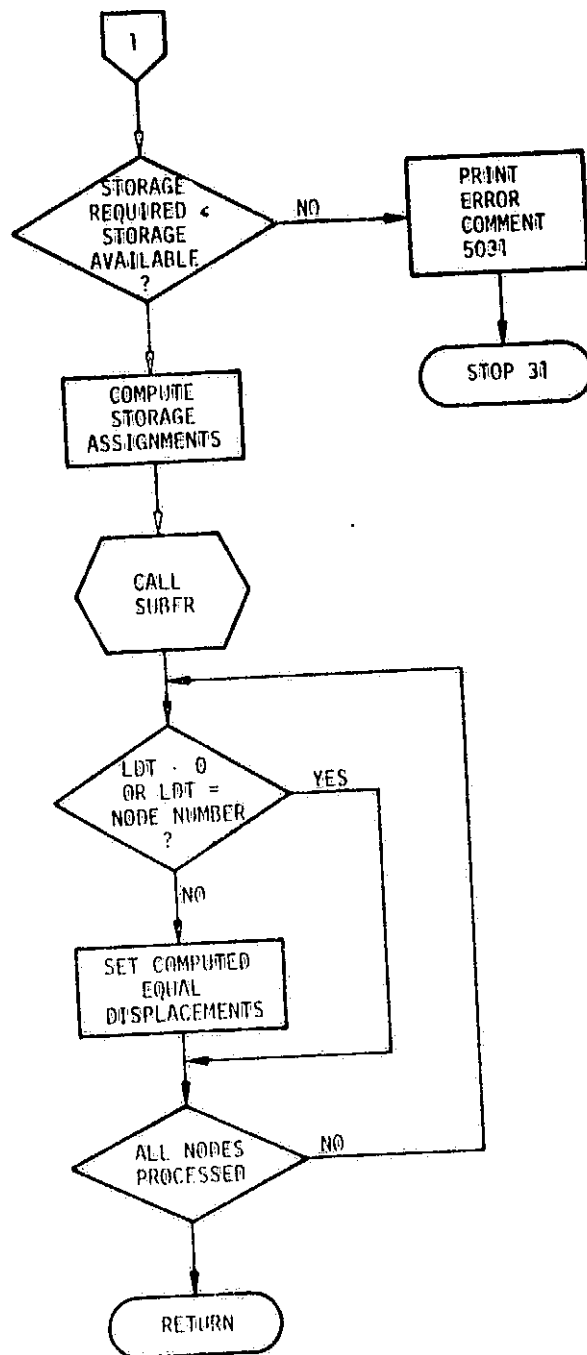
Figure 18.3-8: Substitution Flow

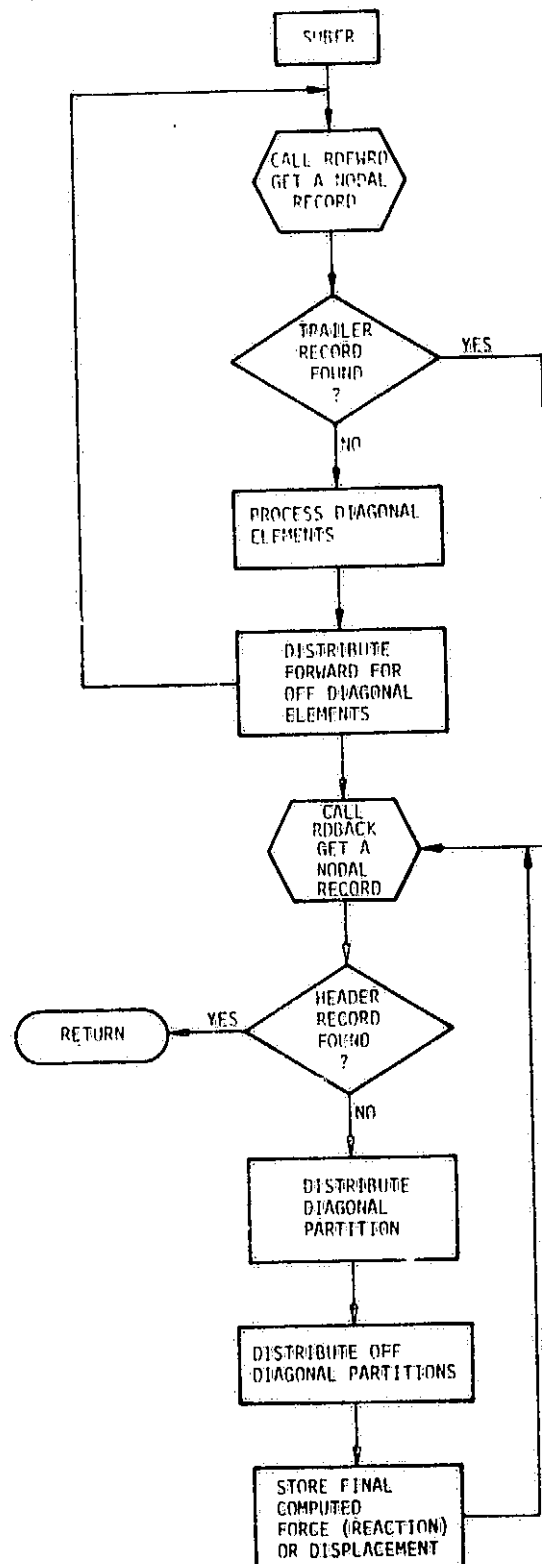
SOLN is the "MAIN" entry subroutine called by the user. Its main functions are to 1) set prescribed equal known (input) displacements, and add known (input) forces on computed equal displacement nodes, 2) to compute storage assignments for the arrays needed in the process, and 3) after substitution is completed, set computed equal displacements. Storage assignments are in common block/JLB/' and are passed via the calling sequence arguments.

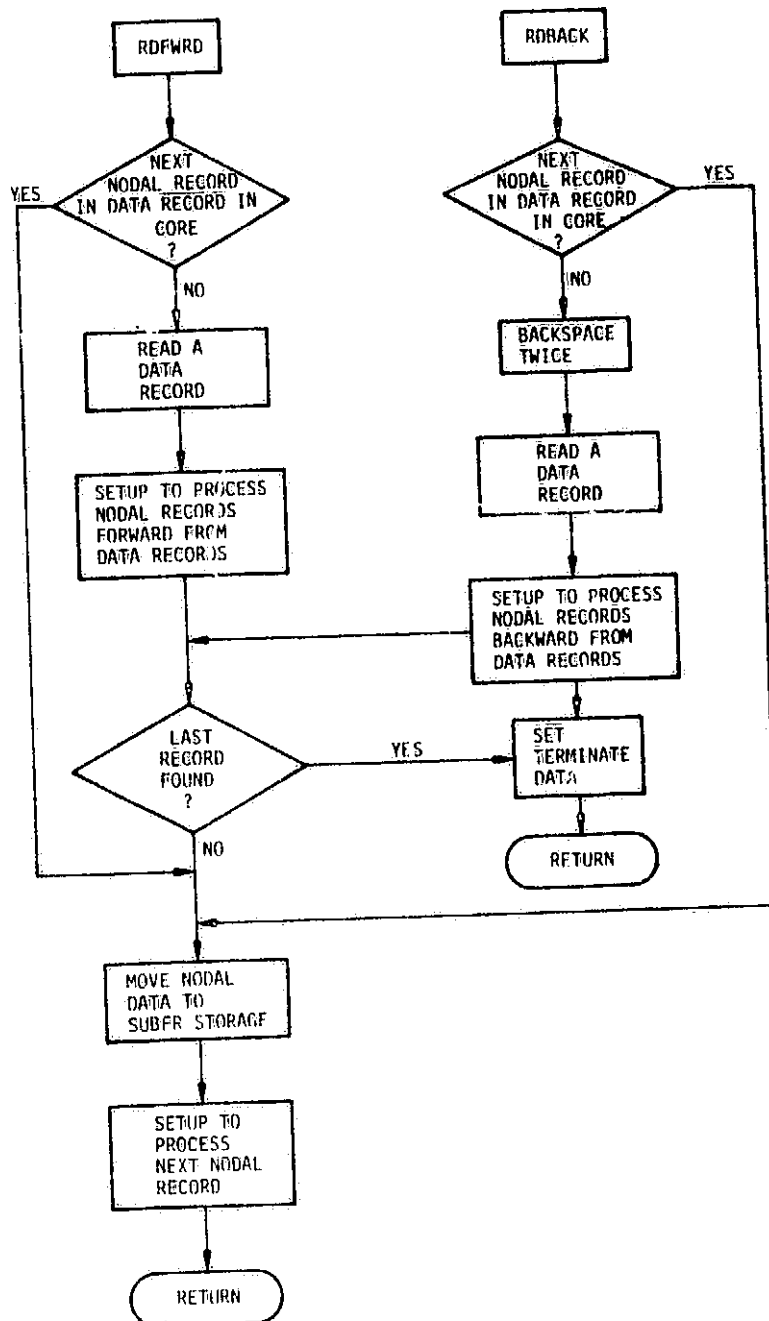
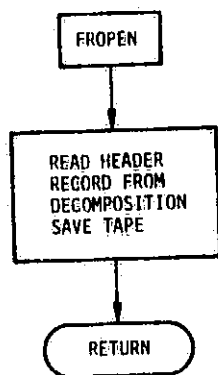
SUBFR is the forward/backward substitution routine performing the actual substitution. RDFWRD/RDBACK/RFOPEN is one routine with three entry points. All three read the decomposition save tape. RFOPEN reads the header record, returning the maximum bandwidth and length of a data record to SOLN for use in computing storage assignments. RDFWRD and RDBACK each read a node record. RDFWRD reads the tape for forward substitution and RDBACK reads the tape backwards for backsubstitution.



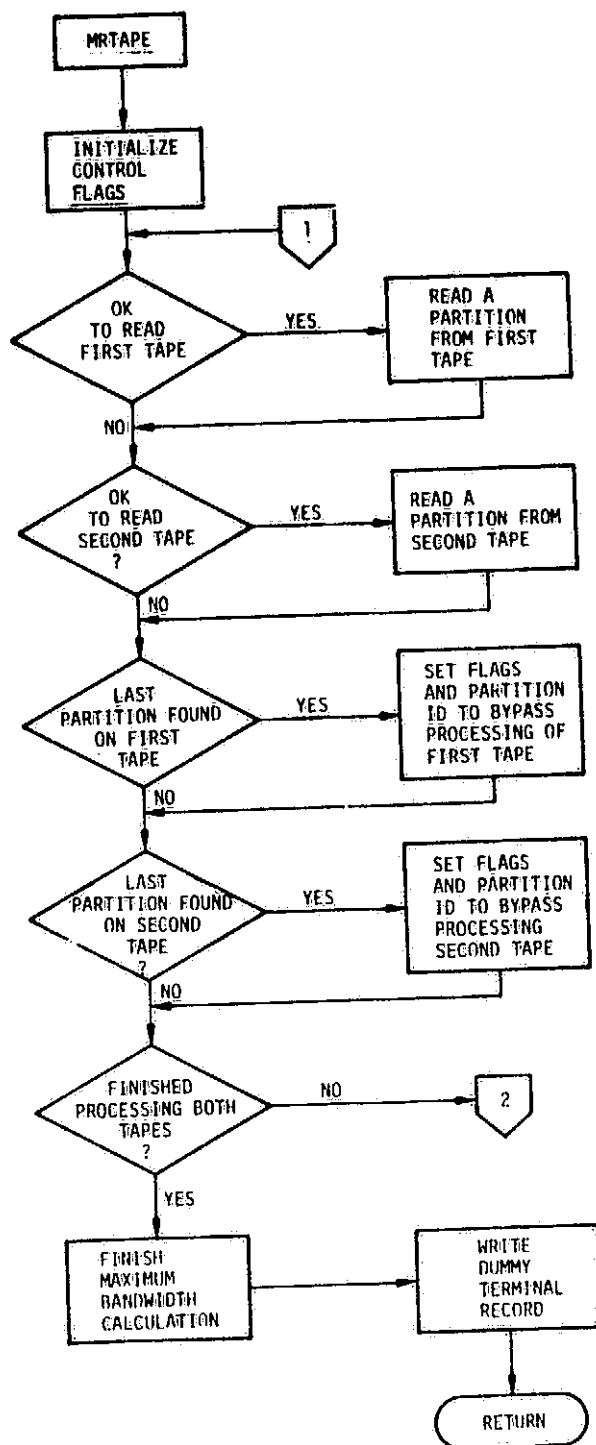


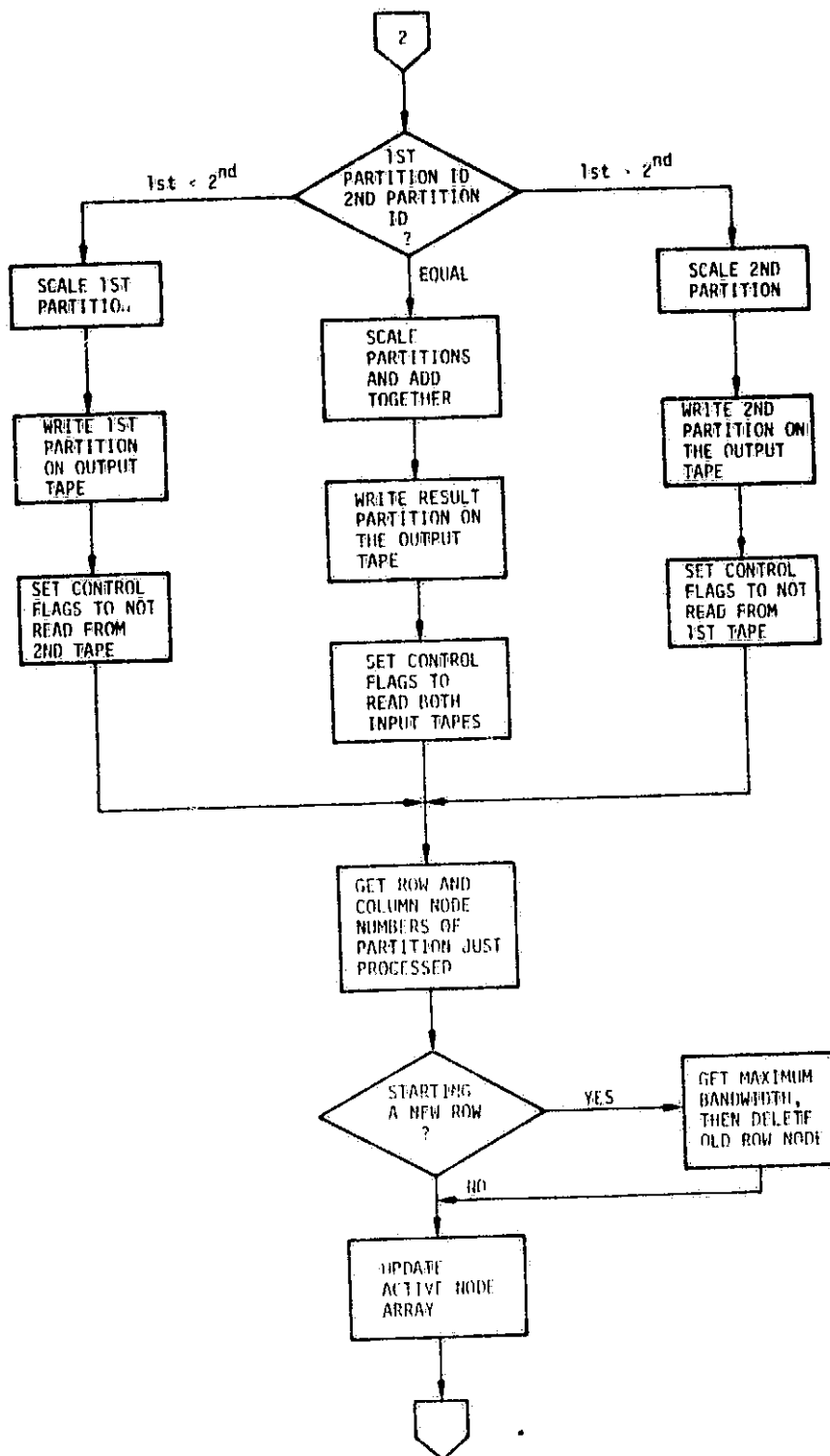


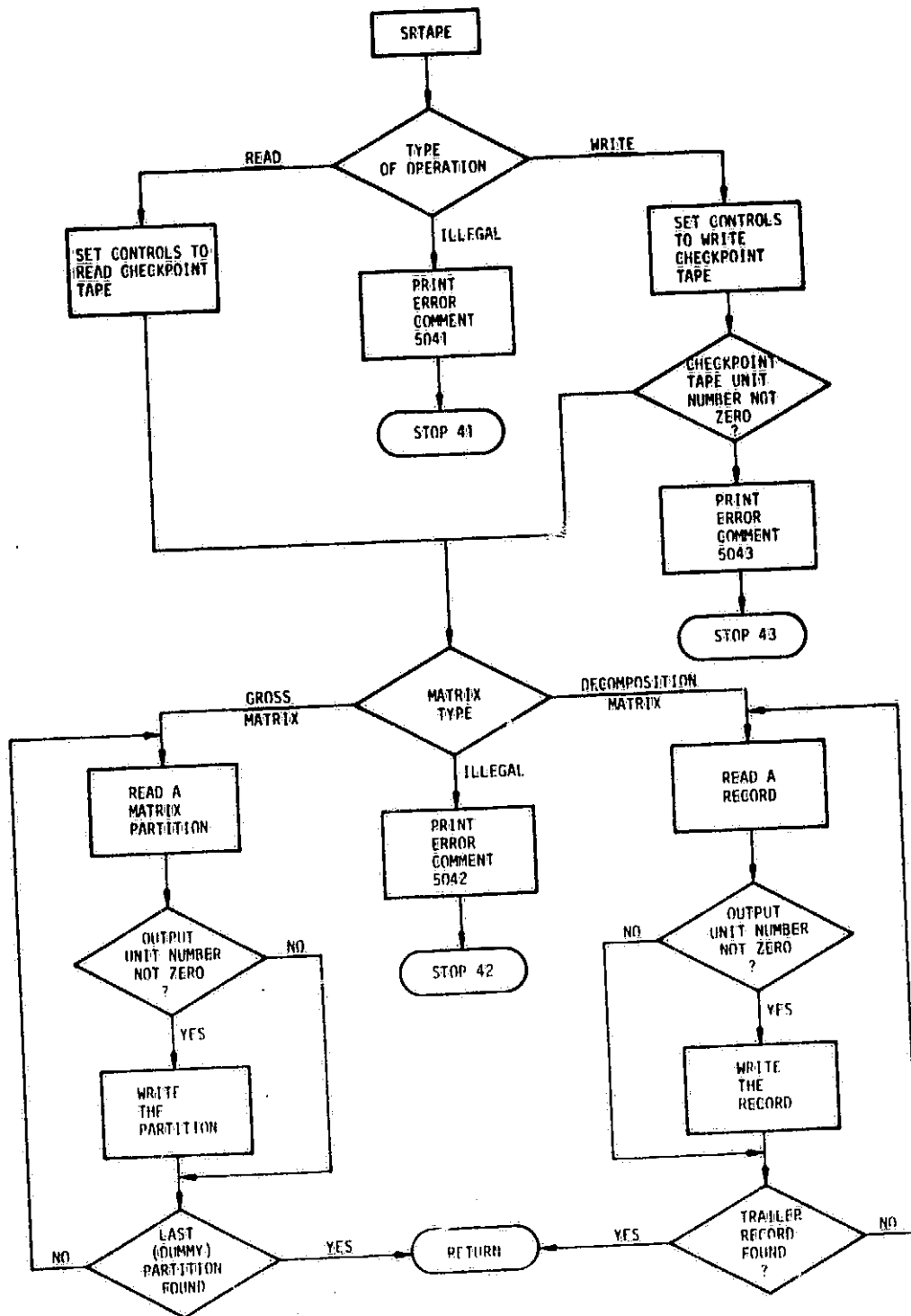




### 18.3.5 SPECIAL ROUTINES







## 19.0 SUBROUTINE AND COMMON TABLES

This section contains two tables. One lists all the subroutines called by the BOPACE subroutines and the other table lists all the common referenced by the BOPACE subroutines.

### 19.1 SUBROUTINE - SUBROUTINE TABLE

| <u>Subroutine</u> | <u>Subroutines called by subroutine in left column</u>       |
|-------------------|--|
| BLKDTA            |  |
| BIGS              | READ1, READTM, READTP, READTC, READC, READM, READ2,<br>READ3 |
| BLOCK             |  |
| COSHAP            |  |
| CSYS              | CSYS1, CSYS2   |
| CSYS1             |  |
| CSYS2             |  |
| DECOMP            | DIC  |
| DEFORM            |  |
| DELETE            |  |
| DIAG              |  |
| DIC               | SAVER, BLOCK, SEARCH, INTER, DIAG, DROW, DOTHER, DELETE      |
| DOTHER            |  |
| DPFORM            | DEFORM   |
| DROW              |  |

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|        |  |
|--------|--|
| DUMMY  | SEARCH   |
| DYVAL  |  |
| EDSHAP |  |
| ERCOMP |  |
| FORCE  | ROTK   |
| GENR8  | KFORM, ROTK  |
| GENR8  | GENR8  |
| HEAD   |  |
| INTER  |  |
| ITER   | YVAL, DYVAL, ZVAL, ITER1   |
| ITER1  |  |
| KFORM  | EDSHAP, COSHAP, YVAL, DEFORM, DPFORM   |
| MAIN   | READRS, READO, MERGE, DECOMP, SRTAPE, BIGS, READ4,<br>READ5, YVAL, SOLN, STRAIN, ITER, ERCOMP, MRTAPE, HEAD,<br>OUTPQ, OUTEL |
| MERGE  | GENR8, MERSOR, DUMMY   |
| MERSOR |  |
| MRTAPE | SEARCH   |
| OUTC   |  |
| OUTE   |  |
| OUTEL  | OUTE, OUTP, OUTS, OUTC, OUTG   |
| OUTG   |  |
| OUTP   |  |
| OUTPQ  |  |
| OUTS   |  |
| RDFRWD |  |



|        |              |
|--------|--------------|
| READC  |              |
| READM  | CSYS, VTET   |
| READRS |              |
| READTC | YVAL         |
| READTM |              |
| READTP |              |
| READO  |              |
| READ1  |              |
| READ2  |              |
| READ3  |              |
| READ4  |              |
| READ5  |              |
| ROTK   |              |
| ROTQ   |              |
| SAVER  |              |
| SEARCH |              |
| SOLN   | SAVER, SUBFR |
| SRTAPE |              |
| STRAIN | ROTQ         |
| SUBFR  | RDFRWD       |
| VTET   |              |
| YVAL   |              |
| ZVAL   |              |

19.2 SUBROUTINE-COMMON TABLE

| <u>Subroutine</u> | <u>Common Referenced by Subroutine in Column One</u>  |
|-------------------|---|
| BLKDTA            | BRICKC, SIZES, ELDATO   |
| BIGS              | ELDATO, ELDAT1, ELDAT2, GEN7, GENP0, GENP7, GENP8,<br>GENP9, GENP10, GENP11, GENP12, GENP13, GENP14, GENP15,<br>GENP16, GENP17, GENC1, GENC2, JLB |
| BLOCK             |   |
| COSHAP            |   |
| CSYS              |   |
| CSYS1             |   |
| CSYS2             |   |
| DECOMP            | JLB   |
| DEFORM            |   |
| DELETE            |   |
| DIAG              |   |
| DIC               |   |
| DOTHER            |   |
| DPFORM            | GENP0, GENP7, GENP8, GENP9, GENP11, GENP12, GENP13,<br>GENP14, GENP15, GENP16, GENP17   |
| DROW              |   |
| DUMMY             |   |
| DYVAL             |   |
| EDSHAP            |   |
| ERCOMP            |   |

|        |   |
|--------|---|
| FORCE  | ELDAT0, ELDAT1, ELDAT2  |
| GENER8 | ELDAT0, ELDAT1, ELDAT2  |
| GENR8  |   |
| HEAD   |   |
| INTER  |   |
| ITER   | GEN7, GENP0, GENP7, GENP8, GENP9, GENP10, GENP11,<br>GENP12, GENP13, GENP14, GENP15, GENP16, GENP17, GENC1,<br>GENC2, ELDAT0, ELDAT1, ELDAT2  |
| ITER1  |   |
| KFORM  | IOUNIT, BRICKC, GEN1, GEN7, ELDAT0, ELDAT1, ELDAT2  |
| MAIN   | IOUNIT, JLB, ELDAT0, ELDAT1, ELDAT2, GEN1, GEN7, GEN8,<br>GENP0, GENP1, GENP2, GENP3, GENP4, GENP5, GENP6, GENP7,<br>GENP8, GENP9, GENP10, GENP11, GENP12, GENP13, GENP14,<br>GENP15, GENP16, GENP17, GENC1, GENC2, SIZES |
| MERGE  | JLB   |
| MERSOR |   |
| MRTAPE | JLB   |
| OUTC   |   |
| OUTE   |   |
| OUTEL  | JLB, ELDAT0, ELDAT1, ELDAT2   |
| OUTG   |   |
| OUTP   |   |
| OUTPQ  |   |
| OUTS   |   |
| RDFRWD |   |
| READC  |   |

|        |                        |
|--------|------------------------|
| READM  |                        |
| READRS |                        |
| READTC |                        |
| READTM |                        |
| READTP |                        |
| READO  |                        |
| READ1  |                        |
| READ2  |                        |
| READ3  |                        |
| READ4  |                        |
| READ5  | ELDAT0, ELDAT1, ELDAT2 |
| ROTK   |                        |
| RQTQ   |                        |
| SAVER  |                        |
| SEARCH |                        |
| SOLN   | JLB                    |
| SRTAPE | JLB                    |
| STRAIN | ELDAT0, ELDAT1, ELDAT2 |
| SUBFR  | JLB                    |
| VTET   |                        |
| YVAL   |                        |
| ZVAL   |                        |

## 20.0 FILE USAGE

BOPACE uses Fortran I/O to access a number of files. Some of the files are defined by the user and the others are defined in the program. A current list of files listed by logical unit number follows:

| <u>Unit Number</u> | <u>Description</u>   | <u>Defined by</u> |
|--------------------|--|-------------------|
| 5                  | Input card file  | READRS            |
| 6                  | Output printer file  | READRS            |
| UIN1               | Type I input data  | user              |
| UIN2               | Type II input data   | user              |
| UOUT               | Major output file  | user              |
| UINRS              | Restart tape   | user              |
| UOUTRS             | Checkpoint tape  | user              |
| UNITE1(=11)        | Merged elastic stiffness matrix  | MAIN              |
| UNITE2(=12)        | Decomposed elastic stiffness matrix  | MAIN              |
| UNITP1(=13)        | Merged total Jacobian matrix. Used only when the input variable SCODE is equal to 3, 4 or 5.     | MAIN              |
| UNITP2(=14)        | Decomposed total Jacobian matrix. Used only when the input variable SCODE is equal to 3, 4 or 5. | MAIN              |
| UNITS1(=15)        | Scratch file used by the Gauss wavefront merge and decomposition routines.                       | MAIN              |

|             |                                |        |
|-------------|--------------------------------|--------|
| UNITS2(=16) | Same as UNITS1                 | MAIN   |
| IEDAT(=1)   | Element data                   | BLKDTA |
| IEDIN(=2)   | Input integration point data   | BLKDTA |
| IEDOUT(=3)  | Updated integration point data | BLKDTA |

Logical units IEDIN and IEDOUT are initially defined by the block data program BLKDTA. However, these files are swapped in BOPACE whenever the integration point data as defined by common ELDAT2 is updated. These exchanges of unit numbers occur in MAIN, BIGS and OUTEL.

## 21.0      OVERLAY

The overlay of BOPACE was designed to minimize loading of segments and to maximize the size of common JLB for a given core size. A schematic of the overlay is shown in Figure 21.0-1.

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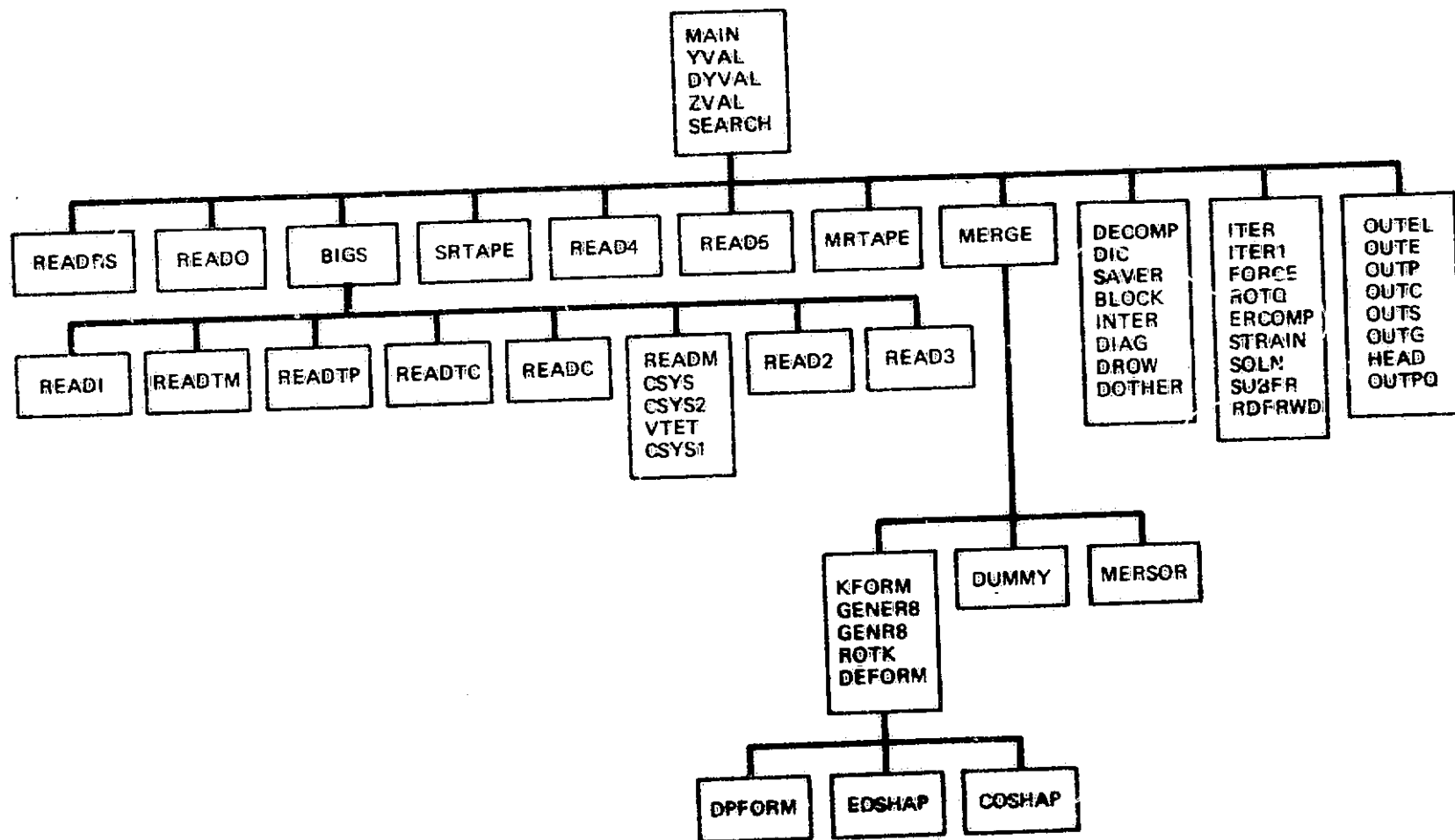


Figure 21.0-1: Overlay Schematic